S N Bose
The Man and His Work



Collected Scientific Papers

S N Bose : The Man and His Work

Part I : Collected Scientific Papers

CENTRE FOR BASIC SCIENCES 1986

S. N. BOSE NATIONAL

1894-1974

विज्ञानेन परिपश्यन्ति धीराः

S N Bose (1894-1974), best known for his seminal contribution to quantum statistics, left behind a substantial body of original work in the form of papers in a wide range of fields including chemistry, spectroscopy, thermoluminescence, statistics, group theory, mathematical physics and Einstein's unified theories, collected in the first part of this volume, with annotative introductions on the papers by distinguished scientists from a later generation.

Bose played a significant role in laying the foundations of science teaching and research at the two major universities of eastern India in the first half of the century, Calcutta and Dhaka (now in Bangladesh). It was in Dhaka in 1924 that he devised the new counting method for light-quanta which reconciled Einstein's concept of light-quanta with Planck's law. Within the next six months Einstein wrote three papers showing that Bose's method, when slightly extended to material atoms, yielded the correct quantum theory of ideal gases. The new counting method has come to be known as the Bose-Einstein statistics. Particles whose behaviour is described by the Bose-Einstein statistics have been named bosons after Bose.

As a teacher, Bose developed a fairly comprehensive model for education in India that focussed on the dissemination of a scientific culture in the country through the regional vernaculars; and was equipped to negotiate with the developmental needs of the nation. His lectures and addresses and miscellaneous pieces, collected in the second part of this volume (several of them translated from Bengali for the first time) provide glimpses into his various emotional and intellectual concerns, passions and obsessions, and friendships, documented in the long biography that is the most exhaustive to date, and brings together a lot of material gathered from various sources including the Einstein papers stored at Jerusalem and the archives of Dhaka University, and rarely seen pieces of correspondence.

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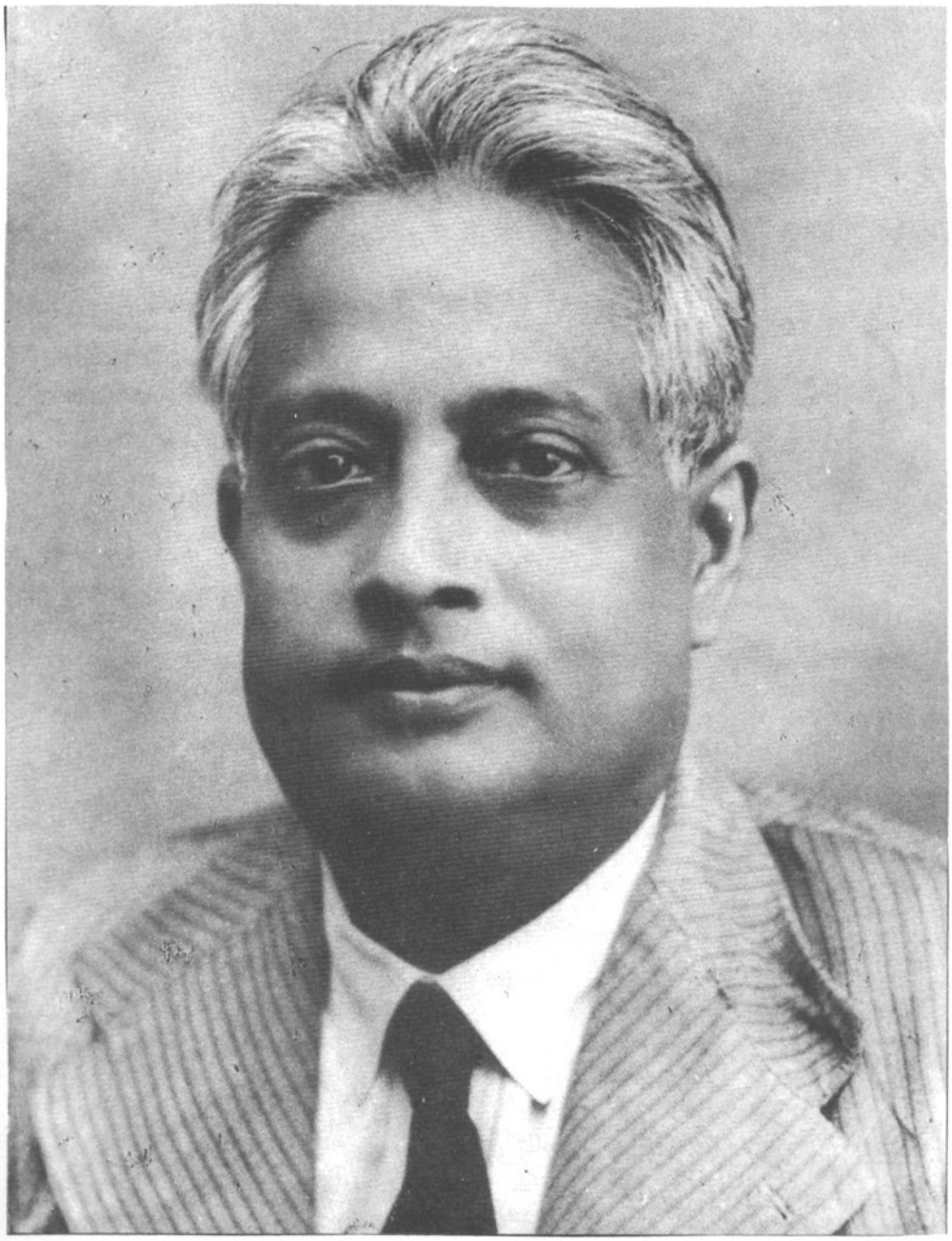
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— for giving us access to information, material and documents in their holdings

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and in particular Rathindranath and Benu Bose, Nilima Mitra, Ruchira Mitra, Ramendranath Bose, and Jayanti Chattopadhyay

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S N Bose : Introductions to scientific papers

Foreword

The Satyendra Nath Bose National Centre for Basic Sciences was established by the Department of Science and Technology, Government of India, in 1986 as a memorial to the Indian scientist Satyendranath Bose. On the occasion of his birth centenary in 1994, the Centre has prepared the present volume incorporating his scientific papers, some selected miscellaneous pieces and addresses, and a biography. Bose wrote and published in four languages : Bengali, English, German and French. Though we have included the original German and French articles, we have provided their English translations also. His writings in Bengali have been collected and brought out by Bangiya Vijnan Parishad, an institution he helped to establish. We have included translations of some of these articles which may be of interest to a general reader.

Several biographical accounts of S N Bose were available. We found that the list of scientific papers was incomplete in all of them, and that not enough attention had been given to his work in Dhaka after his return from Europe. Two published papers — one in German (1927) and another in English (1929) from this period — are included in our collection. Enakshi Chatterjee and Santimay Chatterjee have given a fuller account of his activities in Dhaka — establishing physics laboratories, and helping develop modern research in physics and chemistry.

In some respects S N Bose's career is the story of the triumph and tragedy of Indian science in the first half of the twentieth century. His brief encounter with Europe in the twenties assured his place among the great scientists. He tried to strengthen the scientific base in India but he found little financial and spiritual sustenance. He himself described these two aspects very well, when he said that he was like a comet which came once but never returned, and he seemed to be living on the Moon.

Some comments by later Indian workers about the scientific papers are also included; these hopefully will help an inquisitive young research worker in placing the

scientific papers in the right perspective.

A long list of acknowledgements preceding this foreword covers most of the individuals and organizations who have helped us in several ways. Special thanks are due to the Department of Science and Technology, Government of India, for financial support to the S N Bose National Centre for Basic Sciences for activities in the Centenary year, of which the present publication is one.

The volume is divided into two parts — the first part aimed at a more specialist readership, containing his scientific papers, with introductory commentaries by later workers in the specific fields; and the second part designed for a more general readership, with the exhaustive biography, his public lectures and addresses, and miscellaneous pieces (several of them translated from Bengali for the first time).

C K MAJUMDAR
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1

The Equation of State of a Real Gas

1 *On the Influence of the Finite Volume of Molecules on the Equation of State* (1918)
2 *On the Equation of State* (1920)

In a doctoral dissertation to the University of Leiden in 1873, J D van der Waals proposed a modified equation of state of a real gas. He included two effects : the excluded volume effect due to strong range repulsion and an intrinsic pressure due to weak long range attraction. The work was praised by J C Maxwell who proposed the well-known Maxwell construction to remove the unstable part of the isotherms. The van der Waals equation was enormously successful in explaining critical phenomena, critical indices and the liquefaction of gases. By 1910 experimentalists, however, began to find deviations from the predictions; two constants were not enough, and people started looking for improvements. In 1901 K Onnes introduced the virial coefficients to represent the isotherms. But the simplicity of the approach of an equation of state with a few parameters was too attractive : more than fifty equations of state have been proposed. M N Saha and S N Bose used thermodynamics, especially the Boltzmann formula for entropy, in their characteristic way to arrive at their equation of state. The Boltzmann formula would appear again in Bose's other works. The modern theory of a gas at high density starts from the cluster development of H D Ursell and J E Mayer (J E Mayer and M G Mayer, *Statistical Mechanics*, J Wiley and Sons, New York 1977, Second edition, pp.229 ff.).

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2

Classical Mechanics

1 ***The Stress-Equations of Equilibrium*** (1919)
2 ***On the Herpolhode*** (1919)

The equilibrium of isotropic elastic solid bodies is discussed in the well-known book by A E H Love, *A Treatise On the Mathematical Theory of Elasticity,* Fourth Edition, Dover, New York 1927, ch V. Here in pp. 134-5, J H Michell's equations are given. V Cerutti's method is described in ch X, pp. 237-40. The problem of the sphere is discussed in ch XI. After the publication of the paper in 1919 Bose lost interest in the problems of elasticity and never returned to them in published work.

The force-free motion of a body about a fixed point is discussed in books of classical dynamics (eg E T Whittaker, *A Treatise on the Dynamics of Particles and Rigid Bodies,* Fourth edition, Cambridge University Press, Cambridge 1961) in terms of elliptic functions. Poinsot's construction gives a simple geometrical description of the motion. Bose proved a geometrical property without using elliptic functions. The same method was also found by W van der Woude.

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3

Quantum Theory

1 *On the Deduction of Rydberg's Law from the Quantum Theory of Spectral Emission* (1920)
2 *Plancks Gesetz und Lichtquantenhypothese* (1924)
3 *Wärmegleichgewicht im Strahlungsfeld bei Anwesenheit von Materie* (1924)
4 *A Note on Dirac Equations and the Zeeman Effect* (1943)

Bose's contribution to Quantum Theory consists of four papers, two of which are on atomic physics and two are about quantum statistical mechanics. This covers the period 1920-1943. Of these, the best known is *the* paper on Bose statistics for the photon gas (*Z f Physik* **26**, 178 (1924)). A brief critical appreciation of these papers follows:

In the first paper *On the Deduction of Rydberg's Law from the Quantum Theory of Spectral Emission*, the Bohr-Sommerfeld method of quantization of action

$$\oint p_j \, dq_j = n_j \, h$$

is used to generate Balmer terms

$$\frac{N}{m + \alpha + (\beta / m^2)}$$

for any series for any atom. Bose starts with a monopole plus a dipole as the best approximation of the potential seen by a valence electron :

$$V(r) = -\frac{e^2}{r} + \frac{e\ L \cos\ \theta}{r^2}\ .$$

The calculations involve abelian integrals which have to be suitably approximated. Bose also has an effective expression for the Rydberg constant in terms of the fundamental constants, but is either unaware of (or disinclined to cite) A Haas who first obtained the relation (A Haas, quoted in *Introduction to Theoretical Physics*, vol II, Constable, London 1948).

In the paper *A Note on Dirac Equations and the Zeeman Effect* (*Indian J Phys.* **17**, 301 (1943)), written together with K Basu, the problem of the energy levels of a hydrogenic atom in an inhomogeneous magnetic field is solved using Sonine polynomials. The calculations are elegant and straightforward and lead to the quadratic dependence of the perturbed energy levels on the magnetic field.

The celebrated paper *Plancks Gesetz und Lichtquantenhypothese* (*Z f Physik* **26**,168 (1924)) of S N Bose introduces the new photon statistics and deduces the Planck distribution as the equilibrium distribution for photons. Bose considered complexions of photons as the primary characterization of the state rather than which photon had which energy. Photons were treated as strictly indistinguishable.

With the modification that Bose introduced into the calculation we obtain the correct thermodynamics of the photon gas. That one step was the basis of the new synthesis between the wave and the particle properties of photons, and with it, the foundations of quantum field theory. After Bose's paper came an avalanche of developments: the extension of Bose's theory to particles of arbitrary mass and non-zero chemical potential by Einstein, the Fermi-Dirac statistics for electrons, the quantization of the electromagnetic field by Heisenberg and Pauli, and quantum electrodynamics by Dirac.

To put Bose's synthesis in its proper setting it is good to recall that in the nineteen-twenties there were two items of unfinished business, one regarding photons as particles and the other concerning statistical mechanics of identical particles. In both cases uneasy makeshift solutions were generally accepted instead of definitive solutions. In the old dichotomy between particles and waves for describing light there were highly persuasive arguments on both sides, but it was thought that a crucial experiment was the determination of the relative speed of light in two media with different refractive indices. It appeared that if light consisted of particles, the speed in the optically denser medium should be greater; if it consisted of waves,this speed should be lesser. Fizeau's experiment on the speed of light in water thus seemed to find evidence definitively for the wave theory! Yet, what of the postulated photons? How do we reconcile the notion of a photon with its discrete momentum and energy to its lesser speed in water than in air? We must conclude that photons may be particles, but they do not behave as particles are naïvely expected to behave. A revision of the concept of a particle ought to be made. We now know that extended particles also bend towards the normal in a medium in which it moves slower; so Fizeau's experiment could not really distinguish between extended particles and waves.

The other item of unfinished business is even older. It concerns itself with the statistical mechanics of identical particles. In calculating the partition function and the entropy, one finds that the entropy is not a strictly additive quantity: when we mix two volumes of an ideal gas at the same temperature and pressure the resultant entropy is larger than the sum of the two entropies.This Gibbs' paradox shows that such a collection of identical particles is not a satisfactory model for an ideal gas. Instead of heeding this warning signal people 'fixed' the trouble by an ad hoc procedure in dividing the partition function by the factorial of the number of particles, thus condoning the Maxwell--Boltzmann statistics. The genuine need for a reexamination of the implications of the strict identity of the particles was not appreciated until Bose, three decades yet to be born.

Elementary particles were originally introduced as the stuff from which the world was made. They were to be immutable entities. But the photon was clearly an entity which could be created or destroyed. Where does a photon come from and where does it go? And how can we really understand creation and destruction? What is the implication of strict identity of photons? In what sense and to what extent can we think of light as a collection of photons? All these questions were answered at one stroke by Bose, who asked us to consider the many-photon states to be counted as states with equal probability. Photons were thus particles all right, but particles for which strict identity was to be recognized by considering as distinct only those cases in which the distribution of photons over phase cells were distinct.

Photons thus became nothing but levels of an underlying field. Creation or destruction of photons is merely a 'movement' of the field. Photons are then manifestations of the potentialities of the radiation field: the dichotomy between the field and the particle thus ceases. Two have become one.

Automatically the embarrassment of the Gibbs paradox is resolved: the paradox was just telling us that the strict identity of particles must be taken into account. But if photons are but the differences between the levels of the radiation field, they are all identical! And the process of creation and destruction is thought of as a change in the state of the field, the 'motion' of the field. If we have equations of motion of the field, we have the means of describing the creation and destruction of photons. It took two more years for Heisenberg and Pauli to write down the equations of motion for the radiation field and another year for Dirac to construct a theory of the emission and absorption of photons. In Dirac's work the oscillators of Planck were at last identified. The formulation of the equations of motion of the electromagnetic field had still unsatisfactory features. Many others contributed to the resolution of this problem, among them Dirac, Heisenberg and Pauli, Fermi and Gupta.

In the course of his work on the quantum theory of radiation Dirac introduced the now familiar notion of creation and destruction operators which increase or decrease the number of quanta in a state. These creation and destruction operators, introduced as the operator coefficients of the quantized field operator, do not commute with each other but instead satisfy a commutation relation which transcribes the commutation relations between field quantities as formulated by Heisenberg and Pauli. Dirac had

already discovered that the commutator bracket in quantum mechanics was the natural analogue to the Poisson bracket in classical mechanics. It reaffirms the Bose hypothesis that photons obey the Bose statistics.

When Bose advanced his hypothesis the only species of elementary particles that were identified were the electron, the proton, the neutron and the photon. Of these only the photon obeys Bose statistics. The photon number is not covered and it is a zero mass particle. Both of these impart special characteristics to the statistics of photons. Among the Bose systems available then was Helium. In this case the particle number is conserved; and the particles are nonrelativistic. So we need to extend Bose's ansatz. This was done by Einstein in the same year. To conserve the particle number we have to introduce a non-zero chemical potential. For photons the chemical potential is zero. In the case of an ideal Bose gas with non-zero chemical potential there is a critical temperature below which a finite fraction of the gas condenses into a single quantum state. This condensed phase, discovered by London, should exhibit superfluid properties; and London suggested that superfluid helium should be related to this Bose-Einstein condensation phenomenon.

Einstein arranged for the translation and publication of Bose's paper on the statistics of photons and added a remark endorsing it as 'substantial progress.' Both in his original letter to Einstein and in his subsequent correspondence Bose addressed the great man as 'teacher' and accords him great respect; and that is as it ought to be. It is in the definition of the teacher, as understood in the classical Indian tradition, that he remove all the doubts of the student and weld his understanding into a harmonious unity: such a teacher is the one worthy of adoration.

> To that Teacher who removes all my doubts, welds my vision into a unity and thus enables me to gaze on secret knowledge; to that One my homage.

Einstein does not seem to have told Bose how his theory could be extended to a theory of ideal Bose gases by introducing a chemical potential and making use of a general energy–momentum relation. Einstein formulated this extension in one of his papers.

Bose followed up this paper by another more detailed and more ambitious paper. In his first paper, Bose had used a 'static derivation' of an equilibrium configuration as the most probable configuration. In the second paper, he used a 'dynamic derivation' in which the equilibrium configuration is the one in which transitions into and out of each state balance each other. For the special model of a two-level Bohr atom and monochromatic radiation, Einstein had shown (*Phys Z* **18**, 12 (1917)) how one can understand the Planck distribution when one takes into account both the stimulated and the spontaneous emission on the one hand and stimulated absorption on the other.

Bose generalized this to arbitrary atoms with arbitrary numbers of discrete energy levels and radiation of all possible frequencies: the essential elements in the derivation are the conservation of energy in collisions and the ratio of the rates of emissions and absorption. These were correctly computed by Bose. (Unfortunately, for some strange reason, Bose seems to have different values for the absolute transition rates which prompted Einstein to append a critical comment to the paper.)

In these two papers done before the birth of quantum mechanics as we know today, Bose laid the foundations of a quantum theory of the electromagnetic field.

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4

Chemistry

1 *Messungen der Zersetzungsspannung in nichtwässerigen Lösungsmitteln* (1927)
2 *Reaction of Sulphonazides with Pyridine* (1943)

To an outsider it might seem odd that a theoretical physicist of Bose's stature, who was intensely trying to understand the basic laws of physics, would now and then take time off and spend days in a dingy chemical laboratory. One might reasonably expect a theoretical physicist like him to get interested in the nature of the chemical bond or in the properties of electrolytic solutions — a problem fruitfully tackled by his friend and classmate J C Ghosh; but strangely, he showed no interest in any of these theoretical challenges. Rather, he liked to synthesize and analyze chemicals useful to contemporary society. He was obviously guided by a nationalistic feeling. A part of his interest in down-to-earth chemistry might have been inherited from his father who founded a small chemical industry. His close association in his formative days with Acharya P C Ray, who championed the cause of Indian chemical industries, might also have played a role.

In Dhaka University he set up a working organic chemistry laboratory and encouraged his students to synthesize a number of important drugs like emetine, sulpha drugs, etc.Most of these works are either not published at all or published without his name. Only occasionally did his name appear, such as in *Science and Culture* with P K Dutta, where the reaction between sulphonazides and pyridine was studied. His interest in organic chemistry continued after his return to Calcutta University. In an

article in *Science and Culture* **10**, 1974, p. 295, A Chatterjee has recalled some of Bose's forays in organic chemistry, particularly alkaloid chemistry.

At Dhaka, Biswas and Bose developed a simple instrument to measure the polarization voltage and decomposition voltage of some electrolytes in non-aqueous solvents, where a simple manual commutator was used to alternate the direction of current and thus reduce polarization errors at the electrodes. At Calcutta University he set up an X-ray laboratory and helped chemists with structural problems. Professor J N Mukherjee, the noted colloid chemist and classmate of Bose, prepared a map of clays of different parts of India. The structural aspects of these clays were studied in Bose's laboratory. He encouraged the organic chemists to determine molecular structures through X-ray analysis.

Although Bose's works in chemistry do not carry the mark of a genius, these show the versatility of the man and his eagerness to solve the problems of the country as well as his colleagues and students.

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3 *Germanium in Sphalerite from Nepal* (1950)
4 *Extraction of Germanium from Sphalerite Collected from Nepal—Part I* (1950)
5 *Extraction of Germanium from Sphalerite Collected from Nepal—Part II* (1950)

The invention of the transistor brought to reality the inventor's dream of realizing an electron device by controlling the motion of electrons available in ample quantity inside a solid. Power was no longer required to be wasted in obtaining controllable electrons in vacuum. The active device for electronics could now be made much smaller and the life expectancy of electronic equipment made much larger.

The first transistor was made with polycrystals of germanium (element number 32 in the Periodic Table) discovered by the German chemist Winkler in 1886 and named after his country. As germanium transistors were expected to replace vacuum tubes in all electronic circuits, S K Mitra concluded in his Presidential Address at the Forty-second Indian Science Congress held in 1955 at Baroda that 'the future stage, if one may venture to make a prophecy, will be the era of the uses and applications of the element germanium.' But germanium was mostly recovered from chimney dust collected from refineries of lead, titanium and zinc in the USA and in the UK from chimneys of all kinds of industries using coal which in Britain had a large germanium content. There were not many other sources of germanium and so scientists were exploring all possible sources. It is no wonder that Bose, being aware of all current developments, endowed with a very sharp foresight, and committed to the national interests, encouraged his student R K Datta to look for sources of germanium in minerals. This work resulted in three letters, in which is described the procedure used

for extracting germanium from sphalerite collected from Nepal and the suitability of the ores as a source of germanium. The three letters should be considered as important original contributions to the technology of germanium and establishing sphalerite as a good source of germanium. The only pity is that the work was not apparently followed up, and no semiconductor industry grew in India, although Bose did identify the problem within a year of the invention of the transistor.

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5

Spectroscopy

Beryllium Spectrum in the Region λ 3367-1964 (1929)

The spectrum of the beryllium atom is similar to that of the helium atom and should have been easier to analyze. It is well-known that He I shows singlets and triplets and He II (with one electron detached) shows doublets. But early observations did not resolve the triplets clearly. One sees the same kind of controversy in Be I and Be II in this paper.

The way the spectra were clarified by earlier workers seems almost miraculous today. We also note that after returning from Europe in 1926, Bose organized a spectroscopic laboratory at Dhaka. (Another paper published in 1927 deals with experimental work in electrochemistry.)

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6

Statistics

1 *On the Complete Moment-coefficients of the D^2-statistic* (1936)
2 *On the Moment-coefficients of the D^2-statistic and Certain Integral and Differential Equations Connected with the Multivariate Normal Population* (1937)

Though Statistics — as a scientific method — was known in India even at the beginning of the present century, significant statistical studies and researches were initiated during the twenties by the late Professor P C Mahalanobis. While working as a Professor of Physics at Presidency College, Calcutta, Mahalanobis took up several data-analytic studies on a wide variety of real-life problems and followed them up to develop several theoretical models and inferential tools. He could inspire quite a few brilliant young men in physics and mathematics to work on theoretical as well as applicational aspects of Statistics. Some of those drawn to Statistics — directly or indirectly by Professor Mahalanobis — later changed their initial academic pursuits to concentrate on Statistics. Others, like Professor Satyendranath Bose, continued with their original vistas but made remarkable contributions to statistical theory and absorbed statistical concepts and tools in their subsequent works.

While working on some anthropometric data on statures of Anglo-Indians, Professor Mahalanobis (1936) came up with an idea of generalized distance between two populations — more or less on the lines of Karl Pearson's coefficient of racial likeness (1921). He defined the famous D^2-statistic for this purpose in terms of the mean vectors (of the characteristics measured in the two groups) and the covariance matrix. It was intended to be a quantity determined entirely in terms of the sample values of the variates. And

for judging the significance or otherwise of such a sample value of the statistic its sampling distribution had to be worked out. R C Bose (1936) derived an exact distribution of a modified form of the D^2-statistic in which population variances and covariances were substituted for the corresponding sample estimates. He used the expression of the density function (in terms of Bessel functions) to obtain the moment coefficient, which turned out to be the same as those calculated earlier by Professor Mahalanobis (1936), using approximate methods.

Fascinated by the new idea of generalized distance and the nuances of the mathematics used by R C Bose, S N Bose (then working in the University of Dacca) looked at the problem (1936) and came up with a recurrence formula for moments of the modified D^2-statistic to obtain the exact moments without using the density expression and investigated some properties of the moment coefficients. He could also offer a different equation which can yield moments of even fractional orders (expressible not by a polynomial but by an infinite series) — entities that have found many important uses later.

Professor Bose continued his interest in the mathematics of this problem. In his 1937 *Sankhyā* paper he started with some algebraic identities among the moments of the modified D^2-statistic deduced from their differential forms and derived an integral equation connected with these moments. He investigated the fundamental differential equation satisfied by the multivariate normal distribution in its various forms — spherical, parabolic and product. He also offered a series solution of the differential equation.

The contents of the two papers in Sankhyā (1936 and 1937) speak of the depth of S N Bose's vision about a new problem and its mathematical fallouts. It may be pointed out, however, that the results derived by S N Bose have not created a big impact on statistical aspects of multivariate analysis, since the basic shortcoming of the modified D^2-statistic considered by him renders the results somewhat less useful. The differential equation satisfied by the multivariate normal distribution has not found its way into later investigations on characterizations. In fine, the papers by S N Bose on D^2-statistic have been rarely cited or used.

S P MUKHERJEE
Centenary Professor of Statistics
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7

The Ionosphere

1 *Anomalous Dielectric Constant of Artificial Ionosphere* (1937)
2 *On the Total Reflection of Electromagnetic Waves in the Ionosphere* (1938)

Various theories of the propagation of radio waves through the ionosphere were developed during 1930-1938 based on either the ray treatment or the wave treatment. The refractive index comes out in general to be a complex quantity which is a function of the electron number density and collision frequency which are functions of height. Consequently, the solution of Maxwell's equations becomes highly complex. Appleton deduced certain conditions of reflection by assuming that the refractive index must vanish.

However, since the refractive index is complex, other criteria were formulated by various workers (D R Hartree, *Proc Camb Phil Soc* **25**, 47,1929: ibid. **27** 143, 1931; *Proc Roy Soc* A **131**, 428, 1931; Fosterling and Lassen, *Ann d Physik* **18**, 26, 1933; H G Booker, *Proc Roy Soc* A **155**, 235, 1936; M N Saha and R N Rai, *Proc Nat Inst Sci Ind* **3**, 359, 1937; M N Saha, R N Rai and K B Mathur, *Proc Nat Inst Sci Ind* **4**, 53, 1938).

R N Rai (*Proc Nat Inst Sci Ind* **3**, 307, 1937) suggested that a better criterion of reflection would be that the group velocity of the wave vanishes. This gave him a new condition in addition to those of Appleton, a condition that was experimentally confirmed by Pant and Bajpai (*Science and Culture* **2**, 409, 1937), L Harang (*Terr Mag* **40**, 29, 1937) and R Jonaust, Abadie and Joigny (*L'onde electrique* **16**, 185, 1937).

Experiments also indicated that apart from totally reflecting electromagnetic waves the ionosphere also partially reflected and partially transmitted these waves. More-

over, for very long waves, the complex refractive index can change within one wavelength and reflection can occur even though the refractive index is not zero. These features could not be explained by any of the theories.

Both Appleton's and Rai's conditions of reflection were obtained by neglecting the effect of damping, an essential feature of the physical process. The questions that arose were: (1) do both these conditions follow as consequences of the Maxwell-Lorentz theory and (2) what are the conditions of reflection when collisional damping cannot be neglected? These are the problems that Bose set about to tackle in the 1938 paper.

Instead of Maxwell's equations with a complex refractive index, Bose used the microscopic equations of the Maxwell-Lorentz theory and the method of characteristics used by Hadamard, Debye and others. The method is general and can also be used for the propagation of light in a material medium. However, Bose's results were expressed in symbols unfamiliar to workers in ionospheric physics. It was M N Saha and K B Mathur (*Ind Jour Phys* **13**, 251, 1939) who gave a critical assessment of Bose's results in a form easily comprehensible to them. They showed that Bose's treatment gives the same results as those of previous workers for the case of vertical propagation. However, in the presence of collisions the nature of the complex refractive index and polarization became complicated, and Bose's conditions do not give accurate results.

In 1960 H K Sen and A A Wyller (*J Geophys Rev* **65**, 3931, 1960) used the Chapman-Enskog method and the Boltzmann equation to obtain the effect of the velocity dependence of the collision frequency in a closed analytic form.

Earlier in 1937 Bose and S R Khastgir had investigated the conditions under which the value of the dielectric constant of an ionized medium exceeds unity (anomalous behaviour) and carried out simple experiments with ionized air in a discharge tube (artificial ionosphere) to show that the dispersion formula alone cannot explain the observed anomalies.

EDITORS

8

Mathematical Physics

1 *Studies in Lorentz Group* (1939)
2 *The Complete Solution of the Equation*:
$$\nabla^2\varphi - \frac{\partial^2\varphi}{c^2\partial t^2} - k^2\varphi = -4\pi\rho(xyzt) \quad (1941)$$
3 *On an Integral Equation Associated with the Equation for Hydrogen Atom* (1945)

The three papers are on various problems in mathematical physics: the first deals with the properties of the matrix group O (4,C) in its defining representation, with consequences for the physical Lorentz group SO (3,1); the second develops a particular technique, involving integration in the complex plane à la Sommerfeld, for solving the inhomogeneous Klein-Gordan equation; and the third is concerned with the Schrödinger equation for the Coulomb problem, analyzed as an integral equation in momentum space. These papers were written in 1939, 1941 and 1945 respectively. A few introductory comments regarding each are presented.

It is most easily seen from the infinitesimal approach, provided that one works with complex linear combinations of the basis elements, that the Lie algebra of SO(3,1) formally splits into two commuting SU(2) or angular momentum like algebras. This helps in the construction of all the finite dimensional irreducible matrix representations of SO(3,1), and the connection of SL(2,C) also becomes clear. One sees on the way the possibility of formally expressing a finite element as a product of two commuting factors, in the complex sense. To realize all this without using infinitesimal techniques, however, requires considerable ingenuity. This is what is attempted in the 1939 paper of Bose. He in fact examines the

much larger group of all complex orthogonal transformations in four dimensions, namely the group O(4,C). The key idea is to deal with symmetric elements of O (4,C) and find ways of characterizing and factoring them; and for antisymmetric matrices, exploiting the properties of self-dual and antiself-dual sets of matrices. It is the latter that actually lead to the commuting structure referred to above. From the discussion of O(4,C), Bose descends to the case of SO(3,1) by imposing the requisite reality conditions, and thus connects up the spinor approach based on SL(2,C), as typified in the familiar statement $(1/2,1/2) = (1/2,0) \times (0,1/2)$: the four-vector representation of the Lorentz group is the product of the two basic spinor representations.

Turning to the second paper, it is basically an exercise using relativistically invariant solutions of the Klein-Gordan equation to solve a problem for which H J Bhabha in an earlier paper had presented a solution in a particular form. At the time the paper was written, namely 1941, the technology associated with the wave equation using Stückelberg-Feynman functions, invariant Green's functions, retarded and advanced ones, etc., were presumably not yet widely known and used. The interest in Bose's paper lies in its use of complex contour integration methods, a favourite of Sommerfeld, to get solutions of the inhomogeneous Klein-Gordan equation in Kirchhoff-like form.

The third (1945) paper is devoted to a discussion of the quantum mechanical Coulomb problem in momentum space. It has been known for a very long time, through the work of V Fock and V Bargmann, that if the Schrödinger equation for energy eigenfunctions is expressed in momentum space, it has the form of an integral equation, and moreover it makes the higher symmetry in the problem manifest. Thus the SO(4) symmetry for bound states, and SO(3,1) for scattering states, can be explicitly exhibited by suitable energy-dependent choices of variables. (It is curious, however, that Bose does not refer to Fock's and Bargmann's papers at all). Bose however proceeds somewhat differently realizing that the energy-dependent choice of variables would discriminate between bound and scattering states. He exploits methods of Hobson involving solid harmonics and operator calculus to explicitly solve the integral equation, after reducing it essentially to a radial problem (in momentum space). An interesting feature of his expressions is that the quantization of the bound state energies, and also the fact that the angular momentum quantum number is bounded above essentially by the principal quantum number, both arise from the requirement that the momentum space eigenfunction be single-valued, rather than that it be normalizable. It is the latter condition that is normally used in a configuration space solution to the problem. His method of course leads to both bound and scattering state wave functions. At the end of the paper, a completeness statement involving the former alone is developed. One presumes therefore that this is distinct from the usual physical notion of completeness of eigenfunctions of the Hamiltonian operator, since that would have brought in the scattering states also.

All three papers show a taste and knack for clever algebraic and analytic manipulations, even though the problems addressed are circumscribed in scope.

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Indian Institute of Science, Bangalore.

9

Unified Field Theory

1 *Les identités de divergence dans la nouvelle théorie unitaire* (1953) [30.3.53]
2 *Une théorie du champ unitaire avec* $\Gamma_\mu \neq 0$ (1953) [18.7.53]
3 *Certaines consequences l'existence du tenseur g dans le champ affine relativiste* (1953) [18.7.53]
4 *The affine connection in Einstein's new unitary field theory* (1954) [29.9.52]
5 *Solution d'une équation tensorielle intervenant dans la théorie du champ unitaire* (1955) [Receipt date not mentioned]

The papers are listed in chronological order. The journal receipt date is given within brackets.

The general theory of relativity established itself as a satisfactory description of gravitation. The only other interaction that was then known was the electromagnetic, and a natural sentiment was to bring the electromagnetic and gravitational interactions under one umbrella — to be more precise, to geometrize the electromagnetic field as well.

Einstein, however, demanded much more from a unified field theory. He was committed to the field idea and was never reconciled to quantum mechanics with its basic indeterminacy. He expected that matter itself would ultimately resolve into a field and, as singularities mean a breakdown of the field concept, there should be no singularities in a truly unified field theory. Again, when Kaluza's five-dimensional geometry appeared, the idea was that our observations were limited to four dimensions

and thus were essentially of an incomplete nature. One might wonder whether this incompleteness could account for quantum indeterminacy.

What, by the way, is a unified theory ? We may quote Einstein's definition :

> Neither the field equation nor the Hamiltonian can be expressed as a sum of several invariant parts but are formally unified entities.

There were a host of unified field theories — one may compare the ever-changing theories with the changing models of cars. But they all had one similarity. The 4-dimensional Riemannian geometry has ten independent variables (the metric tensor components g_{ik}), just sufficient for the gravitational field whose source, the energy–momentum–stress tensor, has also ten components. So, to incorporate the electromagnetic field, one must have some additional geometric variables. These additional variables were introduced in a number of ways, e.g.

(a) In the Weyl theory, it was postulated that the norm of a vector changes in a parallel displacement according to the law.

$$\frac{\delta l}{l} = \phi_i \, dx^i$$

and the vector ϕ_i was identified with the electromagnetic potential vector.

(b) In Kaluza's five-dimensional formalism the metric tensor has 15 components.

(c) The fundamental tensor g_{ik} was assumed to be complex, so that there were 20 variables.

(d) The fundamental tensor was assumed to be non-symmetric and the affinities Γ^i_{kl} which define parallel displacement were also taken to be non-symmetric. There were thus altogether 80 variables.

Over a number of years different forms of the non-symmetric theory were developed by Schrödinger and Einstein. One particular form gained wide publicity as it was announced by Einstein first in the popular press in December 1949 and then presented in the 1950 edition of his book *Meaning of Relativity* as an Appendix. In his papers Bose used the adjective 'new' before unified field theory without indicating specifically which particular theory he had in mind. It seems clear that he referred to this theory.

We give a table to pinpoint the salient points of this theory as compared to the general theory of relativity :

	GTR	UFT
1.	Symmetric metric tensor $g_{\mu\nu}$ and symmetric affinities $\Gamma^\alpha_{\mu\nu}$.	Non-symmetric $g_{\mu\nu}$ and $\Gamma^\alpha_{\mu\nu}$. [What is the metric tensor ? No specific statement is made in the book but it became common to identify the symmetric part of $g_{\mu\nu}$ with the metric tensor and the antisymmetric part was related to the electromagnetic field, as we shall presently see.]

2. $g_{\mu\nu;\alpha} = 0 \rightarrow$

$$\Gamma^{\mu}_{\nu\sigma} = \frac{1}{2} g^{\mu\alpha} (g_{\alpha\nu,\sigma} + g_{\alpha\sigma,\nu} - g_{\nu\sigma,\alpha})$$

$g_{\mu\nu;\alpha} = 0$

(*i.e.* $g_{\mu\nu,\alpha} - g_{\rho\nu}\Gamma^{\rho}_{\mu\alpha} - g_{\mu\rho}\Gamma^{\rho}_{\alpha\nu} = 0$).

[The solution of these 64 equations, i.e. expressing Γ's in terms of $g_{\mu\nu}$ and their derivatives, was an extremely difficult problem and was given independently by Bose, Einstein and Kaufman, Hlavaty and Tonnelat. Later there were also other workers in the field including R S Mishra.]

3. Field equations obtained from the variational principle

$$\delta \int g^{\mu\nu} R_{\mu\nu} \sqrt{-g}\ d^4x = 0$$

where $R_{\mu\nu}$ (the Ricci tensor) is

$R_{\mu\nu} = \Gamma^{\sigma}_{\mu\alpha}\Gamma^{\alpha}_{\sigma\nu} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} + \Gamma^{\alpha}_{\mu\alpha,\nu} - \Gamma^{\alpha}_{\mu\nu,\alpha} = R_{\nu\mu}$.

The variation can be performed in two ways :

(1) $g^{\mu\nu}$'s are to be varied and the relation between Γ's and $g_{\mu\nu}$'s assumed.

(2) $g^{\mu\nu}$ & Γ's are independently varied—the relations between Γ's and $g_{\mu\nu}$'s appear as one set of field equations.

Field equations obtained from the variational principle

$$\delta \int g^{ik} u_{ik} \sqrt{-g}\ d^4x = 0,$$

$g^{\mu\nu}$'s & Γ's varied independently. The tensor u_{ik} is

$$u_{ik} = R_{ik} - \frac{1}{3}\left[\left(\Gamma_{i,k} - \Gamma_{k,i}\right) + \Gamma_i \Gamma_k\right]$$

$\Gamma_i = \Gamma^{l}_{\underset{\vee}{il}}$ and

$$R_{ik} = \Gamma^{s}_{il}\ \Gamma^{l}_{sk} + \frac{1}{2}\left[\Gamma^{s}_{\underline{is},k} + \Gamma^{s}_{\underline{ks},i}\right] - \Gamma^{s}_{ik}\Gamma^{t}_{\underline{st}} - \Gamma^{s}_{ik,s}$$

where underlining indicates the symmetric part and the hook the antisymmetric part. The tensor u_{ik} is selected by some ad hoc conditions introduced by Einstein which may be summarized in the form

$u_{ki} = u_{ik}\ (\tilde{\Gamma})$

$u_{ik} = R_{ik}\ (\Delta)$

where

$\Delta^{l}_{ik} = \Gamma^{l}_{ik} - \frac{1}{3}(\Gamma_i \delta^{l}_{k} - \Gamma_k \delta^{l}_{i})$ so that $\Delta^{l}_{\underset{\vee}{il}} = 0$. The conditions were not justified by any theoretical consideration. *Bose therefore sought to work with a different variational principle.* However, this led to a much more complicated set of equations which involved two arbitrary constants.

4.	**Field equations** $R_{\mu\nu} = 0$. **The equations satisfy four differential identities.**	**Field equations** $R_{ik} = 0$, (a) $g_{ik;l} = 0$, (b) $\Gamma_i = 0$. (c) **The equation $\Gamma_i = 0$ was introduced ad-hoc on the ground that this does not cause an over-determination. The equations (b) and (c) together give $(\sqrt{-g}\, g^{ik})_{,k} = 0$. This led Einstein to identify g^{ik} with the dual of the electromagnetic field tensor—there is no free magnetic pole.**
5.	**If fields other than gravitational were present, additional terms were introduced in the Lagrangian and the field equations then became** $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = kT_{\mu\nu}$.	**The electromagnetic field is already taken care of. No mention is made of other fields. Unification is thus not complete.**
6.	**Physical conclusions drawn all agree with observations.**	**No physical conclusions could be drawn. Indeed, Pauli remarked (1958), 'Whether the field equation. . . can actually be connected with physics at all is rather doubtful.'**

Before going over to the subject matter of these papers, it is somewhat interesting to note that at least four of these papers were produced within the span of less than a year. That was something unusual in Bose's career. Never before had he written so many papers in a year and never before had he contributed as many as five papers in a single field. One may wonder whether these works kindled Bose's enthusiasm to a degree he had never experienced in his whole life. And mark, five papers in four different journals — was it all fun with Bose !

The first paper concerned the divergence identities in the unified field theory we have just outlined. Long before, Hilbert (1924) had proved that the set of equations obtained from a variation principle would not be independent but there would be four divergence identities satisfied by them. For non-symmetric theories Einstein obtained the identities by a method which was rather complicated. Bose showed that they could be obtained in a somewhat more general manner by following Hilbert's procedure.

In the second paper Bose used a much more complicated Lagrangian. Einstein's Lagrangian was obtained on the basis of somewhat arbitrary arguments and then the equation $\Gamma_i = 0$ appended, justifying this by the rather curious reason that it would not make the equation set overdetermined. Bose's equations were naturally more complicated and were not supplemented by the ad hoc condition $\Gamma_i = 0$. However, the equation system involved two arbitrary constants and hence they were not quite appealing. Neither Bose nor any of his students investigated these equations.

THE INSTITUTE FOR ADVANCED STUDY
PRINCETON, NEW JERSEY

October 4th,1952

SCHOOL OF MATHEMATICS

Professor S.N.Bose
University College of Science
92, Upper Circular Road
Calcutta,9, India.

Dear Professor Bose:

Thank you for your letter of September 20th. I am glad to see that you are interested in this theory and that you have devoted so much work and penetration to the solution of the equation

$$g_{(ik,l)} = 0$$

I believe, to be sure, that the solution of those equations is not of great help toward the answer of the question: Do the singularity-free solutions of the equation system have physical meaning? Are there at all singularity-free solutions which correspond to the atomistic character of matter and radiation? It seems to me that the mathematical methods available at present are not powerful enough to answer this question.

However, I am now firmly convinced that the equation-system

$$g^{(ik}_{;l} = 0 \qquad \Gamma^{i}_{il} = 0$$

$$R_{(ik)} = 0 \qquad R_{ik,l} + R_{kl,i} + R_{li,k} = 0$$

represents the only formally natural generalization of the relativistic law of gravitation.

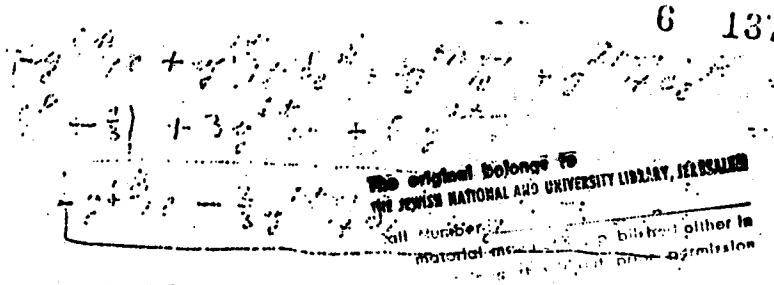
With kind regards,

sincerely yours,

A. Einstein.

Albert Einstein.

Einstein's letter of 4 October 1952 to Bose


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Lieber Herr Bose!

Ich erhielt heute Ihren freundlichen Brief, in welchem Sie mich nach meiner Ansicht über Ihre letzte Arbeit fragen. Diese habe ich schon vor etwa 10 Tagen erhalten.

Statt nun auf irgend welche Einzelheiten einzugehen, möchte ich nur folgendes sagen. Wenn man die Hamiltonsche Funktion

$$g^{ik}R_{ik}$$

$$\left(R_{ik} = \Gamma^{\alpha}_{ik,\alpha} - \Gamma^{\alpha}_{i\alpha,k} - \Gamma^{\alpha}_{i\tau}\Gamma^{\tau}_{\alpha k} + \Gamma^{\alpha}_{ik}\Gamma^{\tau}_{\alpha\tau} \right)$$

des Variationsprinzips ohne jede Modifikation oder Nebenbedingung bildet, so erhält man als Gleichungensystem

$$\left.\begin{aligned} &R_{ik} = 0 \\ \text{und} \quad &\left(\mathfrak{W}^{ik}_{\ell} \equiv\right) g^{ik}_{,\ell} - g^{i\tau}\Gamma^{k}_{i\tau} - g^{k\tau}\Gamma^{i}_{\tau\ell} + g^{ik}\Gamma^{\tau}_{\tau\ell} + g^{i\tau}\Gamma^{k}_{\tau}\delta^{k}_{\ell} = 0 \end{aligned}\right\} (I)$$

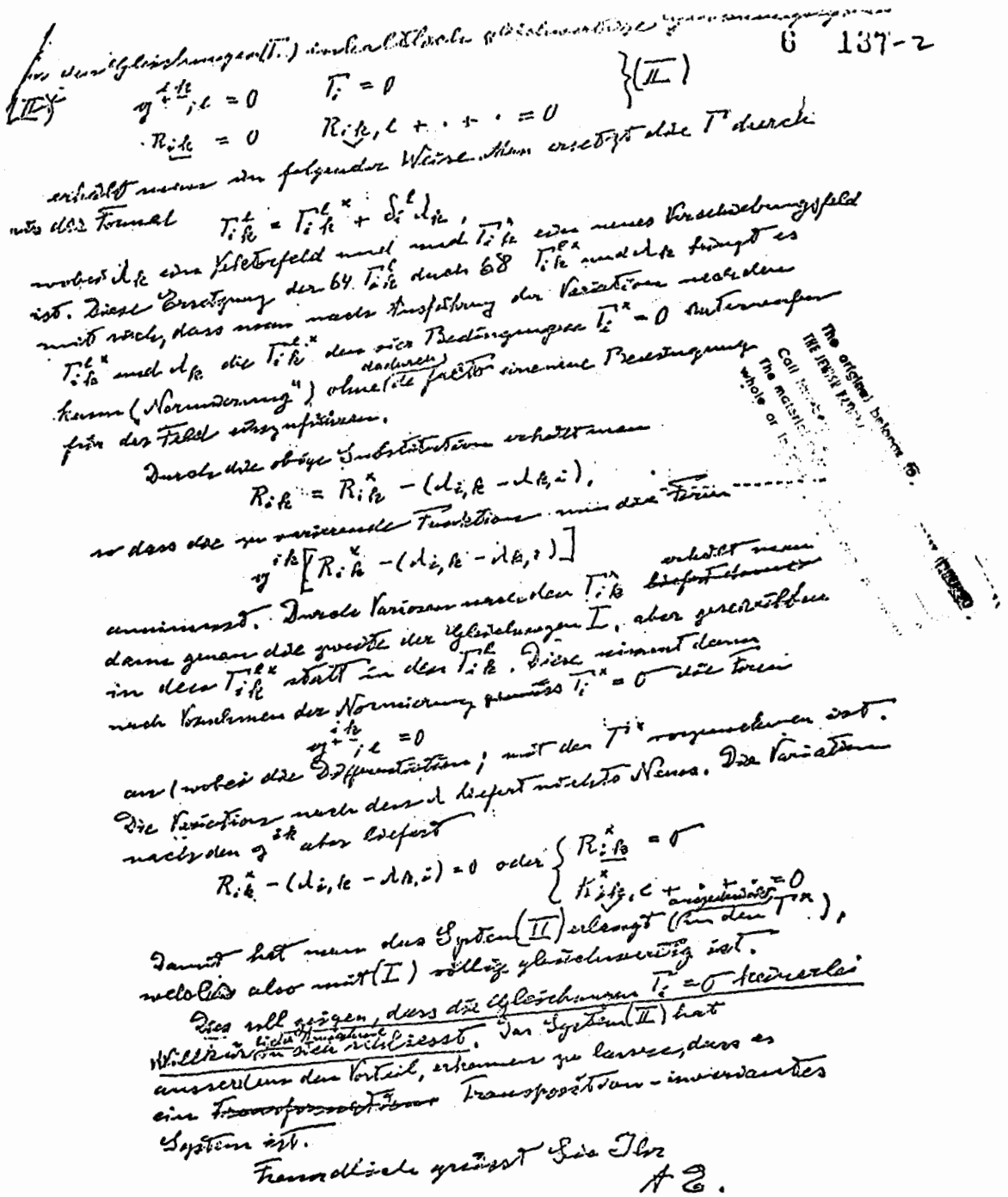
Durch Hier kommt die Bedingung $\Gamma^{i}_{ik} = 0$ vor. Man kann die zweite Gleichung durch Eliminieren des zweiten Gliedes auch auf die Form bringen

$$-g^{ik}_{,\ell} + \left(g^{ik}\Gamma^{\tau}_{\ell\tau} - \tfrac{2}{3}g^{i\tau}\Gamma^{k}_{\tau\ell}\right) = 0.$$

Die Γ_{ℓ} verschwinden hierbei nicht.

Die Gleichungen in der oben angegebenen Form sind lediglich eine Umformung von (I) ohne Änderung des materiellen Gehaltes. Erst in diesen die umgeformten Gleichungen tritt die Bedingung $\Gamma^{i}_{ik} = 0$ und die Herleitung der ersten Gleichung in die Form $R_{ik} = 0$ $R_{ik,\ell} + \cdots = 0$ auf.

Dass die Gleichungen (I) noch einer Umformung bedürfen geht schon daraus hervor, dass von die zweite der Gleichungen (I) die Γ aus den g nicht völlig bestimmt, indem diese Gleichungen die Identität $\mathfrak{W}^{sk}_{s,k} \equiv 0$ erfüllen, welche neben den vier Bianchi – Identitäten bestehen. Es bleibt daher von den Γ eine Funktion (von 4 Variablen) unbestimmt,



Einstein's letter to Bose in Paris from Princeton, dated 22 October 1953, commenting on Bose's paper 'A Unitary Field Theory . . .' for *Le Journal de Physique*; by courtesy of The Jewish National and University Library, Jerusalem.

The remaining three papers are all on the equation

$$g_{\mu\nu,\,\alpha} - g_{\rho\nu}\,\Gamma^{\rho}_{\mu\alpha} - g_{\mu\rho}\,\Gamma^{\rho}_{\alpha\nu} = 0.$$

In the third paper Bose considered the above equation as first order partial differential equations for $g_{\mu\nu}$. Then the condition of integrability is $g_{\mu\nu,\,\alpha\beta} = g_{\mu\nu,\,\beta\alpha}$. This condition reduced to equations involving Γ's and $g_{\mu\nu}$ but free of the derivatives of $g_{\mu\nu}$. They were explicitly worked out by Bose. This was a more or less routine investigation and did not lead to any very impressive result.

In the last two papers in the list Bose was interested in solving the above equations, regarding them as coupled algebraic equations for Γ's, taking the $g_{\mu\nu}$'s as known. The problem is to find the solution not for particular coordinates but in a form which will hold in all coordinate systems. This was then a challenging problem especially in view of a remark by Schrödinger (1947) :

> In the general case it is next to impossible to give a tensorial solution of these equations.

The adjective 'tensorial' in Schrödinger's remark is not quite correct for Γ's are themselves not tensors ; what he meant was that the solution must hold in all coordinate systems. However, Schrödinger was soon proved to be unduly pessimistic. Mme Tonnelat, Hlavaty in 1953, Bose, Einstein and Kaufman in 1954 and later on many workers including Mishra gave solutions. Although Bose's paper was published somewhat later than the first papers of Tonnelat and Hlavaty, Bose's first paper was communicated in September 1952, earlier than the publications of Tonnelat and Hlavaty. Indeed, all three were ignorant of the works of others and their methods were also different. Again, while Einstein had received a pre–publication manuscript from Bose and was aware of the work of Hlavaty, he and Kaufman proceeded without splitting $g_{\mu\nu}$ and Γ^{i}_{kl} into their symmetric and anti-symmetric parts, unlike the procedure adopted by Bose and others. Strangely, some later workers like Mavrides, Kichenssamy and even Mishra (1976) comment on the works of Tonnelat and Hlavaty, but they seem to ignore the works of Bose and also of Einstein and Kaufman.

IMPORTANCE OR MOTIVATION OF THESE WORKS

One may wonder what one can gain by these solutions. The first answer is that a pure mathematician's motivation arises simply from the mental desire to solve a problem just as the mountaineer climbs the Everest simply because 'it is there'.

Perhaps some of the eminent mathematicians had the idea that once the Γ's are found in terms of $g_{\mu\nu}$, these values of Γ would be plugged into the expression for $R_{\mu\nu}$ and one could attempt a general solution of the equation $R_{\mu\nu} = 0$. However, even in the much simpler case of general relativity, such a general solution of $R_{\mu\nu} = 0$ cannot be obtained and in reality has not even been attempted. What, therefore, one has to do is either to assume some symmetry and in that case the calculation of Γ's becomes not that difficult (and has been done by Bonnor, Papapetrou and others), or to deduce

some general theorems. Thus to Einstein the crucial problem was: 'Do the singularity-free solutions of the equation system have physical meaning ? Are there at all singularity-free solutions which correspond to the atomistic character of matter and radiation ?' From this viewpoint the solution of those equations is not of great help. [Here those equations refer to the equations whose solution Bose presented in a letter to Einstein and we have just now considered.]

But, if the solution of the Γ's was not important from the point of view of the theory, one may ask: why did then Einstein and Kaufman give a solution ? The fact is, it came as a by-product of their investigation of the following problem. The universe we observe has three space-like and one time-like dimension. So, in order to be physically acceptable, the signature must be 2 or, in other words, the metric tensor determinant must be negative. Einstein and Kaufman considered provisionally $g_{\mu\nu}$ to be the metric tensor and then investigated if this has the correct signature — do the field equations ensure that this signature will be maintained everywhere ? Their answer was in the affirmative, and in course of the proof they found the solution of the equations $g_{ik,l} = 0$.

As we come to the close of our discussion of Bose's works in unified field theories, some disturbing questions come to mind. Did Bose share Einstein's attitude towards quantum mechanics ? Did Bose believe that a viable unified field theory based on geometry can really come about while most physicists thought these attempts were futile ? Bose has left nothing in writing which can throw any light on these matters. In fact, his papers were purely mathematical exercises without any reference whatsoever to physics.

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10

Thermoluminescence

A Report on the Study of Thermoluminescence (1955)

Thermoluminescence means the phenomenon of emission of light (infrared or ultraviolet radiation included) other than pure thermal radiation, by a system under thermal stimulation. It is evidently related to energy states of the physical system, the relative transition probabilities and related electronic processes. When a system is excited by any method and a part of the excitation energy is stored in it, the system is termed thermoluminescent if it releases, on heating, a part or whole of the stored energy in the radiant form. The phenomenon of thermoluminescence exhibits a very large variety of behaviours, corresponding to different substances with variations in impurity contents or other imperfections. Thus, it is not surprising that thermoluminescence in all its details is a comparatively poorly understood phenomenon even in the case of simple materials.

Thermoluminescence appears to have been first observed by Boyle about three hundred years ago. Since the publication of Randall and Wilkins' work in 1945 there has been a steady accumulation of data in this field as well as attempts at theoretical interpretation of the experimental results in the light of the modern theory of solids, which already met with striking success in allied fields. The applicational possibilities of thermoluminescence in geological and archaeological work, dosimetry, or as a research tool in science and industry, have been already demonstrated by the tremendous growth in the number and variety of work in this field in the last fifty years.

Although extensive work has been carried out with thermoluminescence as a research tool, most of such work consists of the determination of trap depths and their changes under varying experimental conditions, but the results are far from satisfactory for a clear understanding of the electronic processes occurring inside the solid. The identification of the trapping and emitting centres, responsible for the different glow peaks, remains as yet a challenging problem in most of the phosphors. It was felt by Bose (and other pioneers in the field) that the physical interpretation of the results of investigations on thermoluminescence would be facilitated if the spectral compositions of the thermoluminescence and afterglow emission could be compared with the luminescence of the phosphor under study. The research workers in this field had been trying to find out the spectral compositions of thermoluminescence and afterglow emission with the help of filters; naturally, the results were indicative of the fact that the spectral composition of the emission undergoes changes during the process of thermoluminescence or afterglow decay in many cases but the data were not of convincing precision.

The duration of thermoluminescence is generally of the order of a minute and afterglow emission, comparatively poor in intensity, may also change in spectral composition as the intensity changes with time. The difficulties of interpretative work were thus essentially due to the lack of a suitable measuring device and experimental techniques. Bose saw the crux of the problem, and designed a rapid scanning spectrophotometer of comparatively high sensitivity to meet the requirements of the experimental workers in this field. The design and fabrication of the scanning spectrophotometer was reported by Bose at the International Conference on Crystallography held in Paris in 1954.

The spectrophotometer is capable of scanning the entire spectral range of visible and near-ultraviolet radiation in 0.9 seconds, and the spectral distribution of emission either in afterglow decay or thermoluminescence can be displayed on the screen of a cathoderay oscilloscope. It was thus possible to observe the changes in the spectral composition of emission during any time-varying process like thermoluminescence. The instrument performs the dual role of a spectrograph and a microphotometer, and can very conveniently be adapted to carry on studies of (i) fluorescence and phosphorescence, (ii) thermoluminescence spectra, (iii) the temperature dependence of luminescence (both fluorescence and phosphorescence), (iv) the correlation of colour centres and other known trapping centres with luminescence, (v) the development of emission centres in the phosphor during X-ray and cathoderay irradiation, and (iv) near-infrared absorption and emission spectra etc. The possibilities of the scanning spectrophotometer have however not been fully exploited till date.

The special features of the work initiated by Bose at the Khaira Laboratory in Physics in Calcutta were the luminescent and related properties of solids and frozen liquids under excitation by low energy electron beams as well as X-rays. The excitation is limited to a thin surface layer in the case of irradiation by low energy electrons, while for X-rays or other high energy excitations the entire volume of the crystals is affected. The two methods of excitation should thus bring out the distinctive differences in the

effects of impurities , self-absorption and energy transfer processes, on the luminescent and allied properties of solids. Some work was also attempted on the luminescence of solids under soft X-ray excitation; and attempts were made to correlate the data on valence band spectra with those on colour centres and luminescence of simple solids.

Adapted from H N Bose, *On the Pioneering Works of Professor S N Bose in the Field of Thermoluminescence,* in the Proceedings of the Seminar on The Scientific Contributions of Professor S N Bose, Cal Math Soc 1943, pp. 111-121.

11

Bose Statistics : a historical perspective

a. Planck's derivation of his law of black-body radiation

In order to appreciate the novelty and importance of Bose's work in its historical perspective, it is necessary first to give a brief account of Planck's original derivation [Planck, 14 December 1900] of the law of black-body radiation,

$$\rho(\nu, T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} \tag{1}$$

which he had discovered empirically a few weeks earlier (7 October 1900). It consisted of *three* steps.

First, he established the relation

$$\rho_\nu = \frac{8\pi\nu^2}{c^3} U_\nu \tag{2}$$

between the energy density ρ_ν of incident radiation at temperature T, whose frequency lies between ν and $\nu + d\nu$, and the average energy U_ν of a resonator of the same frequency ν at temperature T, on the basis of *classical* electromagnetic theory. Comparing (1) and (2), he found U_ν :

$$U_\nu = \frac{h\nu}{e^{h\nu/kT} - 1} \,. \tag{3}$$

In the second step he determined the entropy of these oscillators by integrating $TdS=dU$ where T is taken from (3) as a function of U (for fixed ν). He obtained

$$S = k\left[\left(1+\frac{U_\nu}{h\nu}\right)\ln\left(1+\frac{U_\nu}{h\nu}\right) - \frac{U_\nu}{h\nu}\ln\frac{U_\nu}{h\nu}\right]. \tag{4}$$

The third step was the revolutionary one. He introduced two ideas at this stage that he himself considered as 'acts of desperation'. He assumed that the total energy $U_N = N\,U_\nu$ of N oscillators (resonators) was made up of finite energy elements ϵ such that $U_N = P\epsilon$ with P a large number. Then he searched in Boltzmann's work for a permutation measure W_N (the total number of complexions or distributions) of P discrete equal energy values ϵ among N oscillators that would correspond to the right hand side of (4) when used in $S_N = k\ ln\ W_N$. He found

$$W_N = \frac{(P+N-1)!}{P!\,(N-1)!}. \tag{5}$$

Using $P/N = U_\nu/\epsilon$, $S_N = N\,S$ and applying Stirling's formula, he obtained

$$S = k\left[\left(1+\frac{U_\nu}{\epsilon}\right)ln\left(1+\frac{U_\nu}{\epsilon}\right) - \frac{U_\nu}{\epsilon}\,ln\,\frac{U_\nu}{\epsilon}\right]. \tag{6}$$

Since S is a function of (U_ν/ν) only, it follows from (4) and (6) that

$$\epsilon = h\nu.$$

This is how quantum theory was born. Planck had no more justification for using the formula (5) than that it gave him the result that he was after : '. . . a theoretical interpretation had to be found at any price, however high it might be,' he wrote to Robert William Wood in October 1931. [Mehra and Rechenberg, 1982, **1**, *1*, 50]. Actually, Planck's combinatorial approach differed from Boltzmann's probabilistic method in that Planck associated W_N with S_N, the equilibrium entropy, without maximizing it. For Boltzmann W_N was the number of possible complexions corresponding to the macrostate which can be realized by the largest number of complexions. As pointed out by Ehrenfest, a strict adherence to the accepted principles of statistical mechanics would have led Planck to the classical law of Rayleigh and Jeans! [Ehrenfest, 1905]

b. Einstein's light-quantum hypothesis

In 1905 Einstein argued on the basis of his analysis of energy fluctuations of radiation obeying Wien's law that such radiation had statistical properties similar to material particles and so must consist of discrete light-quanta of magnitude $(R/N_o)\ \beta\nu = h\nu$. [Einstein, 1905] This light-quantum hypothesis was immediately applied by him to give a reasonable explanation of several radiation phenomena, specially the photo-electric effect. Since these conclusions contradicted the classical electromagnetic theory on which Planck attempted to base his theory of heat radiation, Einstein considered

Planck's theory 'in some way a counterpart' to his own theory. He subjected Planck's derivation to a critical analysis and came to the following conclusion :

> We must therefore regard the following law as the basis of Planck's quantum theory of radiation : the energy of an elementary resonator can only assume values which are integral multiples of $(R/N_o)\,\beta\nu$; the energy of a resonator changes in jumps by absorption or emission in integral multiples of $(R/N_o)\,\beta\nu$. . . . If the energy of a resonator can alter only in jumps, then for the evaluation of the average energy of a resonator in a radiation cavity, the usual [electromagnetic] theory cannot be used, for the latter does not admit any distinctive energy values for a resonator. [Einstein, 1906]

Two years later Lorentz also came to the same conclusion [Lorentz to Wien, 6 June 1908; Mehra and Rechenberg, 1982, **1**, *1*, 98]. Planck attempted to modify his theory to take into account the quantum of action *without violating any aspect of classical electrodynamics*, but it gradually became clear from his studies and those of others that it was impossible to establish the theory of black-body radiation entirely on the foundation of Maxwell's classical electrodynamics and the statistical mechanics of Maxwell and Boltzmann.

c. Debye's derivation of Planck's law

In 1910 Debye gave a new derivation of Planck's radiation law in which he sought to avoid the inconsistencies (as viewed from Maxwell's theory) of Planck's earlier derivations as pointed out by Einstein [Debye, 1910]. Instead of using the relation (2) between the radiation density ρ_ν and the average energy of the oscillator U_ν, Debye calculated 'the probability for a given state of radiation and therefore as is well known, the entropy, using the properties of the state alone without employing resonators.' In agreement with Rayleigh and Jeans, he calculated the number $N_\nu\,d\nu$ of elementary states or vibrational modes (*Hohlraum oscillators*) contained in a volume V and obtained

$$N_\nu\,d\nu = \frac{8\pi\nu^2\,V\,d\nu}{c^3}\,. \tag{8}$$

Assuming that an amount of energy $h\nu$ gets distributed over each of these vibrations according to an arbitrary distribution function f_ν, he obtained

$$\rho_\nu\,d\nu = \frac{8\pi h\nu^3}{c^3}\,f_\nu\,d\nu\,. \tag{9}$$

Now, 'analogously to Planck', Debye *assumed* that the probability of distributing $N_\nu f_\nu\,d\nu$ quanta of energy $h\nu$ among $N_\nu\,d\nu$ vibrations was given by

$$\omega_\nu = \frac{(N_\nu\,d\nu + N_\nu f_\nu\,d\nu)\,!}{(N_\nu\,d\nu)\,!\;(N_\nu f_\nu\,d\nu)\,!}\,. \tag{10}$$

He then calculated the distribution function f_ν in the following way. He calculated the

maximum of ω_ν with the constraint that the energy remained constant. From this he calculated the equilibrium entropy S_ν to be $k\ ln\ (\omega_\nu^{max})$. Finally, using the definition of entropy to be $(dS_\nu/d\rho_\nu) = T^{-1}$, he obtained

$$f_\nu = \frac{1}{e^{h\nu/kT} - 1}. \tag{11}$$

Together with (9), this yielded Planck's radiation formula (1).

Two important features of Debye's derivation need to be emphasized as background to Bose's derivation that came fourteen years later. First, it became clear from his derivation that Planck's law follows simply from the assumption that the energy transfer from matter to the heat radiation and vice versa is quantized in units of $h\nu$ and that *no knowledge of the properties of material resonators is needed.* In other words, the quantum hypothesis only determined the transfer of energy from one mode of the radiation to another. This was in keeping with Planck's ideas but not Einstein's. Second, he used Planck's definition of the probabilities ω_ν, without analyzing its significance.

d. Indistinguishability of Planck's quanta

In 1911 Ladislas Natanson [1911] subjected the Planck-Debye combinatorial procedure to a critical analysis and showed that it was equivalent to distributing *P indistinguishable* energy elements ϵ among N 'receptacles of energy' distinguished only by the numbers j of quanta in them so that N_j receptacles contain j quanta each, subject to the constraints $\sum_j N_j = N$ and $\sum_j j N_j = P$. He found

$$\omega_\nu = \frac{(N_\nu\, d\nu)\,!}{\prod_{j=0}^{p} N_j\,!}. \tag{12}$$

Contrary to popular belief, therefore, the *indistinguishability of the quanta had already been tacitly assumed by Planck and Debye and this was first noticed by Natanson already in 1911.*

Ehrenfest and Kamerlingh Onnes [1914] also came to a similar conclusion in 1914. They however claimed that Planck's procedure was equivalent to distributing *P indistinguishable* energy elements ϵ among *N distinguishable* resonators and arrived at the expression

$$\omega_\nu = \frac{(N-1+P)\,!}{P\,!\,(N-1)\,!} \tag{13}$$

which is equivalent to the expressions (10) and (12) when N>>1. They then proceeded to clarify the distinction between Planck's energy quanta *which were statistically not independent* (they called them 'non-disjointed quanta') and Einstein's hypothetical light-quanta which were believed to be *statistically independent of one another* ('dis-

jointed quanta'). The concluded : *'Planck's formal device* (distribution of P energy-elements ϵ among N resonators) *cannot be interpreted in the sense of Einstein's light-quanta.'*

e. The introduction of phase space cells by Planck

Planck himself had come to the view on his own that classical statistical mechanics had to be modified in order to yield his radiation formula (1) rather than that of Rayleigh and Jeans. He reported his important conclusion at the Solvay Congress held in Brussels in 1911. According to Gibbs, the probability of finding a single particle in the element $d^3 p\, d^3 q$ of the six-dimensional phase space is given by

$$\frac{e^{-E/kT}\; d^3p\, d^3q}{\int e^{-E/kT}\; d^3p\, d^3q} \tag{14}$$

where $E = E(p,q)$ is the energy of the system. For a classical one-dimensional oscillator, $E = p^2/2m + \beta\, q^2/2$ and so the average energy of such an oscillator is kT. However, if E can take only the discrete values $E_n = n\,\epsilon = n\,h\,\nu$ with n=0,1,2 . . ., then

$$U = \frac{\sum_n E_n\; e^{-E_n/kT}}{\sum_n e^{-E_n/kT}} = \frac{\epsilon}{e^{\epsilon/kT} - 1}, \tag{15}$$

in agreement with (3). It was this straightforward derivation that led Planck for the first time to make the explicit statement that the energy of an oscillator (and not only its average energy) was an integral multiple of $h\nu$. It also led him to interpret the constant h *as a finite extension of the elementary area in phase space.* The motions of one-dimensional classical oscillators can be described by a family of concentric ellipses of semi-axes $(2E/\beta)^{1/2}$ and $(2\,m\,E)^{1/2}$. Since these oscillators can have any energy E, the ellipses form a continuum. However, if it is assumed that these ellipses are separated from each other so that the nth ellipse encloses an area

$$\int\int dp\, dq = n\,h, \tag{16}$$

then the energies of the oscillators must be integral multiples of $h\nu$. Planck therefore drew the revolutionary conclusion that energy quanta were a consequence of the fundamental condition

$$\int_E^{E+\epsilon}\int dp\, dq = h. \tag{17}$$

This implied a revision of the basic tenets of classical mechanics. He declared : 'The

framework of classical mechanics, even if combined with the Lorentz-Einstein principle of relativity, is obviously too narrow to account for all those physical phenomena which are not directly accessible to our coarse senses . . . One should therefore confine oneself to the principle that the elementary region of probability h has an ascertainable finite value and avoid all further speculation about the physical significance of this remarkable constant.' In other words, all attempts at finding a classical mechanical explanation of h must be abandoned. As we shall see later, to Bose these considerations (with which he was familiar) brought home extremely important lessons that would influence his own seminal contribution to the subject. First, he took serious note of the conclusion that classical electrodynamics was essentially incompatible with Planck's radiation formula. Second, since the quantum states of a particle were not continuously distributed throughout phase space (as in classical dynamics), he concluded that their number could be counted by simply dividing the total volume of phase space by h^3, the volume of an elementary cell. Finally, he took seriously Planck's claim that classical statistical mechanics had to be modified in a special way in order to conform to the requirements of quantum theory.

f. Spontaneous and induced transitions

The next important step was taken by Einstein in 1916. In 1913 Niels Bohr had proposed his model of the atom with stationary states with discrete energy values [Bohr,1913, b,c]. The transitions between any pair of these states were assumed to be caused by the exchange of energy quanta $h\nu$ between the atom and the radiation field described by *classical electrodynamics*. Einstein gave an elegant derivation of Planck's radiation formula by considering Bohr atoms in thermal equilibrium with radiation in a cavity but *without making any use of classical electrodynamics* [Einstein, 1916a]. He used Boltzmann's principle to write the probability W_n for an atom to be in a stationary state with quantum number n in the form

$$W_n = g_n \exp\left(-\frac{\epsilon_n}{k\,T}\right), \tag{18}$$

g_n being the statistical weight of the state. He then assumed that a stationary state n may pass to a stationary state m of higher energy ($\epsilon_m > \epsilon_n$) by absorbing a light-quantum of frequency $\nu_{n\,m}$, the rate of transition being $B_n^m\, N_n\, \rho_\nu$ (N_n being the number of atoms in the stationary state n and B_n^m a proportionality constant characterizing the absorption of radiation of frequency $\nu_{n\,m}$ in the presence of external radiation of density ρ_ν). Transitions from a stationary state m of higher energy to a stationary state n of lower energy by the emission of a light-quantum of frequency $\nu_{m\,n} = \nu_{n\,m}$ could, however, take place in *two* ways. Atoms can emit this radiation *independent* of the external field, their number being $A_m^n\ N_m$ (A_m^n being the constant of proportionality). About this Einstein said : 'One can hardly think of it in any other way except as a radioactive reaction.' It came to be known later as 'spontaneous emission.' Atoms can also be *induced* to emit radiation by the external field, the rate being $B_m^n\, N_m\, \rho_\nu$. Einstein

called this 'negative radiation'. In thermal equilibrium one must have $(A^n_m N_m + B^n_m N_m \rho_\nu) = B^m_n N_n \rho_\nu$. Using (18) to calculate N_n/N_m, one gets

$$A^n_m g_m + \rho_\nu \left[B^m_n g_n \exp\left(\frac{\epsilon_m - \epsilon_n}{kT}\right) - B^n_m g_m \right]. \tag{19}$$

At this stage Einstein had to take his guidance from classical theory by essentially applying the 'correspondence principle' according to which classical theory should be a limiting case of quantum theory. In the classical limit ρ_ν goes to infinity as the temperature T goes to infinity for fixed ν (Rayleigh-Jeans law, $\rho_\nu \propto T$). This gave $B^n_m g_m = B^m_n g_n$. With this condition (19) could be written as

$$\rho_\nu = \frac{A^n_m / B^n_m}{\exp\left[(\epsilon_m - \epsilon_n) / kT\right] - 1}. \tag{20}$$

Then Einstein used Wien's displacement law to infer that A^n_m/B^n_m must be proportional to ν^3, and the Rayleigh-Jeans law in the limit of low frequencies to determine the constant of proportionality to be $8\pi h/c^3$. Thus he obtained the relation

$$A^n_m = \frac{8\pi h \nu^3}{c^3} B^n_m \tag{21}$$

and

$$\epsilon_m - \epsilon_n = h\nu \tag{22}$$

which is Bohr's frequency condition. These two relations when substituted into (20) gave Planck's law. This is a remarkable derivation that uses an admixture of the light-quantum hypothesis and Bohr's atomic theory which Bohr regarded as being imcompatible, Wien's radiation law and therefore implicitly the Maxwell-Boltzman distribution for the light-quanta, and the correspondence principle which was a heuristic principle without any fundamental basis. Einstein concluded by saying : 'I admit freely, of course, that the three hypotheses concerning outgoing and incoming radiation, do not at all become substantiated results by the mere fact that they lead to Planck's radiation formula. But the simplicitly of the hypotheses, the generality with which the consideration can be carried through easily, as well as the natural connection of the consideration employed with the limiting case of Planck's linear oscillator (in the sense of classical electrodynamics and mechanics), persuaded me to regard it as very probable that all this constitutes the fundamental outline of the future theoretical derivation.' Although the hypotheses concerning the outgoing and incoming radiation turned out to be correct to a large extent (though not entirely so, as we shall see later), the future derivation (given by Bose) did not make use of them at all. In fact, they became the

major bone of contention between him and Bose.

By 1916 the light-quantum hypothesis had received a measure of empirical support from Millikan's verification of Einstein's photo-electric equation. Millikan's paper [1916b] appeared in the March 1916 issue of the *Physical Review*. Einstein quickly realized from his new derivation of Planck's law that atoms seemed to interact with radiation as if single atoms collided with light-quanta of energy $h\nu$ and directed momentum $h\nu/c$. He wrote to his friend, Michael Besso :

> The fundamental thing is that the *statistical* consideration, which leads to Planck's formula, has become *unified* and thereby as general as one can imagine, since one has not assumed anything about the nature of the mediating molecules other than the most general quantum idea. From it follows the result (which was not contained in the paper I sent you) that in each energy transfer from radiation to matter the momentum $h\nu/c$ is also transferred to the molecule. Hence we conclude that every such elementary process is a *completely directed* event. With that the light-quanta must be considered as being substantiated. [Einstein to Besso, 6 September 1916; Mehra and Rechenberg, 1982, **1**, *2*, 515]

However, Einstein was not really satisfied. In a subsequent paper he remarked :

> The weakness of the theory lies, on one hand, in the fact that it does not bring us closer to a connection with the undulatory theory, and, on the other hand, in the fact that it leaves the instant and direction of the elementary processes to chance. [Einstein, 1916b, 1917]

g. Reactions to the light-quantum hypothesis: discovery of the Compton effect

The majority of physicists, in fact, did not take the light-quantum hypothesis seriously, even after Millikan's work. Millikan himself remarked in his 1916 paper on the photo-electric effect : 'Yet the semi-corpuscular theory by which Einstein arrived at this equation seems at present to be wholly untenable.' The main difficulty lay, as Einstein himself realized, in accounting reasonably for the interference and diffraction phenomena observed with all kinds of radiation in terms of light-quanta. Evidence in favour of the wave theory was overwhelming and mounting. In 1912 Max von Laue, Walter Friedrich and Paul Knipping had settled the issue in favour of X-rays being electromagnetic waves of extremely short wavelengths through the phenomenon of interference of X-rays in crystals [von Laue, Friedrich and Knipping, 1912]. In 1913 Planck, Nernst, Rubens and Warburg who wished to bring Einstein to Berlin, wrote to the Prussian Ministry of Education:

> That he may sometimes have missed the target in his speculations, as, for example, in his theory of light-quanta, cannot really be held against him. For in the most exact of natural sciences every innovation entails risk. [Clark, 1972, 215]

Bohr had his own reasons for not favouring the light-quantum hypothesis though he realized from the beginning that the laws of electrodynamics had to be abandoned

for a proper description of quantum phenomena. The hypothesis ran counter to his plans to build atomic theory on the basis of the correspondence principle.

The general acceptance of the light-quantum hypothesis eventually came in 1923 through the work of Debye [1923] and Compton [1923]. Compton realized that the shift in the wavelength of X-rays scattered by atoms that he observed could not be understood in terms of the classical wave theory of scattering but only in terms of elementary processes of scattering of light-quanta by the electrons in atoms, in which both energy and momentum were conserved. Debye also arrived at the same conclusion independently. Arnold Sommerfeld who was then visiting the University of Wisconsin as Carl Schurz Professor of Physics came to know about Compton's results. On 21 January 1923 he wrote to Bohr : 'The most interesting scientific news I have learned in America is a work of Arthur Compton in St. Louis. According to it the wave theory of X-rays would finally have to be given up.' [Mehra and Rechenberg, **1**, *2*, 529] Wherever he went, Sommerfeld referred to Compton's work and its importance for the quantum theory. On his return to Germany he continued to advocate Compton's work. Several months later he wrote to Compton : 'Your discovery of the change in wavelength of X-rays also occupies the scientific community in Germany most deeply. I met Einstein and Kossel in August, and we mainly discussed your effect.' [Sommerfeld to Compton, October 1923; Mehra and Rechenberg, 1982, **1**, *2*, 530]

h. Pauli's derivation of Planck's law

Wolfgang Pauli came to know of Compton's discovery in the summer of 1923 when he came to Hamburg on several weeks' leave of absence from Copenhagen. He immediately got interested in the problem of thermal equilibrium between radiation and free electrons. Lorentz had reported in the first Solvay Conference that he could not establish the condition of thermal equilibrium for electrons having a Maxwellian velocity distribution [Lorentz's report in Langevin and de Broglie, 1912]. Pauli wished to re-examine the problem by considering reversible collisions between electrons and directed light-quanta, obeying the laws of conservation of energy and momentum. Pauli [1923] was able to show that thermal equilibrium characterized by Planck's law was possible for electrons having a Maxwellian distribution provided the probability of collisions per unit time was given by

$$A\,\rho_\nu + B\,\rho_\nu\,\rho_{\nu'}\,, \tag{23}$$

where A and B are the coefficients of Einstein's radiation theory [Einstein, 1917] and ρ_ν and $\rho_{\nu'}$ the densities of radiation before and after the collision respectively. Pauli showed that the first term was dominant for radiation satisfying Wien's law while the second term was more important for radiation of longer wavelengths satisfying the Rayleigh-Jeans law. Pauli interpreted this as the effect of interference fluctuations of classical waves. Einstein and Ehrenfest welcomed Pauli's work and clarified and generalized it to cases in which the elementary processes involved more than two light-quanta [Einstein and Ehrenfest, 1923]. In particular, they discussed the 'paradoxical' second term in (23) 'which implies that the number of the elementary processes of scattering occurring per unit time at an electron (which is at quasi rest) increases faster than being proportional to the radiation density ρ' of frequencies ν', which the

radiation quantum possesses after being modified by the elementary process.' They concluded that this term followed from a detailed considertion of all interaction processes between radiation and electrons, including the induced emission of light (Einstein's 'negative radiation').

i. Bose's derivation

While the leading European and American physicists were skeptical, even hostile to Einstein's light-quantum hypothesis, two unknown young physicists in India, M N Saha and S N Bose grasped its importance and started using it with great success. Within two years of Einstein's 1917 paper from which he drew the revolutionary conclusion that a light-quantum carried directed momentum $h\nu/c$, M N Saha, a close friend of Bose, applied it to explain the existence of radiation pressure on objects of molecular size [Saha, 1919]. On the basis of the classical theory Nicholson [1914] and Klotz [1918] had shown that radiation pressure on particles of molecular size should become totally 'evanescent'. Saha applied 'quantum theory in the place of the old continuous theory of light. Instead of assuming that "light" is spread continuously over all points of space,' he supposed that 'they are localized in pulses of energy $h\nu$ . . . Let this pulse encounter a molecule m and be absorbed by it. Then in doing so the molecule will be thrust forward with an impulsive momentum of $h\nu/c$, (c=velocity of light); for we may suppose the pulse to have the mass $h\nu/c^2$ and the momentum $h\nu/c$; the absorption of the pulse by the molecule may be taken as a case of inelastic impact, the whole momentum being communicated to the molecule. The velocity with which the molecule will move forward = $h\nu/mc$.' Saha concluded that '*radiation-pressure may exert an effect on the atoms and molecules which are out of all proportion to their actual sizes.*' This must have been the first application of Einstein's hypothesis that light-quanta carried a directed momentum $h\nu/c$.

In March 1924 Saha visited Dhaka and stayed with Bose. While teaching postgraduate students Bose had keenly felt the need for a derivation of Planck's law free of logical difficulties. Saha drew Bose's attention to the papers of Pauli [1923] and their connection to Einstein and Ehrenfest [1923] and their connection to Einstein's paper of 1917. In an interview with J Mehra [Mehra and Rechenberg, **1**, *2*, 565] Bose recalled : 'What seemed to be happening in Pauli's work was that in order to apply the quantum conditions you had to know exactly what was going to happen afterwards.' Stimulated by his discussions with Saha, Bose began to study carefully the works of Planck [1900,1910], Peter Debye [1910,1923], Einstein [1917], Arthur Compton [1923], Pauli [1923] and Einstein and Ehrenfest [1923]. This resulted in two papers that he wrote in June 1924 [Bose, 1924a, 1924b] and sent to Einstein for his views and for publication in *Zeitschrift für Physik*. Einstein translated both of them into German and had them published in *Zeitschrift für Physik* in 1924. To both of them he added his own comments.

It is clear from these two papers of Bose which he always regarded as integral parts of each other that he clearly recognized the following points :

(1) The fundamental assumptions of the quantum theory are incompatible with the laws of classical electrodynamics [Bose, 1924a].

(2) All previous derivations had made use of relation (2) 'between the density of radiation and the average energy of an oscillator, and they make assumptions

about the number of degrees of freedom of ether, which enters the above equation (the first factor on the right hand side). This factor could however be derived only from the classical theory. This is an unsatisfactory feature in all derivations and it is not surprising that efforts are made again and again to give a derivation free from this logical flaw.' [Bose, 1924a]

(3) Even Einstein's 1916/1917 derivation, although 'remarkably elegant', makes use of 'Wien's displacement law and Bohr's correspondence principle. Wien's law is based on classical theory and the correspondence principle assumes that the quantum theory agrees with the classical theory in certain limiting cases.' [Bose, 1924a].

(4) 'In all cases it appears to me,' wrote Bose [1924a] 'that the derivations are not sufficiently justified from a logical point of view. On the other hand, the light-quantum hypothesis combined with statistical mechanics *(as adapted by Planck to conform to the requirements of quantum theory)* appears sufficient for the deduction of the law independent of classical theory' (our italics). In other words, the incompatibility between Einstein's light-quanta ('disjointed quanta') and Planck's law (implying 'undisjointed quanta') can be removed by adapting the statistical mechanics of light-quanta to conform to the requirements of quantum theory, as proposed by Planck.

(5) 'The problem of thermodynamic equilibrium of radiation in the presence of material particles can however be studied using the methods of statistical mechanics, *independently of any special assumption about the mechanism of the elementary processes on which the energy exchange depends.*' [Bose, 1924b]

Bose therefore proceeded in four steps.

First step: He took Einstein's light-quantum hypothesis seriously and treated black-body radiation as a collection of light-quanta enclosed in a volume V. If there are N_s light-quanta of energy $h\nu_s$ ($s = 0 \quad s = \infty$), the total energy E is

$$E = \sum_s N_s h \nu_s = V \int \rho_\nu \, d\nu \, . \tag{24}$$

ρ_ν can be determined from this simple relation if N_s is known. So, the problem is to determine N_s. 'If we can give the probability for each distribution characterized by arbitrary values of N_s, then the solution is determined by the condition that this probability is a maximum subject to the subsidiary condition' (24) [Bose,1924a]. Bose then proceeded to find this probability.

Second step: Before he could do that, he needed to compute the number of *states* of a light-quantum whose frequency lies between ν_s and $\nu_s + d\nu_s$. In order to be able to do this, he *extended* Planck's idea of dividing up the phase space of a material oscillator into elementary cells of volume h^3 *to the phase space of light-quanta.* In other words, *he extended Planck's 'first quantization' of material oscillators to the radiation field itself.* This was the most novel and crucial input. Emboldened by Compton's discovery, he assumed the momentum of a light-quantum of frequency ν to be $h\nu/c$ in the direction of its motion. He must have also been aware of Saha's success in explaining radiation

pressure on molecular-size objects with the help of this hypothesis [Saha,1919]. The instantaneous *state* of such a light-quantum, he stated, is characterized by a point in six-dimensional phase space which is constrained to lie on the surface of a cylinder defined by

$$p_x^2 + p_y^2 + p_z^2 = \frac{h^2 \nu_s^2}{c^2} . \tag{25}$$

Then the phase space belonging to the frequency interval $d\nu_s$ is

$$\int dx\, dy\, dz\, dp_x\, dp_y\, dp_z = 4\pi V \left(\frac{h\nu_s}{c}\right)^2 \frac{h\, d\nu_s}{c} = 4\pi \frac{h^3 \nu_s^2}{c^3} V d\nu_s . \tag{26}$$

Dividing this by h^3, the 'ultimate elementary region' [Bose to Einstein, 4 June 1924], Bose obtained $4\pi V\nu_s^2\, d\nu_s/c^3$. Then the following statement occurs : 'In order to take into account the *fact* of polarization, it seems however appropriate to multiply this number once again by 2' (our italics) to obtain $8\pi V\nu_s^2\, d\nu_s/c^3$ as the number of cells belonging to $d\nu_s$. No explanation is offered as to how this 'fact of polarization', an essentially *classical* concept, can be understood in terms of the light-quantum hypothesis, although Bose claimed to deduce this factor *independent of the classical electrodynamics* [Bose to Einstein, 4 June 1924]. Bose had always maintained privately that he did offer a quantum theoretic explanation but Einstein removed it from his translation and substituted it by the statement about the polarization factor. In his letter to Bose, Einstein simply wrote : 'You are the first person to derive the factor quantum theoretically, even though because of the polarization factor 2, not wholly rigorously' [Einstein to Bose, 2 July 1924]. What was the explanation that Bose claimed to have offered? It was that *light-quanta carried an intrinsic spin that could take only the values* $\pm h/2\pi$. There is only one recorded evidence of this because Bose's original manuscript in English is missing from the Einstein archives. It appears in a paper by C V Raman and S Bhagavantam [1931]. The paper is entitled 'Experimental Proof of the Photon Spin'. They write :

> In his well-known derivation of the Planck radiation formula from quantum statistics, Prof. S.N. Bose obtained an expression for the number of cells in phase space occupied by the radiation, and found himself obliged to multiply it by a numerical factor 2 in order to derive from it the correct number of possible arrangements of the quantum in unit volume. The paper as published did not contain a detailed discussion of the necessity for the introduction of this factor, but we understand from a personal communication by Prof. Bose that he envisaged the possibility of the quantum possessing besides energy $h\nu$ and momentum $h\nu/c$ also an intrinsic spin angular momentum $\pm h/2\pi$ round an axis parallel to the direction of its motion. The weight factor 2 thus arises from the possibility of the spin of the quantum

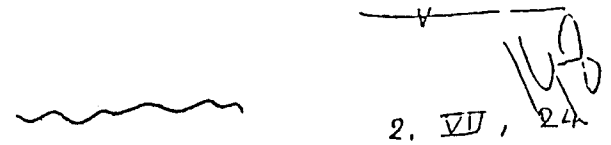
PHYSICS DEPARTMENT.
Dacca University.

Dated, the 4th June 1924.

Respected Sir, I have ventured to send you the accompanying article for your perusal and opinion. I am anxious to know what you think of it. You will see that (I have tried to deduce the coefficient $8\pi\nu^2/c^3$ in Planck's Law independent of the classical electrodynamics, only assuming that the ultimate elementary regions in the Phase-space have the content h^3. I do not know sufficient German to translate the paper. If you think the paper worth publication, I shall be grateful if you arrange for its publication in Zeitschrift für Physik. Though a complete stranger to you, I do not feel any hesitation in making such a request. Because We are all your pupils though profiting only by your teachings through the Your writings. I do not know whether you still remember that Some boy from Calcutta asked your permission to translate your papers on Relativity in English. You acceded to the request, the book has since been published. I was the one who translated your paper on Generalised Relativity

Yours faithfully
S.N. Bose

Bose's letter of 4 June 1924 to Einstein



2. VII, 24

Lieber Herr Kollege!

Ich habe ihre Arbeit über-
-setzt und der Zeitschrift für
Physik zum Druck übergeben.
Sie bedeutet einen wichtigen
Fortschritt und hat mir
sehr gut gefallen. Ihre
Einwände gegen meine Arbeit
finde ich zwar nicht richtig.
Denn das Wien'sche Ver-
schiebungsgesetz setzt die
undulations Theorie nicht voraus
und das Bohr-che korrespondenz-
prinzip ist überhaupt nicht
verwendet. Doch dies thut
nichts. Sie haben als erster
den Faktor quantentheoretisch
abgeleitet wenn auch wegen
des Polarisations-Faktors 2
nicht ganz streng. Es ist
ein schöner Fortschritt.

Mit freundlichen Gruss

(Sd) Ihr A Einstein.

Einstein's postcard of 2 July 1924 to Bose

> being right-handed or left-handed, corresponding to the two alternative signs of the angular momentum. There is a fundamental difference between this idea, and the well-known result of classical electrodynamics to which attention was drawn by Poynting and more fully developed by Abraham that a beam of light may in certain circumstances possess angular momentum . . . Thus, according to the classical field theory, the angular momentum associated with a quantum of energy is not uniquely defined, while according to the view we are concerned with in the present paper, the photon has always an angular momentum having a definite numerical value of a Bohr unit with one or other of the two possible alternative signs.

Why did Einstein remove Bose's explanation? Probably he felt it was too radical and unnecessary at that stage. However, instead of removing it altogether, he could have added a suitable comment to the one he did make at the end of the paper. In fact, he did not hesitate to add a dissenting note to Bose's second paper [1924b] as we shall see later.

Be that as it may, let us proceed with Bose's derivation. Having divided the phase space volume belonging to the frequency range ν_s and $\nu_s + d\nu_s$ by h^3, he wrote : *'the total number of cells must be regarded as the number of possible arrangements of a quantum in a given volume'* (our italics). Classically there are infinitely many ways of arranging or distributing a particle within a phase space cell of finite extent corresponding to the infinitely many points contained within it. However, according to Planck, as we have seen, quantum states are not continuously distributed in phase space but are separated from one another by finite amounts determined by h. Thus, *an elementary cell of volume h^3 can contain at most one quantum state*, and the problem of counting the number of quantum states reduces to the counting of these elementary cells. This is a totally new interpretation of the factor $A_s = 8\pi\,\nu_s^2\,V d\nu_s/c^3$. (In classical wave theory it is the number of transverse standing modes in the interval $d\nu_s$ in a volume V.) In his covering letter to Einstein Bose wrote :

> *Respected Sir,*
>
> *I have ventured to send you the accompanying article for your perusal and opinion. I am anxious to know what you think. You will see that I have tried to deduce the coefficient in Planck's law independent of the classical electrodynamics, only assuming that the ultimate elementary regions in Phase-space have the content h^3.*
>
> [Bose to Einstein, 4 June 1924].

On 12 July 1924 Einstein wrote to Ehrenfest : 'The Indian Bose has given a beautiful derivation of Planck's law, including the constant [i.e. $8\pi\nu^2/c^3$].'

Third Step: Having demonstrated that the factor $8\pi\nu^2/c^3$ in Planck's law corresponds to the total number of *quantum states* of radiation, Bose then proceeded 'to calculate *the thermodynamic probability (macroscopically defined) of a state*' in the sense of Planck (our italics). This is equal to the number of different microscopically defined states by means of which the macroscopic state can be realized. If all the cells (states) were considered to be distinct from one another, the answer would be A_s! However, this

is not the case. *Since each cell is to be counted as a single quantum state,* if follows that the permutations of the light-quanta within a cell do not produce new states. *Consequently, the cells can be partitioned into distinct classes characterized solely by their occupation numbers*. Let there be a particular distribution of N_s quanta among A_s states such that there are p_1^s states that are empty, p_2^s states with one quantum, p_2^s states with two quanta, and so on. Then, *following Boltzmann* and the usual procedure of statistical mechanics for these *states (not the quanta)*, one can immediately write down the possible number of such distributions as

$$W_s = \frac{A_s\ !}{p_o^s\ !\ p_1^s\ !\ p_2^s\ !\ \ldots} \tag{27}$$

which is precisely the expression (12) given by Natanson. The 'thermodynamic probability' of 'the state defined by all p_r^s' is then clearly $W = \prod_s W_s$ which is equivalent to the Debye-Planck expression (10).

Fourth step : Bose then followed the standard procedure, already followed by Debye [1910], of maximizing *ln W* subject to the constraints $E = \sum_s N_s h\nu_s$ and $N_s = \sum_r r p_r^s$ to obtain Planck's law. Notice that the total number of photons $N = \sum_s N_s$ and *there is no constraint on N*. It turns out that the constraint on N_s is not necessary, but Bose did not notice this.

Notice that the only departure of the expression (27) from the usual Boltzmann expression lies not in the product of factorials in the denominator but in putting the multiplicative factor $(g_i)^r$ corresponding to the possible arrangements of r quanta within each cell i having g_i levels equal to unity, as demanded by the division of phase space into elementary cells of volume h^3. (In his second paper [1924b] written more or less at the same time as the paper under discussion and received by Einstein only five days later, Bose used these factors, as we shall see later, for material particles obeying Maxwell-Boltzmann statistics.) This is precisely the adaptation of statistical mechanics by Planck 'to conform to the requirements of the quantum theory' that Bose referred to in the third paragraph of his paper.

Was Bose aware of the analysis of the Planck-Debye definition of the probability by Natanson and by Ehrenfest and Kamerlingh Onnes? One cannot tell. He certainly did not refer to their papers. It is reasonable, however, to assume that he was not aware of the latter work because if he had been, he would have explained how their conclusion, namely that *'Planck's formal device* (distribution of P energy elements ϵ among N resonators) *cannot be interpreted in the sense of Einstein's light-quanta'* [Ehrenfest and Kamerlingh Onnes, 1914, p.873], could be reconciled with his own starting point, namely the combination of the light-quantum hypothesis with Planck's counting device. The explanation, of course, lies in the fact that Planck's law reduces to Wien's law in the limit $h\nu/kT >> 1$, and that it is only in this limit that the light-quanta have Maxwell-Boltzmann behaviour as inferred by Einstein. It is one of those quirks of history that Einstein should have been able to infer the quantum nature of radiation from its approximate classical statistical behaviour in a certain domain and thus missed the true nature of its statistics. It was left to Bose to discover it, possibly

uninhibited by the analysis of Ehrenfest and Kamerlingh Onnes and their conclusion. Hence Einstein's enthusiastic welcome of Bose's work as an important and new support to his favourite light-quantum hypothesis, making it compatible with Planck's law.

j. The birth of a new statistics: Einstein applies Bose's method to ideal gases

Bose's counting of states, of course, implied a *new* statistics for the light-quanta. Neither he nor Einstein seemed to have quite grasped the importance of this initially. In his letter of 2 July 1924 to Bose, Einstein compliments him for being the first to derive the factor A_s quantum theoretically but does not write a word about the new statistics. 'In my opinion,' wrote Einstein in a footnote to Bose's paper, 'Bose's derivation signifies an important advance. The method used here gives the quantum theory of an ideal gas as I will work out elsewhere.' He extended Bose's method to material particles in three communications to the Prussian Academy in Berlin on 10 July 1924, 8 January 1925 and 29 January 1925 [Einstein, 1924, 1925a, 1925b] without informing Bose or asking for his collaboration. Bose came to know of the comment in the footnote shortly before he left for Europe and of the first of the communications to the Prussian Academy on his arrival in Europe in October 1924. In the first paper this is how Einstein described Bose's method :

> [First] : The phase space of an elementary object (here of a monatomic molecule and in Bose's case of a light-quantum), associated with a given (three-dimensional) volume, is divided into "cells" of extension h^3. [Second] : If many elementary objects are present then their (microscopic) distribution, which plays a role in thermodynamics, is determined by the particular manner how the elementary objects are distributed among the cells. [Third] : The "probability" of a macroscopically defined state (in the sense of Planck) is equal to the number of different microscopic states by means of which the macroscopic state can be thought to be realized. [Fourth] : The entropy of the macroscopic state and, therefore, the statistical and thermodynamic behaviour of the system is determined by Boltzmann's theorem relating entropy and probability.' [Einstein, 1924, p.261]

Again, there is no mention of the new statistics! In applying Bose's method to ordinary material atoms, Einstein had to introduce an additional parameter (the chemical potential which vanishes for light-quanta) to take into account the conservation of their number. He drew a very important conclusion from this:

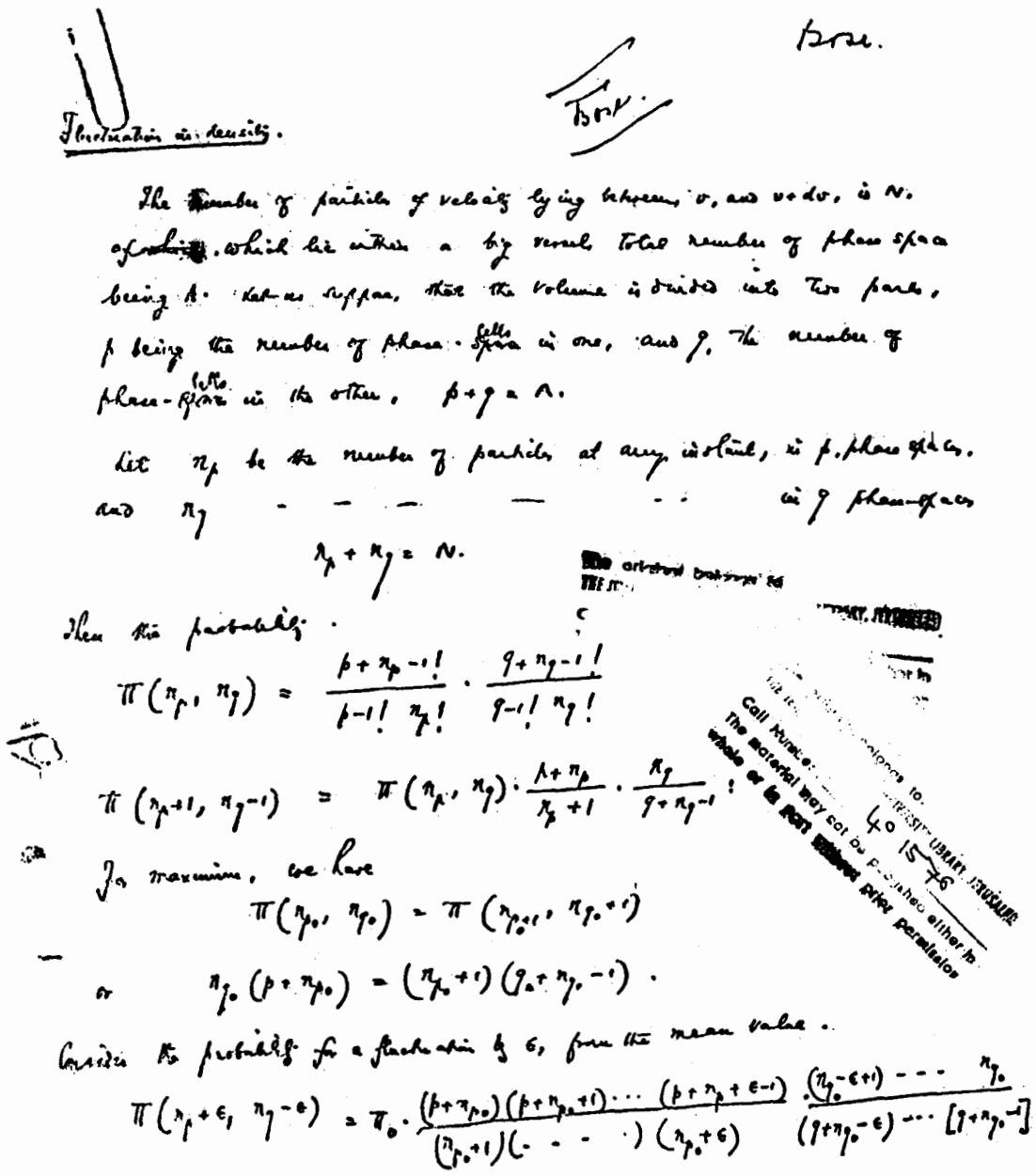
> According to the theory presented here, Nernst's theorem is satisfied in the case of ideal gases. To be sure, our formulae cannot immediately be applied to extremely low temperatures, for we have assumed in their derivation that the p_r^s change only relatively infinitely little if s is altered by 1. Still one recognizes at once that the entropy must vanish at the absolute zero of temperature. The reason is that then *all molecules are in the first cell; and for this state there exists only one distribution of molecules according to our counting method.* Hence our assertion is immediately proved to be correct. [Einstein, 1924, p.265] (our italics)

This is a conclusion of far-reaching consequence, yet there is no mention of any new statistics for the molecules; the emphasis is on Bose's counting method for *states* (one quantum state per cell). It was only six months later on 8 January 1925 that Einstein discussed the difference between the counting methods of Boltzmann and Bose and recognized the fact, already pointed out by Ehrenfest [1924], that *the new statistics for molecules was inconsistent with their statistical independence*. However, while Planck [1925] considered the analogy with radiation a weak point in Einstein's gas theory, the statistics of radiation and that of material particles being completely different in his opinion, Einstein regarded this result differently. He concluded : 'This result represents in itself a support of the view concerning the *deep natural relation between radiation and gas, since the same statistical treatment which leads to Planck's formula establishes — when applied to ideal gases — the agreement with Nernst's theorem.*' [Einstein, 1925a, p.7] (our italics) He was of the opinion that an understanding of the 'mysterious influence' among the molecules would come in the future.

k. The birth of wave mechanics

Based on the new counting method, he showed in his second paper communicated on 8 January 1925 that the mean square energy fluctuation of the gas molecules is given by an expression which is the sum of two terms, one corresponding to the Maxwell-Boltzmann statistics of non-interacting molecules and the other to interference fluctuations associated with wave phenomena. (There exists in the Einstein Archives in the Jewish National and University Library, Jerusalem, a two page calculation done by Bose entitled 'Fluctuation in density' which ends with this fundamental result. The document is undated and exists in the Scientific Correspondence File Folder 'B-Misc.-II'. Bose was in Paris from 18 October 1924 until the first part of October 1925.) From this he immediately drew attention to the importance of Louis de Broglie's Doctoral thesis (which he had heard of from Paul Langevin and asked for a copy which he received and read in December 1924 [Jammer, 1966, 249]) in which de Broglie had attached wave properties to ponderable matter in analogy with the wave-particle duality of radiation. Einstein wrote : 'I shall discuss this interpretation in greater detail because I believe that it involves more than a mere analogy.' He then went on to explore various consequences of de Broglie's hypothesis.

When Erwin Schrödinger whose principal research during his early years in Zurich (1921-27) was on the statistical thermodynamics of ideal gases, read Einstein's first paper [1924a] on the quantum theory of an ideal gas, he also did not understand that the molecules were being treated as being indistinguishable. On 5 February 1925 he wrote to Einstein suggesting a possible error in his probability formula. Einstein replied explaining that the 'quanta or molecules are not treated as independent of one another' and even gave a little diagram to explain the difference between Bose's counting method and that of Boltzmann. He concluded his letter with the emphatic statement: 'There is certainly no error in my calculation.' [Walter Moore, 1989, 183] It was Einstein's remark in his second paper that de Broglie's idea 'involves more than a mere analogy' that really induced Schrödinger to study the 'de Broglie-Einstein undulatory theory' as he called it at that time, and eventually led to his discovery of wave mechanics in 1926 [Schrödinger, 1926a, 1926b and 1926c]. Schrödinger later said that


Fluctuation in density.

Bose.

Bose.

The number of particles of velocity lying between v, and $v+dv$, is N. of which lie either a by usual total number of phase space being h. Let us suppose, that the volume is divided into Two parts, p being the number of phase-cells in one, and q, the number of phase-cells in the other, $p+q = h$.

Let n_p be the number of particles at any instant, in p. phase cells, and n_q - - - - - - - - in q phase-cells

$$n_p + n_q = N.$$

Then the probability.

$$\pi(n_p, n_q) = \frac{p+n_p-1!}{p-1!\ n_p!} \cdot \frac{q+n_q-1!}{q-1!\ n_q!}$$

$$\pi(n_p+1, n_q-1) = \pi(n_p, n_q) \cdot \frac{h+n_p}{n_p+1} \cdot \frac{n_q}{q+n_q-1}$$

In maximum, we have

$$\pi(n_{p_0}, n_{q_0}) = \pi(n_{p_0+1}, n_{q_0-1})$$

or $$n_{q_0}(p+n_{p_0}) = (n_{p_0}+1)(q+n_{q_0}-1).$$

Consider the probability for a fluctuation of ϵ, from the mean value.

$$\pi(n_p+\epsilon, n_q-\epsilon) = \pi_0 \cdot \frac{(p+n_{p_0})(p+n_{p_0}+1)\cdots(p+n_p+\epsilon-1)}{(n_{p_0}+1)(\cdots)(n_{p_0}+\epsilon)} \cdot \frac{(n_{q_0}-\epsilon+1)\cdots n_{q_0}}{(q+n_{q_0}-\epsilon)\cdots[q+n_{q_0}-1]}$$

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Calculation of 'Fluctuation in density' in Bose's hand (Scientific Correspondence File Folder 'B-Misc.-II'); by courtesy of The Jewish National and University Library, Jerusalem.

$$\log \pi\left(n_{p_0}+\epsilon,\ n_{\gamma_0}-\epsilon\right) - \log \pi_0$$

$$= \frac{(\epsilon-1)(\epsilon)}{2}\left[\text{by } \frac{1}{p+n_{p_0}} - \frac{1}{n_{p_0}+1}\right] + \frac{\epsilon(\epsilon-1)}{2}\left[\frac{1}{q+n_{\gamma_0}-1} - \frac{1}{n_{\gamma_0}}\right]$$

$$\text{or} \quad \pi\left(n_{p_0}+\epsilon,\ n_{\gamma_0}-\epsilon\right) = \pi_0 \cdot e^{-\frac{\epsilon^2}{2}\left[\frac{p-1}{(p+n_{p_0})(n_{p_0}+1)} + \frac{q-1}{n_{\gamma_0}(q+n_{\gamma_0}-1)}\right]}$$

$$\therefore \quad \overline{\Delta\epsilon^2} = \frac{1}{\frac{p-1}{(p+n_{p_0})(n_{p_0}+1)} + \frac{q-1}{n_{\gamma_0}(q+n_{\gamma_0}-1)}}.$$

when, the volume containing p phase cells is supposed to be very small compared with the other. $\frac{q-1}{n_{\gamma_0}(q+n_{\gamma_0}-1)}$ can be neglected.

$$\therefore \quad \frac{\overline{\Delta\epsilon^2}}{n_{p_0}^2} = \frac{(p+n_{p_0})\,n_{p_0}}{p\,n_{p_0}^2} = \frac{p+n_{p_0}}{p\,n_{p_0}} = \left(\frac{1}{p}+\frac{1}{n_{p_0}}\right)$$

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'wave mechanics was born in statistics.' (our italics) [Walter Moore, 1989, 188] He also wrote to Einstein on 23 April 1926 : 'By the way, the whole thing would not have started at present or at any other time (I mean, as far as I am concerned) had not your second paper on the degenerate gas directed my attention to the importance of de Broglie's ideas.' [Jammer,1966, 257]

1. Further developments

Meanwhile in July 1925 Werner Heisenberg had sent a paper to *Zeitschrift für Physik* in which he gave a preliminary account of matrix mechanics [W Heisenberg,1925]. Schrödinger proved the complete equivalence of his wave mechanics and the Heisenberg–Born–Jordan matrix mechanics [Schrödinger, 1926d]. A whole train of rapid developments occurred. Pauli discovered his Exclusion Principle within six months of Bose's paper [Pauli, 1925]; Fermi published his paper on the statistics of particles obeying Pauli's principle in early 1926 [Fermi, 1926]; Dirac linked the Bose and Fermi statistics of particles to the symmetry properties of their wave functions and named them 'bosons' and 'fermions' [Dirac, 1926]; in 1927 Dirac invented 'second quantization' of the electromagnetic field [Dirac, 1927] which was soon followed by a similar method for Fermi fields developed by Jordan and Wigner [1928]; later Pauli [1940] proved the spin-statistics theorem in relativistic quantum field theory; Pauli [1927] also applied Fermi-Dirac statistics to the paramagnetism of an electron gas and Sommerfeld [1927,1928] applied it to the electrons in a metal; and Fritz London [1938] suggested that the superfluidity of liquid helium (^{4}He) was related to Einstein-Bose condensation. Thus the developments of Schrödinger's wave mechanics and that of quantum statistical mechanics were directly triggered off by Bose's first paper followed by Einstein's far-sighted extension of his method to ideal gases of *material* particles and his championing of de Broglie's ideas which soon received experimental confirmation in the hands of Davisson and Germer [1927] and G P Thomson [1927].

Years later this is how Schrödinger summed up the position :

> The two equivalent ways of looking at (7.19) either as counting the number of quantum states of a particle, or as counting the number of wave-mechanical proper vibrations of the enclosure, interest us for this reason. The second attitude makes us think of the 'n_s particles present in state α_s' as of a proper vibration (or a 'hohlraum' oscillator to use a customary phrase) in its n_sth quantum level. (This attitude really corresponds to so-called second quantization or field quantization.) n_s becomes a quantum number and the stipulation that the system of quantum numbers
>
> $$n_1, n_2, n_3, \ldots, n_s, \ldots,$$
>
> determines only one state of the gas, not a class of
>
> $$\frac{n!}{n_1!\, n_2! \ldots .\, n_s! \ldots}$$
>
> states, ceases to be a strange new adoption, and comes into line with the ordinary view about quantum states and their statistical weight (viz. equal

for any two of them).

It is the first, the particle attitude, that has suggested the term 'new statistics' which is frequently used. And that is why this idea of new statistics did not, originally, arise in connexion with heat radiation, because here the wave point of view was the historical one, the classical one—nobody thought of any other at the outset. The wave picture was considered to be (and historically was) the classical description. The quantization of the waves therefore duly appeared to be a 'first' quantization and nobody thought of anything like 'second quantization'.

Not until the idea of photons had gained considerable ground did Bose (about 1924) point out that we could, alternatively to the 'hohlraum' oscillator statistics, speak of photon statistics, but then we had to make it 'Bose statistics'. Very soon after, Einstein applied the same to the particles of an ideal gas. And thereupon I pointed out that we could also in this case speak of ordinary statistics, applied to the wave mechanical proper vibrations which correspond to the motion of the particles of the gas [Schrödinger, 1952, 49-50].

We wish only to point out, as explained above, that Bose did not actually 'speak of photon statistics' — it was *implied* by his counting of quantized photon *states*, and this only became gradually clear with Einstein's application of Bose's counting method to ideal gases, although Ehrenfest and Natanson had already pointed this out in 1911 and 1914 respectively in relation to Planck's counting method of December 1900 which Bose showed was applicable to light-quanta.

m. Did Bose make a 'shot in the dark'?

Some recent widely read scientific biographies have given rise to the impression that Bose made 'a shot in the dark' with three assumptions all of which turned out to be correct but none of which he tried to justify 'by arguments of any kind; they seemed to appear intuitively to his mind, perhaps because his thoughts were uncluttered by the ongoing controversies of the European physicists.' [W. Moore, 1989, 181; A. Pais, 1982, 428] The assumptions were that photons were (a) massless particles capable of existing in two states of polarization, (b) indistinguishable and (c) their number was not necessarily conserved. The record must be put right. That photons are massless particles with energy $h\nu$ and directed momentum $h\nu/c$ was proposed by Einstein and only used by Bose. That they exist in two possible 'polarization states' was *inferred* by Bose from phase space considerations which yield for the number of quantum states between ν_s and $\nu_s + d\nu_s$ the factor $4\pi V\nu_s^2\, d\nu_s/c^3$ rather than $8\pi V\nu_s^2\, d\nu_s/c^3$, the factor that occurs in Planck's formula. Actually, as we have already discussed, in order to account for this factor of 2, Bose had proposed that photons carried one unit of intrinsic spin which could only take the values ± 1. Whoever had heard of particles with polarization? Moreover, 'polarization' was a wave concept which Bose wanted to avoid. However, Einstein quietly dropped it and the statement regarding the polarization factor was most probably inserted by him. Since Bose's original manuscript in English

is missing from the Einstein archives and Bose did not keep a copy, there is no way to verify this.

How about 'indistinguishability'? Again, as we have seen, although Bose did not explicitly use the word 'indistinguishable' which is by itself rather vague, he did use Planck's division of phase space into elementary cells of volume h^3 to justify his counting method for *states* (which implied indistinguishability of the quanta), namely that there cannot be more than one quantum state in an elementary cell. 'In any case,' he wrote, 'the total number of cells must be regarded as the number of possible arrangements of a quantum in the given volume.' What is not usually appreciated in usual expositions of Bose statistics is that Bose did not count the number of ways of distributing individual photons over a set of states. Instead, he calculated the *thermodynamic probability* of a *macroscopic state* by counting the number of ways in which the macroscopic state can be thought to be realized through given distributions of the number of photons over the micro-states (phase space cells).

Finally, what about non-conservation of photons? This is a non-issue. Since, like Debye [1910], Bose considered the statistical thermodynamics of an *open* system of radiation in thermal equilibrium and did not include the molecules of matter with which this radiation constantly exchanged energy (which he did in his second paper which we shall turn to presently), there was no question of conservation of energy quanta even if treated as massless particles. In his 1917 derivation of Planck's law Einstein did not impose the conservation of light-quanta which he certainly regarded as massless particles interacting with molecules. Since Bose was not doing anything new or different in this respect, there was no original or deep comment he could have made on this issue. Nevertheless, it is true that Bose, like most others who have made revolutionary contributions, did not, and could not be expected to, understand the full implications of their discoveries.

n. Bose's second paper (1924)

We now turn to his second paper [1924] on *'Thermal Equilibrium in Radiation Field in the Presence of Matter'*, also translated into German probably by Einstein himself. This paper, completed on 14 June 1924, i.e. more or less at the same time as the first paper, consists of two parts. In the first part Bose derived general conditions for statistical equilibrium of a system consisting of matter and radiation, *independent of any special assumptions about the mechanism of the elementary radiative processes.* In the second part he proposed a new expression for the probability of these elementary radiative processes which differed from that of Einstein. Einstein considered this hypothesis not to be 'applicable' to elementary radiative processes and gave two reasons for it in a footnote to the paper. As a consequence, the paper has been largely ignored. However, Bose remained firmly of the opinion for the rest of his life that Einstein had not done him justice.

Bose started off by giving a quick critical review of the most important derivations of Planck's law until then given by Debye [1910], Einstein [1917], Pauli [1923], and Einstein and Ehrenfest [1923] on the scattering of radiation by electrons. Bose considered Debye's 1910 derivation not to be 'completely independent of classical electrodynamics' since he derived the factor $8\pi\nu_s^2 V d\nu_s/c^3$ from the normal modes of the ether.

He considered the derivations of Einstein [1917], Pauli [1923] and Einstein and Ehrenfest [1923] to be dependent on *ad hoc* special assumptions about the probability of elementary radiative processes. 'The problem of thermodynamic equilibrium of radiation in the presence of material particles,' he wrote, 'can however be studied using the methods of statistical mechanics, independently of any special assumption about the mechanism of the elementary processes on which the energy exchange depends. In this way we obtain a general relation which is valid for all special assumptions about the elementary processes and their probabilities.' This is in keeping with Kirchhoff's law. He then went on to say: 'If it is possible to write down the thermodynamic probability for any particular distribution of quanta of radiation and the probability for any arbitrary energy distribution among the particles, then the thermodynamic probability for the bigger system consisting of matter and radiation is simply the product of the two probabilities. The condition of equilibrium is simply that the resulting probability must be a maximum.' For the thermodynamic probability for radiation, Bose used the expression

$$W = \prod_{\nu} \frac{(A_\nu + N_\nu\, d\nu)\,!}{A_\nu\,!\, N_\nu\, d\nu\,!} \tag{28}$$

where $A_\nu = 8\pi V\nu^2\, d\nu/c^3$ which, he stated, had been derived earlier. As references he quoted his own paper and that of Debye [Debye 1910]. (It is curious that the reference to his previous paper is given as 'The author, to appear in *Phil Mag*'. This is because Bose had submitted his paper first to the *Philosophical Magazine*. Not hearing from them for a while, he sent a copy to Einstein. While translating Bose's second paper, the translator obviously forgot to change the reference of the earlier paper to *Zeitschrift für Physik* **26**, 178-181, 1924). Although expression (28) is indeed the Ansatz used by Debye, it does not appear in Bose's first paper! Instead, the corresponding expression that appears in his first paper is (27), the form given by Natanson. As we have seen earlier, the two are mathematically equivalent, but Bose did not care to explain this, probably because this result must have been fairly well known in the literature in those days.

For the thermodynamic probability for material particles. Bose writes:

> This can be found easily. We want to generalize our assumptions a little so that the case of the Bohr atom with discrete energy levels as well as the case of translational energy of particles can be included. *Let the phase space be divided into cells. For every cell there is a probability g that a particle occupies it. The g's are in general equal except for the case of Bohr's atoms. The thermodynamic probability for any arbitrary distribution* n_1, n_2 etc among the different cells is,

$$\frac{N\,!\; g_1^{n_1}\; g_2^{n_2}\; \ldots}{n_1\,!\; n_2\,!\; \ldots} \tag{29}$$

(our italics). Unlike in the case of radiation (expression (27), Bose now uses the factor $\prod_i g_i^{n_i}$ for material particles and obtains the classical Maxwell-Boltzmann distribution. In other words, he treats material particles as in classical (*distinguishable*) statistical mechanics but light-quanta according to quantum (*indistinguishable*) statistical mechanics.

Finally, he writes down the thermodynamic probability for the total system as the product of (28) and (29) :

$$W = \prod_s \frac{(A_s + N_s)!}{A_s!\, N_s!} \prod_t \frac{g_T^{n_T} N!}{n_T!} \tag{30}$$

with

$$\Sigma N_s h\nu_s + \Sigma n_T E_T = E \tag{31}$$

and

$$\Sigma N_T = N. \tag{32}$$

He then considers an elementary process in which a particle passes from the rth cell to the sth cell while a light-quantum of frequency ν changes into a light-quantum of frequency $\nu.'$ The stationarity of W subject to the conditions (31) and (32) gives

$$\frac{n_r}{g_r} \prod_\nu \frac{N_\nu}{N_\nu + A_\nu} = \frac{n_s}{g_s} \prod_{\nu'} \frac{N_{\nu'}}{N_{\nu'} + A_{\nu'}} \tag{33}$$

where

$$\Sigma\, h\nu' - \Sigma h\nu + E_s - E_r = 0\,. \tag{34}$$

Bose then demonstrated that (33) indeed generalized the results of Pauli [1923] and Einstein and Ehrenfest [1923] without the need for any *ad hoc* assumptions regarding the elementary radiative processes designed to give Planck's formula. In particular, in the case of Bohr's atoms considered by Einstein in 1917, equation (33) reduces to

$$\frac{n_r}{g_r} \frac{N_\nu}{N_\nu + A_\nu} = \frac{n_s}{g_s} \tag{35}$$

which is Einstein's condition derived on the assumption that atoms in higher energy levels make transitions to lower energy levels in two ways:

1. A spontaneous transition as in radioactivity *independent of the state of the external radiation field.*
2. An induced transition whose probability depends on the state of the external radiation field.

Transitions from lower to higher energy levels always take place by induced absorption whose probability depends on the state of the external radiation field. Further, Einstein had to postulate certain relations (namely, the opposite limiting Wien and Rayleigh-Jeans laws) among these transition probabilities to obtain Planck's law. Bose's derivation is independent of all these assumptions. It is for this reason that Einstein took the trouble to have the paper translated into German and communicated it to *Zeitschrift für Physik*. However, his disagreement with Bose started from the *next* part of the paper in which Bose proposed a new expression for the probability of an interaction between a particle and a quantum of radiation.

Bose's starting point was the following observation which he considered to be fundamental :

> *. . . even in a collision no interaction is as probable as the occurrence of any special interaction. . . From the classical theory. . . one would expect that there is some interaction whenever a quantum and a material particle come together. Therefore here it is a question of a departure from classical theory.*
>
> The assumption made above, that even in a collision no interaction need occur, is quite analogous to the assumption of the stability of stationary states which is so fundamental to Bohr's theory of line emission and can be traced back to the same origin—*the probability of existence of the stationary states of the particles*. It is interesting to remark in this connection that in the analogous case of a collision of an electron with an atom, experiments show that the electron goes through the atom without changing either the interatomic or its own motion. (our italics)

Bose was obviously referring to the Ramsauer–Townsend effect. He saw a simple way of realizing the non-classical possibility in Planck's cellularization of phase space. Let p_r be the number of cells with r quanta. 'In order that the radiation and particle may interact, it must be in a cell occupied by a quantum. The particular cell which the molecule under consideration occupies will pass through all possible states if we observe it long enough, that is, sometimees it is empty, sometimes occupied by 1 quantum, sometimes by 2 quanta etc. The duration of these events will be finally proportional to the equilibrium values of p_o, p_1 etc.*When r quanta and a particle are together, then either an exchange of energy takes place or nothing happens.*' (our italics) Therefore $r + 1$ different events are possible, namely the exchange of 1 quantum of energy, 2 quanta etc or no exchange at all. Therefore the total number of possible events is

$$p_o + 2p_1 + 3p_2 + \ldots = A_\nu + N_\nu d\nu_s = \sum_r (r+1)\, p_r . \tag{36}$$

The number of cases in which interaction or energy exchange occurs is

$$p_1 + 2p_1 + 3p_3 + \ldots = N_s d\nu_s = \sum_r r p_r . \tag{37}$$

Consequently the probability of an interaction is

$$P = \frac{\Sigma r p_r}{\Sigma (r+1) p_r} = \frac{N_s \, dv_s}{A_s + N_s \, dv_s} . \tag{38}$$

This is Bose's second fundamental result.

Having derived this result, Bose then considered Einstein's 1917 problem, namely Bohr atoms exchanging energy with the radiation field. In order to be consistent with the fundamental equilibrium condition (35), he found it was sufficient to consider (a) transitions from lower to higher states through absorption of quanta of characteristic frequency ν and (b) transitions from higher to lower states to be spontaneous, i.e. independent of the external radiation field. *The additional assumption of induced emission (negative radiation) processes was not necessary*. Let the strength of induced absorption be β. Then the probabillity of induced absorption is

$$\beta \frac{N_\nu \, d\nu}{A_\nu + N_\nu \, d\nu} . \tag{39}$$

Let the coefficient of spontaneous emission be α. Then for equilibrium one gets

$$n_r \beta \frac{N_\nu \, d\nu}{A_\nu + N_\nu \, d\nu} = \alpha \, n_s \tag{40}$$

which agrees with (35) provided $g_r \beta = g_s \alpha$. A substitution of the value of n_r / n_s then gives Planck's law. Bose then treated the cases considered by Pauli [1923] and Einstein and Ehrenfest [1923] and showed that the fundamental equilibrium condition (33) followed from the probability law (38) without requiring any further assumptions.

In a note added to the paper Einstein gave two reasons why he thought Bose's hypothesis about the probability of elementary radiative processes was not 'applicable'. In his letter of 3 November 1924 Einstein summarized the two reasons. 'Your principle is not compatible with the following two conditions:

1) The absorption coefficient is independent of the radiation density.
2) The behaviour of a resonator in a radiation field should follow from the statistical laws as a limiting case.' [Einstein to Bose, 3 November 1924]

Let us see to what extent these criticisms were fair. As far as the first point is concerned Einstein was referring to Beer's law. Bose's probability law, however, predicted a dependence of the absorption coefficient on the radiation density , decreasing with it. 'If the behaviour had been such,' wrote Einstein in the note, 'then it would certainly have already been discovered in the case of infra-red radiation from hot light sources.' This is hard to believe. The departure from classical behaviour predicted by Bose's principle should occur only when $N_\nu / A_\nu << 1$, i.e. for very low intensity radiation. It is now well known that the departures from classical behaviour predicted by quantum optics are not seen even when the intensity of light is extremely low, *unless the source happens to be of a special kind* (like a single-photon source). The

reason is that light from all classical sources is in a coherent state in which the number fluctuation is extremely large (Poissonian or higher). Therefore only light from 'single-photon states' (Fock states) and 'squeezed states' and other such 'sub-Poissonian situations' show departures from classical behaviour. [Aspect, 1987; Loudon, 1983] Such light sources were certainly unknown in 1924.

The second point concerns the 'correspondence principle' (not necessarily Bohr's) which in this case implies that a radiation field can transfer positive as well as negative energy to a resonator, depending on the phase. 'The probabilities of both these transitions must depend on the density of radiation, that is, on N_ν as opposed to Bose's hypothesis,' wrote Einstein. The discovery of masers and lasers after the Second World War eventually vindicated Einstein's contention. It was nevertheless eventually an *empirical* rather than a sound theoretical refutation. Although the 'correspondence principle' was used as a heuristic tool with great success by Einstein, Planck and later the Copenhagen school prior to the discovery of quantum mechanics around 1925/1926, it is no longer regarded as a fundamental principle. In fact, quantum mechanics itself is known not to satisfy this principle. [Liboff, 1980; Cabrera and Kiwi, 1987; Bohm and Hiley, 1985] For example, the nodes of the eigen-solutions inside a box do not disappear for arbitrarily high quantum numbers so that a classical particle cannot pass through such points. Also the cross-section for hard sphere scattering has no classical limit. Einstein was aware of this and used it against quantum mechanics as well.

o. Bose's third (unpublished) paper (1925)

Although Bose was unhappy, particularly with the first point ragarding Beer's law, he accepted the second point, modified his stance and wrote a third paper which he sent to Einstein from Paris on 27 January 1925. He sent him a letter under separate cover in which he wrote :

> *It seems that the hypothesis of negative Einstrahlung stands, which, as you have yourself expressed, reflects the classical behaviour of a resonator in a fluctuating field. But the additional hypothesis of a spontaneous change, independent of the state of the field, seems to me not necessary. . .*
>
> *I am rather anxious to know your opinion about it. I have shown it to Professor Langevin here and he seems to think it interesting and worth publishing.*

It was never published. Neither did Bose keep a copy of the paper, nor is there one in the Einstein Archives as far as one can ascertain. So, the precise contents and details will remain unknown. Bose arrived in Berlin in October 1925 and discussed his new ideas with Einstein. Einstein was so convinced of the necessity and reality of the two independent processes of emission of light from atoms (spontaneous and induced) that he could not agree with Bose that one of them, the former, was unnecessary to assume in deriving Planck's law. To Bose, the process of emission of light appeared as *a single physical process* and the division into two independent processes appeared artificial. The assumption of one, he claimed, would automatically imply the other in thermal equilibrium, *given the special statistical property of the radiation field.* Initially Bose started with spontaneous emission as input but later changed his position. It was clear to Bose that Planck's law was *a consequence of the special statistical property of the radiation field itself and was independent of the mechanisms of energy transfer with*

17 Rue de Sommerard
Paris Ve.
27th January '25.

Revered Master.

I received your kind note of 3rd Nov. in which you mention your objection against the elementary law of Probability. I have been thinking about your objection all along, and to do not answer it satisfactorily. It seems to me that there is a way out of the difficulty, and I have written down my idea in the form of a paper which I enclose herewith under a separate cover. It seems that the three hypothesis of Negative-Einstallung, Slauds, which as you have yourself expressed, reflect the classical behaviour of a resonator in a fluctuating field. But the additional hypothesis of a spontaneous change, independent of the state of the field seems to me not necessary. I have tried to look at the radiation field from a new standpoint, and (have sought to separate the propagation of Quantum of energy from the propagation of electro-magnetic influence.) I feel vaguely that some such separation is necessary

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Bose's letter of 27 January 1925 to Einstein

if Quantum theory is to be brought in [illegible] the Generalised Relativity theory.

The views about the radiation-fields, which I have ventured to put forward seem to be very much like what Bohr has recently expressed in Phil. Mag. 1924. But it is only a guess, as I cannot say honestly to have exactly understood all what he means to say, about the Virtual fields and Virtual oscillators.

I am rather anxious to know your opinion about it. I have shown it to Prof. Langevin here and he seems to think it interesting, and worth publishing.

I cannot exactly express how grateful I feel for your encouragement, and the interest you have taken in my papers. Your first postcard, came at a critical moment, and it has more than any other made this sojourn to Europe possible for me. I am thinking of going to Berlin at the end of this month, where I hope to have your inestimable help and guidance.

Yours sincerely

S. N. Bose.

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atoms and molecules. This was indeed his capital contribution to physics and forms the substance of his first paper and the first part of his second paper. To Einstein, on the other hand, it appeared that Planck's law was a consequence of special mechanisms of energy transfer between the radiation field and molecules and that thermal equilibrium was impossible without both induced and spontaneous emission. This was indeed the lesson he derived from his 1916/1917 papers which he considered to contain 'the fundamental outline of the future theoretical derivation.' Einstein was compelled to regard spontaneous emission as being independent of the state of the radiation field and an *inherent* property of atoms and molecules because in 1917 he was unaware of the special quantum statistical property of the radiation field. It is strange that even after championing Bose's method he failed to see Bose's point about spontaneous emission. To Bose it was clear that it depended on the 'environment' in which the atom or molecule was placed. He wrote in his second paper [1924] :

> From classical theory one would expect that no stationary states are possible and that an interaction or coupling (Bohr) between ether and the excited atom and the radiation connected with it is always present. On the other hand, in order to explain spectral emission we must assume it to be possible that no interaction (emission) occurs. Because of the existence of stationary states we are therefore led to assign to every stationary state a probability coefficient or a mean life-time.

Recall the expression (38) for the fundamental probability P. It can be written as $\bar{n} / (\bar{n} + 1)$ where $\bar{n} = N_\nu / A_\nu$ is the average number of quanta per cell. For $\bar{n} >> 1$, P tends to unity, the classical limit in which stationary states are not possible. It follows from this that the average life-time of a stationary state (and therefore the probability of spontaneous emission) depends on the average radiation density $\bar{n}$. Bose used to recount to his students Einstein's objection to this. 'Suppose there were only one hydrogen atom in the universe in an excited state,' Einstein once remarked to Bose, 'don't you think it would radiate spontaneously and come down to its ground state?' Bose did not disagree, but nevertheless, felt that such a *gedanken* situation did not capture the situation in a Hohlraum or cavity. It was Bose's intuition that eventually turned out to be right in the end, not Einstein's. Firstly, in Dirac's method of 'second quantization' (Dirac, 1927) which is widely used in quantum field theory today, one starts with a classical electromagnetic field in which the equality of the probabilities of induced emission and absorption (determined by the Fourier coefficients in the plane wave expansion) is built in, and obtains the spontaneous emission term as a *consequence* of the fundamental commutation rules. Although this is not exactly what Bose had proposed, it fits in with his basic contention that spontaneous emission is a consequence of the quantal property of the radiation field *itself* and need not be introduced as an independent hypothesis concerning the radiating atoms. Einstein's initial reaction to Dirac's contribution was also decidedly negative. [Pais, 1982, 441]

Secondly, in quantum electrodynamics spontaneous emission is not a property of an isolated atom but of an atom-vacuum system and can be significantly inhibited or enhanced by placing the atoms in a suitable environment. In quantum electrodynamics the vacuum is not empty and devoid of everything—it is full of virtual particles

appearing and disappearing, interacting with one another; it provides a non-trivial physical 'environment' to an atom. A whole new branch of quantum optics called 'cavity quantum electrodynamics' has developed since about 1987 utilizing dramatic changes in spontaneous emission rates to construct new kinds of microscopic masers that operate with a single atom and a few photons or with photons emitted in pairs in a two-photon transition. This is what two of the pioneers in the field have to say :

> Ever since Einstein demonstrated that spontaneous emission must occur if matter and radiation are to achieve thermal equilibrium, physicists have generally believed that excited atoms inevitably radiate. Spontaneous emission is so fundamental that it is usually regarded as an inherent property of matter. *This view, however, overlooks the fact that spontaneous emission is not a property of an isolated atom but of an atom-vacuum system.* The most distinctive feature of such emission, irreversibility, comes about because an infinity of vacuum states is available to the radiated photon. If these states are modified — for instance, by placing the excited atom between mirrors or in a cavity– spontaneous emission can be greatly inhibited or enhanced.
>
> Recently developed atomic and optical techniques have made it possible to control and manipulate spontaneous emission. [Haroche and Kleppner, 1989]

In his third unpublished paper Bose had gone further. He wrote in his letter of 27 January 1925 to Einstein :

> *I have tried to look at the radiation field from a new standpoint and have sought to separate the propagation of Quantum of energy from the propagation of electro-magnetic influence. I seem to feel vaguely that some such separation is necessary if Quantum theory is to be brought in line with Generalized Relativity theory.*
>
> *The views about the radiation field, which I have ventured to put forward, seem to be very much like what Bohr has recently expressed in May Phil Mag 1924. But it is only a guess, as I cannot say honestly to have exactly understood all he means to say about virtual fields and virtual oscillators.*

This letter shows that Bose was one of the first persons to have formulated the idea of an 'empty wave' (an electromagnetic wave propagating in space and time but carrying no energy-momentum). He was certainly the first person to have intuitively seen its relationship with the principle of General Relativity, although the connection is not clear. Einstein already had the idea of an empty wave in his light-quantum hypothesis according to which 'a ray of light expands starting from a point, the energy does not distribute on ever increasing volumes, but remains constituted of a finite number of energy quanta localized in space and moving without subdividing, and unable to be absorbed or emitted partially.' [Einstein, 1905]

If the localized quanta carry all the energy and momentum, what happens to the electromagnetic waves which can produce interference? There was no clear-cut answer. The problem was so acute that Einstein referred to these waves as *Gespensterfelder* (ghost waves) guiding the photons. [Bohr, 1945, 206] Louis de Broglie also had a similar

idea in his concept of phase or pilot waves 'guiding the propagation of the energy' and enabling a 'synthesis of the waves and the quanta.' [de Broglie, 1923, 549] Slater also arrived at a similar notion. As he recalled : 'A number of scientists — W [illiam] F [rancis] G [ray] Swann, among others — had suggested that the purpose of the electric field was not to carry a continuously distributed density of energy, but to guide the photons in some manner. This was the point of view which appealed to me, and during my visit at the Cavendish Laboratory in the fall of 1923, I elaborated on it. [Slater, 1975, 9] Slater who believed in the existence of light-quanta as well as electromagnetic waves found the following way out of the difficulty : 'I decided you could only have a statistical connection between them because you couldn't set up a vector to represent the photon in electromagnetic theory : that you would have the intensity of the wave governing the probability of finding the photon there. And I wanted to have the wave emitted during the stationary state so as to get it emitted over a long enough period so that it would have a suitable spectral distribution.' [Slater, loc.cit., 30]

When Slater arrived in Copenhagen on 21 December 1923 to collaborate with Hendrik Kramers and Niels Bohr, he was prevailed upon by them to relinquish Einstein's light-quantum hypothesis but to retain his idea of electromagnetic waves emitted by oscillators during their stationary states and carrying no energy. They coined the name *virtual oscillators* for them. The collaboration resulted in the famous Bohr–Kramers–Slater proposal [Bohr, Kramers, Slater, 1924] which Bose referred to in his letter to Einstein. In the BKS proposal the conservation of energy (and momentum) was abandoned for individual transition processes in atoms but was retained as a statistical concept. Most interestingly, they wrote : 'the transitions which in [the Einstein theory of 1917] are designated as spontaneous are, in our view, induced by the virtual field.' Bose must have noted this with satisfaction and this must have been one reason why he regarded the BKS proposal to be 'very much like' his own. The theory was however quickly falsified by experiments carried out by Bothe and Geiger [Bothe and Geiger, 1925] and Compton and Simon [Compton and Simon, 1925a, b] which established the conservation of energy and momentum in individual processes. Nevertheless, their ideas were reformulated and generalized by Born [1926] in his well known statistical interpretation of the wave function in quantum mechanics, an idea now universally accepted. This is what Heisenberg wrote about its historical antecedent :

'The probability wave of Bohr, Kramers and Slater. . . was a quantitative version of the old concept "potentia" in Aristotelian philosophy. It introduced something standing in the middle between the idea of an event and the actual event, a strange kind of physical reality just in the middle between possibility and reality.

Later, when the mathematical framework of quantum theory was fixed, Born took up this idea of the probability wave and gave a clear definition of the mathematical quantity in the formalism.' [Heisenberg, 1958, 29]

On 16 July 1946 , twenty-one years after he received Bose's letter of 27 January 1925 from Paris, Einstein wrote to Schrödinger about the latest status of the unified field theory (of electromagnetism and gravitation based on the General Principle of Relativity):

'It is indeed correct that the energetics occurs quite differently here compared to Maxwell-Poynting. I consider, however, that in this respect the Maxwell theory is really

false on account of quantum actualities. In light the energy exists in something like quasi-singularities. *The wave field as such should therefore not be the site of energy. One must simply require this and be happy that in the new theory the transverse wave field is indeed present but as such transports no energy.'* (Our italics) [Moore, 1989, 427]

There is no mention of Bose, his letter or his third paper which Einstein must have rejected outright, although the facsimile of Bose's letter supplied by The Jewish National and University Library, Jerusalem, shows that he had marked the phrase 'have sought to separate the propagation of Quantum of energy from the propagation of electro-magnetic influence' within parentheses.

Louis de Broglie's original interpretation of Schrödinger's equation [de Broglie, 1927] was given a firmer basis by Bohm [1952]. In the de Broglie-Bohm 'casual interpretation' particles are thought to exist independent of observation, embedded in a wave field ψ which satisfies the Schrödinger equation, with a density (over the statistical ensemble) of value $|\psi|^2$. One can show that this approach can explain the interference pattern observed in double-slit experiments without abandoning the notion of well-defined particle trajectories. [Philippidis, Dewdney and Hiley, 1979] Experiments for testing certain consequences of 'empty waves' have been proposed [Selleri, 1990, ch. 4] and some have already been carried out [Zou, Grayson, Wang and Mandel, 1992]. However, such experiments have not yet ruled out all versions of 'empty wave' theories [Vigier and Holland, 1991].

Bose remained ever grateful to Einstein (whom he always addressed as 'Sir' or 'Master') for his encouragement and the interest he had taken in his first paper. Nevertheless, deep within himself, he felt a resentment that rankled all through his life. Three of his most perceptive and original ideas were scotched by Einstein :

1. The idea of the photon spin which is now established beyond doubt. (That Bose proposed the idea in his first paper in 1924 cannot as yet be established beyond every shred of doubt, although circumstantial evidences are strong.)
2. The idea that spontaneous emission is not an inherent property of an isolated atom, totally independent of the radiation field, also confirmed by experiments since 1987.
3. The idea of the propagation of electromagnetic influence carrying no quanta of energy, an area of current theoretical and experimental activity.

Bose could not publish a single paper during his two years in Europe at a time when physics was going through a turmoil and one of its most exciting periods. He returned to Dhaka a disheartened man and did not publish anything in theoretical physics for the next twelve years !

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S N Bose : Collected scientific papers

On the Influence of the Finite Volume of Molecules on the Equation of State.† *By* MEGH NAD SAHA, *M. Sc., and* SATYENDRA NATH BASU, *M. Sc., Lecturers on Mathematical Physics, Calcutta University.*

It is wellknown that the departure of the actual behaviour of gases from the ideal state defined by the equation $p = \frac{NK\theta}{v}$ is due to two causes : (1) the finiteness of the volume of the molecules, (2) the influence of the forces of cohesion, i.e. the attractive forces amongst the molecules. van der Waals was the first to deduce an equation of state in which all these factors are taken into account; according to van der Waals, we have

$$p = \frac{NK\theta}{v-b} - \frac{a}{v^2} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where $b = 4\times$ volume of the molecules, a defines the forces of cohesion.

In all subsequent modifications of this equation (Clausius, Dieterici, or D. Berthelot) the changes which have been proposed all relate to the influence of the cohesive forces; the part of the argument dealing with the finiteness of molecular volumes is generally left untouched.

But it has been found that the results of experiments do not agree with the predictions of theory if we regard a and b as absolute constants. Accordingly it has been proposed to regard both a and b as functions of volume and temperature.*

But before proceeding to these considerations, it is necessary to scrutinize whether the influence of finite molecular volumes is properly represented by the term b. From theoretical considerations, the conclusion has been reached that this is not the case. The argument is as follows :

According to Boltzmann's theory,

$$\text{the entropy } S = K \log W + C,$$

where K = Boltzmann's gas-constant, W = probability of the state. Let us now calculate the probability that a number N of molecules originally confined within the volume V_0 and possessing finite volumes, shall be contained in a volume. V. Neglecting the influence of internal forces, the probability for the first molecule is $\frac{V}{V_0}$, for the second molecule the probability is $\frac{V-\beta}{V_0-\beta}$, where $\beta = 8\times$ volume of a single molecule, for when

† Communicated by the Authors.

*Compare van der Waals, *Proc. Amst.* 1916 ; Van Laar, *Proc. Amst.* Vol. xvi, p. 44.

the first molecule is in position, the space enclosed by a concentric sphere of double the radius of the molecule will not be available for the second molecule. The available space is therefore $V-\beta$, whence the probability is $\frac{V-\beta}{V_0-\beta}$. Introducing similar considerations for the rest of the molecules, we have

$$W = \frac{V}{V_0} \cdot \frac{V-\beta}{V_0-\beta} \cdot \frac{V-2\beta}{V_0-2\beta} \cdots \frac{V-\overline{N-1}\beta}{V_0-\overline{N-1}\beta} \quad \ldots (2)$$

We are, of course, neglecting those cases in which partial overlapping of the regions occupied by two or more molecules occurs; for the number of such cases can at best be a small fraction of the total number. Even cases of actual association do not include these, for in that case, two discrete molecules become merged into one, without their outer surfaces being actually in contact.

From the relations $S = K \log W + C$

and $$\left(\frac{\partial S}{\partial V}\right)_u = \frac{p}{\theta}$$

we can easily verify that

$$p = -\frac{K\theta}{\beta} \log \frac{V-n\beta}{V}$$

$$= -\frac{R\theta}{2b} \log \frac{V-2b}{V} \quad (R = NK) \quad \ldots (3)$$

As a first approximation, when b is small compared to v, we obtain $p = \frac{NK\theta}{v}$ (Boyle-Charles-Avogadro Law), and as a second approximation we obtain

$$p = \frac{NK\theta}{v-b} \text{ (van der Waals correction).}$$

We also note that

$$pV = NK\theta . \frac{x}{1-e^{-x}} \text{ where } x = \frac{\beta p}{K\theta} \quad \ldots (4)$$

To account for the infiuence of internal forces, we multiply, following the lead of Dieterici, the above expression (3) by $e^{-\frac{a}{NK\theta v}}$ having the same significance as before.

From this equation of state, we can easily verify the following results for the critical point :

Critical volume, $$V_c = \frac{2c}{e-1} b = 3.166b,$$

$$K = \frac{NK\theta}{p_c V_c} = 3.513.$$

The corresponding values of V_c from the van der Waals and the Dieterici equations are (3b, 2b) respectively, and of K are $\left(\frac{8}{3} = 2.66, \frac{e^2}{2}\ 3.695\right)$ respectively.

As a matter of fact, for the simpler gases, the value of 'K' obtained in this paper agrees better with the experimental results that the Dieterici value $\frac{e^2}{2}$ we have for oxygen* $K = 3.346$, for nitrogen† $K = 3.53$, for argon‡ $K = 3.424$, for xenon** $K = 3.605$. We need not consider the van-der-Waals value $\frac{8}{3}$, for it fails entirely.

The most serious drawback to Dieterici's equation is, according to Prof. Lewis (vide Lewis's Physical Chemistry, vol. ii. p. 117) that it makes b or the limiting volume $\frac{V_c}{2}$, while the limiting volume, obtained by the extrapolation of Cailletet-Mathias mean density line to the temperature $\theta = 0°\text{K}$ is about $\frac{V_c}{4}$. The value of b obtained in this paper, viz., $\frac{V_c}{3{\cdot}16}$ therefore agrees better with this value.

It is yet premature to predict what influence this investigation will have on the speculations concerning the variability of the volume of molecules with temperature. A more detailed investigation dwelling upon this point, and the application of the formula (4) to Amagat's (pv, p) curves, will be communicated shortly. Meanwhile we point out that the factor $e^{-\frac{a}{NK\theta v}}$ has been introduced into the expression for 'p' only as a provisional measure, though it is considered that this step, though not quite exact, is one in the right direction. In the next paper an attempt will be made to introduce energy into probability calculations.

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Note added in proof—On consulting the literature on the subject, we noticed that in several papers in the Amsterdam Proceedings (*vide* vol xv, p. 240et *seq.*), Dr. Keesom of Leyden had also made attempts to deduce the equation of state from Boltzmann's entropy principle. But, in the expression (2) for W, he introduces, before differentiation, an approximation in which terms up to second order in $\frac{b}{v}$ are retained only. In this way, he arrives at the van der Waals' form $v-b$ for the influence of finite molecular volumes. In obtaining our present equation of state (4), no such approximation has been made.

(M. N. SAHA *and* S. N. BASU.)

*Mathias and K. Onnes, *Proc. Amst.* Feb. 1911.

†Berthelot, *Bull. de la Soc. France de Phys.* 167 (1901)

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The Stress-Equations of Equilibrium

BY
SATYENDRANATH BASU.

(*Read April 6th*, 1919)

It was shown by Mitchell that the six stress-coefficients in an isotropic medium satisfy six equations of the type :

$$\nabla^2 X_x + \frac{1}{1+\sigma}\frac{\partial^2 \odot}{\partial x^2} = 0, \quad \nabla^2 Y_z + \frac{1}{1+\sigma}\frac{\partial^2 \odot}{\partial y \partial z} = 0,$$

... ...

... ...

These equations however have not been used for solving the general problems of Elasticity. It is shown here, that the equations can be successfully integrated in the case of a semi-infinite body bounded by a plane. In the case of the sphere the equtions can be conveniently transformed, in a different form, which then admit of integration in an infinite series of spherical harmonics.

(1) *The semi-infinite solid bounded by* $z = 0$

The surface tractions X_z Y_z Z_z, are supposed to have given values over the plane $z = 0$

Consider the equations

$$\nabla^2 X_z + \frac{1}{1+\sigma}\frac{\partial^2 \odot}{\partial x \partial z} = 0, \quad \nabla^2 Z_z + \frac{1}{1+\sigma}\frac{\partial^2 \odot}{\partial z^2} = 0,$$

$$\nabla^2 Y_z + \frac{1}{1+\sigma}\frac{\partial^2 \odot}{\partial y \partial z} = 0.$$

Since $\odot$ is a harmonic function the general solution can be written as

$$X_z = -\frac{1}{2(1+\sigma)} z \frac{\partial \odot}{\partial x} + X_{z0},$$

$$Y_z = -\frac{1}{2(1+\sigma)} z \frac{\partial \odot}{\partial y} + Y_{z0}, \qquad \ldots \quad (1)$$

$$Z_z = -\frac{1}{2(1+\sigma)} z \frac{\partial \odot}{\partial z} + Z_{z0},$$

where X_{z0}, Y_{z0}, Z_{z0}, are harmonic functions which have given values $X_z Y_z Z_z$, over the plane $z = 0$.

The functions are therefore uniquely determined; they are in fact:

$$X_{z0} = \frac{1}{2\pi} \frac{\partial}{\partial z} \int\int \frac{X_z}{r}\, dxdy, \quad Y_{z0} = \frac{1}{2\pi} \frac{\partial}{\partial z} \int\int \frac{Y_z}{r}\, dxdy,$$

$$Z_{z0} = \frac{1}{2\pi} \frac{\partial}{\partial z} \int\int \frac{Z_z}{r}\, dxdy,$$

also since

$$\frac{\partial X_z}{\partial x} + \frac{\partial Y_z}{\partial y} + \frac{\partial Z_z}{\partial z} = 0$$

we have from (1)

$$-\frac{1}{2(1+\sigma)} \cdot \frac{\partial \odot}{\partial z} + \frac{1}{2\pi} \frac{\partial}{\partial z} \left(\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} \right) = 0,$$

where

$$L = \int\int \frac{X_z}{r}\, dxdy, \; M = \int\int \frac{Y_z}{r}\, dxdy, \; N = \int\int \frac{Z_z}{r}\, dxdy,$$

and

$$\odot = \frac{1+\sigma}{\pi} \left[\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} \right].$$

Thus X_z, Y_z, Z_z, $\odot$ are all determined.

The solution may be afterwards completed, and U, V, W found out as in Cerrutti's method.

(2) *The problem of the sphere.*

Consider the three equations

$$\nabla^2 X_x + \frac{1}{1+\sigma} \frac{\partial^2 \odot}{\partial x^2} = 0,$$

$$\nabla^2 X_y + \frac{1}{1+\sigma} \frac{\partial^2 \odot}{\partial x \partial y} = 0,$$

$$\nabla^2 X_z + \frac{1}{1+\sigma} \frac{\partial^2 \odot}{\partial x \partial z} = 0.$$

Multiplying by x, y, z, and adding we have

$$\left[x\nabla^2 X_x+y\nabla^2 X_y+z\nabla^2 X_z \right]+\frac{1}{1+\sigma}\left[x\frac{\partial^2\Theta}{\partial x^2}+y\frac{\partial^2\odot}{\partial x\partial y}+z\frac{\partial^2\odot}{\partial x\partial z} \right]=0,$$

or,

$$\nabla^2(xX_x+yX_y+zX_z)+\frac{1}{1+\sigma}\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+z\frac{\partial}{\partial z}\right)\frac{\partial\odot}{\partial x}-2\left(\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}\right)=0.$$

Now since

$$\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}=0,$$

we have

$$\left.\begin{aligned} &\nabla^2(rX_r)+\frac{1}{1+\sigma}\frac{\partial}{\partial x}\left(r\frac{\partial\odot}{\partial r}-\odot\right)=0. \\ \text{Similarly we have}\quad &\nabla^2(rY_r)+\frac{1}{1+\sigma}\frac{\partial}{\partial y}\left(r\frac{\partial\odot}{\partial r}-\odot\right)=0, \\ &\nabla^2(rZ_r)+\frac{1}{1+\sigma}\frac{\partial}{\partial z}\left(r\frac{\partial\odot}{\partial r}-\odot\right)=0, \end{aligned}\right\} \qquad (2)$$

where $r\dfrac{\partial\odot}{\partial r}-\odot$ is also a harmonic function.

The form of the equations is exactly similar to the preceding equations.

(3) *The sphere of radius a, has given tractions X_r, Y_r, Z_r, over the surface.*

Assuming $$\Theta=\sum_n \Theta_n,$$

where $\odot_n$ is a solid homogenoous harmonic of the nth degree,

$$r\frac{\partial\odot}{\partial r}-\odot=\Sigma\,(x-1)\,\odot_x$$

also remembering that

$\nabla^2(r^2-a^2)F_n=2(2n+3)F_n$, where F_n is a solid homogeneous harmonic of the nth degree, we see the solutions of the equation (2) can be expressed in the form

$$rX_r=-\frac{1}{2(1+\sigma)}\sum\frac{(n-1)}{2n+1}(r^2-a^2)\frac{\partial\odot_n}{\partial x}+aX_{r0},$$

$$rY_r = -\frac{1}{2(1+\sigma)} \Sigma \frac{n-1}{2n+1}(r^2-a^2)\frac{\partial \odot_n}{\partial y} + aY_{r0},$$

$$rZ_r = -\frac{1}{2(1+\sigma)} \Sigma \frac{n-1}{2n+1}(r^2-a^2)\frac{\partial \odot_n}{\partial z} + aZ_{r0},$$

where X_{r0}, Y_{r0}, Z_{r0}, are harmonic functions which have given values over the surface of the sphere $r = a$ and hence are completely determined. If

$$X_{r0} = \Sigma X_n, \quad Y_{r0} = \Sigma Y_n, \quad Z_{r0} = \Sigma Z_n;$$

we have

$$rX_r = -\frac{(r^2-a^2)}{2(1+\sigma)} \Sigma \frac{n-1}{2n+1} \frac{\partial \odot_m}{\partial x} + a \Sigma X_n$$

$$rY_r = -\frac{(r^2-a^2)}{2(1+\sigma)} \Sigma \frac{n-1}{2n+1} \frac{\partial \odot_n}{\partial y} + a \Sigma Y_n$$

$$rZ_r = -\frac{(r^2-a^2)}{2(1+\sigma)} \Sigma \frac{n-1}{2n+1} \frac{\partial \odot_n}{\partial z} + a\Sigma Z_n.$$

Again

$$\frac{\partial}{\partial x}(rX_r)+\frac{\partial}{\partial y}(rY_r)+\frac{\partial}{\partial z}(rZ_r)$$

$$=x\left[\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}\right]+y\left[\frac{\partial Y_x}{\partial x}+\frac{\partial Y_y}{\partial y}+\frac{\partial Y_z}{\partial z}\right]$$

$$+Z\left[\frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}+\frac{\partial Z_z}{\partial z}\right]+X_x+Y_y+Z_z = \odot,$$

it follows that

$$\odot = -\frac{1}{(1+\sigma)} \Sigma \frac{n-1}{(2n+1)}\left[x\frac{\partial \odot_n}{\partial x}+y\frac{\partial \odot_n}{\partial y}+z\frac{\partial \odot_n}{\partial z}\right]$$

$$+a\Sigma\left(\frac{\partial x_n}{\partial x}+\frac{\partial y_n}{\partial y}+\frac{\partial z_n}{\partial z}\right)$$

or

$$\odot = -\frac{1}{1+\sigma} \Sigma \frac{n(n+1)}{2n+1} \odot_n + a\Sigma\left(\frac{\partial X_n}{\partial x}+\frac{\partial Y_n}{\partial y}+\frac{\partial Z_n}{\partial z}\right).$$

So that

$$\Sigma \odot_n \left[\frac{(1+\sigma)(2n+1)+n(n-1)}{(2x+1)(1+\sigma)} = a\Sigma\psi_n, \right.$$

where

$$\psi_n = \frac{\partial X_{n+1}}{\partial x} + \frac{\partial Y_{n+1}}{\partial y} + \frac{\partial Z_{n+1}}{\partial z}.$$

So that

$$\odot_n = \frac{a(2n+1)(1+\sigma)}{(1+\sigma)(2n+1)(+n(n-1)} \cdot \quad \psi_n .$$

Thus X_r, Y_r, Z_r, $\odot$ are all determined in terms of the known value of X_r, Y_r, Z_r, on the surface.

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On the Herpolhode

By
Satyendranath Basu

(*Read August 31st, 1919*)

M. de Sparre has shown that the horpolhode of Poinsot contains no point of inflexion; his proof was based upon the properties of elliptic functions. Various other proofs of the theorem have since been given by Mannheim, Saint-Germain[2], Routh[3], Le-Cornu[4] and others.

The following simple proof of the theorem is based solely on the dynamical considerations of the Poinsot motion; hence it might appear interesting and instructive.

Let A, B, C be the three principal moments of inertia of a body fixed at a point 0; let 0I and 0I′ be the two instantaneous axes, at time t and $t+dt$, I, I', etc., on the invariable plane trace out the horpolhode, length of 0I, 0I′, etc. being proportional to the resulting angular velocity at all times.

At a point of inflexion of the horpolhode, there will be a stationary tangent, i.e., the total change of the vector II' will be then along its own length.

Remarking that II′ has components proportional to $\dot{w}_1\,dt$, $\dot{w}_2\,dt$, $\dot{w}_3\,dt$, along the three moving principal axes, the condition reduces to

$$\frac{\ddot{w}_1-\dot{w}_2w_3+\dot{w}_3w_2}{\dot{w}_1}=\frac{\ddot{w}_2-\dot{w}_3w_1+\dot{w}_1w_3}{\dot{w}_2}=\frac{\ddot{w}_3-\dot{w}_1w_2+\dot{w}_2w_1}{\dot{w}_3}=\lambda \quad \text{say.} \quad \ldots \quad (1)$$

We have from Euler's equations

$$A\dot{w}_1=(B-C)w_2w_3,$$
$$B\dot{w}_2=(C-A)w_3w_1$$
$$C\dot{w}_3=(A-B)w_1w_2;$$

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[1] An exactly similar proof of the theorem discussed in this paper has been given by Prof. W. Van der Woude in a paper entitled "Over de Herpoloide Van Poinsot", published in Nieuw Archief voor Wiskunde, Tweede Reeks, Deel XII, Eerste Stuk, pp. 94, 95 (1919). It appears however from an enquiry that this journal was not received in the library of the Society before the beginning of November, 1919 —S. K. B.

[2] Compte Rendu—1885

[3] Advanced Dynamics.

[4] Bulletin de Soc. Math. de France—1906.

from which

$$A\ddot{w}_1 = (B-C)(\dot{w}_2 w_3 + \dot{w}_3 w_2)$$
$$B\ddot{w}_2 = (C-A)(\dot{w}_3 w_1 + \dot{w}_1 w_3)$$
$$C\ddot{w}_3 = (A-B)(\dot{w}_1 w_2 + \dot{w}_2 w_1)$$

Substituting in (1) we have

$$\frac{\dot{w}_3}{w_3}(A+B+C) - \frac{\dot{w}_2}{w_2}(C+A-B) = \lambda(B-C)$$

and two similar equations.

So that

$$\frac{\dot{w}_1}{w_1}(B-C)(B+C-A) + \frac{\dot{w}_2}{w_2}(C-A)(C+A-B) + \frac{\dot{w}_3}{w_3}(A-B)(A+B-C) = 0$$

But remembering that

$$Aw_1\dot{w}_1 + Bw_2\dot{w}_2 + Cw_3\dot{w}_3 = 0$$
$$A^2 w_1\dot{w}_1 + B^2 w_2\dot{w}_2 + C^2 w_3\dot{w}_3 = 0$$

we have

$$\frac{w_1\dot{w}_1}{BC(B-C)} = \frac{w_2\dot{w}_2}{CA(C-A)} = \frac{w_3\dot{w}_3}{AB(A-B)}$$

hence substituting, we get as a necessary condition for the point of infiexion

$$\frac{BC(B-C)^2(B+C-A)}{w_1^2} + \frac{CA(C-A)^2(C+A-B)}{w_2^2} + \frac{AB(A-B)^2(A+B-C)}{w_3^2} = 0$$

which is obviously impossible because $B+C-A$, $C+A-B$, $A+B-C$, are all positive, so that the left-hand side is an essentially positive quantity.

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ON THE EQUATION OF STATE.

To the Editors of the Philosophical Magazine.

Sirs, —

In a paper published in the 'Philosophical Magazine' for August 1918 it has been shown that from Boltzmann's theory of entropy we can arrive at an equation of state

$$p = -\frac{R\theta_c}{2b}\log\left(1-\frac{2b}{V}\right) = e^{-\frac{a}{NK\theta v}} \qquad \ldots \quad (1)$$

Amongst other applications of this theorem it was shown that the value of the critical constant $K = \frac{R\theta_c}{p_c V_c}$ would be $\frac{2e}{e-1} = 3.53$, instead of van der Waals' $\frac{8}{3}$ and Dieterici's $\frac{e^2}{2} = 3.69$.

A table was given showing that in the case of the most of the elementary gases the value 3.53 corresponded better with experimental results than either $\frac{8}{3}$ or 3.69.

The list comprised He, N_2, O_2, Xe, but not Hydrogen, about which the experimental data were not satisfactory.

Recently the critical data for Hydrogen have been re-determined in the Laboratory of Prof. K. Onnes (vide Proc. K. Akad. Wetenschappen. Amsterdam, Vol. xx. 1917).

It has been found that

$$\theta_c = 33°.18\text{K}, \quad d_c = .0310, \quad p_c = 12.8 \text{ atmospheres};$$

from these data, $K = 3.35$.

It is superfluous to add that this value of K is in much better agreement with the value 3.53 than with either the value $\frac{8}{3} = 2.66$ or $\frac{e^2}{2} = 3.69$, thus corroborating the belief expressed in the aforesaid paper that the equation (1), though not final, marks a step in the right direction.

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MEGH NAD SAHA.
SATYENDRA NATH BASU.

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On the Deduction of Rydberg's Law from the Quantum Theory of Spectral Emission. By* SATYENDRA NATH BASU, *M. Sc., University Lecturer in Physics, University College of Science, Calcutta.*

It is well known that Rutherford's model of the atom has been fruitful in explaining many facts connected with atomic radiation. In the simplest case of hydrogen, with a nucleus consisting of a single positive charge, and an electron, Dr. Bohr has successfully applied the quantum theory to explain the Balmer series of hydrogen spectra. The mathematical problem of finding the spectral series for any atomic system has since been clearly formulated by Sommerfeld, and the quanta condition has been generalized in a form suitable for system with any number of degrees of freedom. If $q_1, q_2, q_3, \ldots q_n$ are co-ordinates to fix the position of the electron responsible for emission, and $p_1, p_2, p_3, \ldots, p_n$ are the corresponding generalized momenta, any statical path, according to Sommerfeld, is characterized by the conditions

$$\int p_1 dq_1 = n_1 h, \quad \int p_2 dq_2 = n_2 h, \quad \int p_n dq_n = n_n h$$

where n's are whole numbers and h is Planck's constant, the integral being extended generally over the complete orbit. The radiation is supposed to take place when the electron jumps from one statical path to another. The difference in energy, at the same time, flows away in the form of a homogeneous radiation of frequency ν, which can be calculated from the Bohr equation $h\nu = W_1 - W_2$. Sommerfeld has successfully applied this conception in explaining the fine structure of hydrogen lines. It is clear, however, that the problem of theoretically calculating the spectrum of any atom other than hydrogen is beset with difficulties of a formidable nature. It is exactly analogous to the dynamical problem of "n" bodies, where only in favourable cases we are able to find solutions. Nevertheless, from a purely experimental standpoint, we know that the visible radiation from any element can be classified in definite series. The frequency of any line in the series can be expressed as a difference of two terms, each of which has the form

$$\frac{N}{(m+\alpha+\beta/m^2)^2},$$

where m is a whole number and α and β are two constants depending upon the element and the nature of the series. So that if we are to explain the formation of the series

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* Communicated by the Author.

+Bohr, Phil. Mag. July 1913.

**Sommerfeld, Ann. der Physik, li (1916).

from theoretical considerations following Bohr and Sommerfeld we must look upon each member multiplied by "h" as giving the energy of the atomic system when the radiating electron moves in a definite statical path. The complexity of the inner atomic field under which the radiating electron moves is to be looked upon as bringing in the terms involving α and β. So it seems interesting to see what will be the corresponding expression for energy in a system by which the complex nature of the internal field may be approximately represented. In the case of any atom we have, in general, a condensed nuclear charge of $+ne$ (where n is the atomic number) surrounded by rings of electron at different distances. The number of electrons in total must be also equal to n in order to secure that the atom is electrically neutral in the ordinary state.

In X-ray emission the electron displaced comes from the inner rings; in the case of visible radiation, however, we have reasons to think that the displaced electron responsible for radiation comes from the outermost ring—the valency electrons, as they have been designated by Sommerfeld. When excited for radiation, we can suppose that the electron in the outermost ring is removed to a greater distance from the centre than the others, so that the force acting upon it may be regarded as the resultant of the various forces exerted by the central charge and the remaining electrons. The potential at any point can be regarded as given by

$$-\frac{ne^2}{r}+e^2\sum_{s=n-1}\frac{1}{r_s}$$

where r is the distance from the centre and r_s is the distance from the s-th electron. If we neglect the influence of the moving electron upon the arrangement of the others surrounding the nucleus, it is clear that the potential can be approximately represented as $-\frac{e^2}{r}+\frac{L\cos\theta}{r^2}$. The resultant field might be looked upon as due to a single positive charge together with a doublet of strength L in a certain fixed direction which we take as our Z-axis. If we neglect the disturbing effect of the outer electron, L may be taken to be approximately fixed in direction and in magnitude in the small interval of time during which the active emission takes place.

We may, therefore, take as our model a system consisting of a positive charge and a doublet of strength L. We proceed to calculate the energy in a statical path on the above simplified hypothesis.

The kinetic energy of the moving electron is obviously

$$T=\frac{1}{2}[m\dot{r}^2+mr^2\dot{\theta}^2+r^2\sin^2\theta\,\dot{\phi}^2];$$

the potential energy

$$V = -\frac{e^2}{r} + \frac{eL\cos\theta}{r^2}.$$

Two integrals can be at once written down :

$$mr^2 \sin^2\theta\dot{\phi} = c_1,$$

$$m[\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2] - \frac{2e^2}{r} + \frac{2Le\cos\theta}{r^2} = -W.$$

To get another integral, we write

$$\frac{d}{dt}(mr^2\dot{\theta}) - mr^2\sin\theta\cos\theta\dot{\phi}^2 = \frac{eL\sin\theta}{r^2},$$

or

$$mr^2\dot{\theta}\frac{d}{d\theta}(mr^2\dot{\theta}) - \frac{c_1^2\cos\theta}{\sin^3\theta} - meL\sin\theta = 0.$$

Integrated, it gives

$$(mr^2\dot{\theta})^2 + \frac{c_1^2}{\sin^2\theta} + 2meL\cos\theta = c_2.$$

The expressions for three impulses mr, $mr^2\dot{\theta}$ and $mr^2\sin^2\theta\dot{\phi}$ can now be written down : in terms of the constants of integration we have

$$mr^2\sin^2\theta\dot{\phi} = c_1$$

$$mr^2\dot{\theta} = \sqrt{c_2 - \frac{c_1^2}{\sin^2\theta} - 2meL\cos\theta}$$

$$mr = \frac{1}{\sigma}\sqrt{-Wmr^2 + 2me^2r - c_2}.$$

The quanta conditions can be written down as

$$\int mr^2\sin^2\theta\dot{\phi}\,d\phi = n_1h, \qquad \ldots \quad (1)$$

$$\int mr^2\dot{\theta}\,d\theta = n_2h, \qquad \ldots \quad (2)$$

$$\int m\dot{r}dr = n_3h, \qquad \ldots \quad (3)$$

$-W$ being twice the total energy of the system. The integrals are to be extended over the whole range within which the expression within the square root remains positive.

The integration.

From (1) we have obviously

$$c_1 = \frac{n_1 h}{2\pi};$$

of the two remaining expressions, (3) can be integrated most easily : in fact,

$$n_3 h = \int \frac{1}{r} \sqrt{-Wmr^2 + 2me^2 r - c_2}\ dr$$

gives after integration

$$n_3 h = 2\pi \left[e^2 \left(\frac{m}{W} \right)^{\frac{1}{2}} - c_2^{\frac{1}{2}} \right].$$

The second integral can be written as

$$\frac{n_2 h}{2} = I = \int \sqrt{(c_2 - c_1^2) - c_2 x^2 - 2meLx(1-x^2)}\ \frac{dx}{1-x^2}$$

by putting $\cos\theta = x$.

The right-hand side is to be integrated throughout the region, when the cubic remains positive. It cannot be integrated in finite terms; an approximation suitable for our purpose can, however, be made, assuming $2meL$ to be small compared with $(c_2 - c_1^2) = A$. To see what this means we are to remember that $c_2 - c_1^2$ is of the dimension of h^2; so that $2meL$ must be small compared with h^2, or L must be small compared with $\frac{h^2}{me}$. Now, if α, $-\beta$, and γ are taken as the three roots of the cubic, the cubic can be written down as

$$D(\gamma - x)(\alpha - x)(x + \beta),$$

where γ is the greatest of the positive roots and $D = 2meL$. The limits of the integral are obviously α and $-\beta$,

$$I = \int_{-\beta}^{\alpha} \sqrt{A - c_2 x^2 - Dx(1-x^2)}\ \frac{dx}{1-x^2}.$$

Hence

$$\frac{\partial I}{\partial D} = \frac{1}{2} \int_{-\beta}^{\alpha} \frac{-x\,dx}{\sqrt{D}[(\gamma - x)(\alpha - x)(x+\beta)]^{\frac{1}{2}}},$$

or

$$\frac{\partial I}{\partial D} = \frac{1}{2\sqrt{D}} \left[\int_{-\beta}^{\alpha} \frac{(\gamma - x)^{\frac{1}{2}} dx}{(\alpha - x)^{\frac{1}{2}}(\beta + x)^{\frac{1}{2}}} - \gamma \int_{-\beta}^{\alpha} \frac{dx}{(\gamma - x)^{\frac{1}{2}}(\alpha - x)^{\frac{1}{2}}(x+\beta)^{\frac{1}{2}}} \right].$$

Supposing

$$\alpha - x = (\alpha+\beta)\cos^2\theta,$$
$$\beta + x = (\alpha+\beta)\sin^2\theta,$$

we get

$$\frac{\partial I}{\partial D} = \frac{1}{2\sqrt{D}}\left[2\int_0^{\pi/2}[\gamma+\beta-(\alpha+\beta)\sin^2\theta]^{\frac{1}{2}}\,d\theta - 2\gamma\int_0^{\pi/2}\frac{d\theta}{[(\gamma+\beta)-(\alpha+\beta)\sin^2\theta]^{\frac{1}{2}}}\right]$$

$$= \frac{1}{2\sqrt{D}}\left[(\gamma+\beta)^{\frac{1}{2}}E\left(\pi/2, \sqrt{\frac{\alpha+\beta}{\gamma+\beta}}\right) - \frac{\gamma}{(\gamma+\beta)^{\frac{1}{2}}}K\left(\pi/2, \sqrt{\frac{\alpha+\beta}{\gamma+\beta}}\right)\right],$$

where E and K are the usual elliptic integrals, defined by

$$E(\pi/2,k) = \int_0^{\pi/2}\sqrt{1-k^2\sin^2\theta}\,d\theta$$

$$K(\pi/2, k) = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}.$$

On the assumption that D is small, we have

$$\gamma = \frac{c_2}{D} + \frac{D(c_2-A)}{c_2^2},$$

$$\alpha = \sqrt{\frac{A}{c_2}} - \frac{D}{2c_2^2}(c_2-A),$$

$$\beta = \sqrt{\frac{A}{c_2}} + \frac{D}{2c_2^2}(c_2-A).$$

So that, expanding E and K and making necessary approximations, we have finally

$$\frac{\partial I}{\partial D} = \frac{\pi D}{4c_2^{5/2}}\left[c_2 - \frac{3A}{2}\right].$$

Also

$$I = \int\sqrt{(c_2-c_1^2)c_2x^2}\,\frac{dx}{1-x^2} = \pi\,[c_2^{\frac{1}{2}}-c_1].$$

So that we have

$$\frac{n_2 h}{2} = \pi(c_2^{\frac{1}{2}} - c_1) + \frac{\pi}{16}\frac{(2meL)^2}{c_2^{5/2}}(3c_1^2 - c_2).$$

Now, collecting all the quanta conditions, we have

$$c_1 = \frac{n_1 h}{2\pi},$$

$$c_2^{\frac{1}{2}} - c_1 + \frac{(meL)^2}{4c_2^{5/2}}(3c_1^2 - c_2) = \frac{n_2 h}{2\pi},$$

$$e^2\left(\frac{m}{W}\right)^{\frac{1}{2}} - c_2^{\frac{1}{2}} = \frac{n_3 h}{2\pi}.$$

So that we have

$$c_2^{\frac{1}{2}} + \frac{(meL)^2}{4c_2^{5/2}}\left(\frac{3n_1 h_1^2}{4\pi^2} - c_2\right) = \frac{(n_1+n_2)h}{2\pi}$$

and

$$e^2\left(\frac{m}{W}\right)^{\frac{1}{2}} = \frac{n_3 h}{2\pi} + c_2^{\frac{1}{2}}.$$

Assuming

$$c_2^{\frac{1}{2}} = \frac{yh}{2\pi}$$

we have

$$\frac{yh}{2\pi} + \frac{(meL)^2(2\pi)^5}{4y^5 h^5}\left[\frac{3n_1^2 h^2}{4\pi^2} - \frac{y^2 h^2}{4\pi^2}\right] = \frac{(n_1+n_2)}{2\pi}h$$

or

$$y + \left(\frac{2\pi^2 meL}{h^2}\right)^2 \frac{1}{y^5}(3n_1^2 - y^2) = n_1 + n_2.$$

Calling

$$\left(\frac{2\pi^2 meL}{h^2}\right)^2 = \alpha,$$

we have

$$e^2\left(\frac{m}{W}\right)^{\frac{1}{2}} = \frac{h}{2\pi}(n_3 + y),$$

where

$$y + \frac{\alpha}{y^5}(3n_1^2 - y^2) = n_1 + n_2;$$

we have approximately

$$y = (n_1+n_2) + \frac{\alpha}{(n_1+n_2)^3}\left[\ 1-\frac{3n}{(n_1+n_2)^2}\ \right].$$

So that

$$W = \frac{4\pi^2 e^4 m}{h^2\left[\ n_1+n_2+n_3+\ \frac{\alpha}{(n_1+n_2)^3}\left(\ 1-\frac{3n_1^2}{(n_1+n_2)^2}\right)\ \right]^2}.$$

So that the energy in the statical path

$$= -\frac{2\pi^2 e^4 m}{h^2\left[\ n_1+n_2+n_3+\ \frac{\alpha}{(n_1+n_2)^3}\left(\ 1-\frac{3n}{(n_1+n_2)^2}\right)\ \right]^2}$$

if we suppose that the spectrum is due to ionized atoms in which the field can be approximately represented as a central charge of $E = pe$ and a doublet of different strength L'.

We have, by a similar reasoning, energy in a statical path

$$= -\frac{2\pi^2}{h^2}\frac{e^2E^2m}{\left\{\ n_1+n_2+n_3+\ \frac{\alpha'}{(n_1+n_2)^3}\left(\ 1-\frac{3n_1^2}{(n_1+n_2)^2}\right)\right\}^2}$$

$$= \frac{-p^2Nh}{\left\{\ n_1+n_2+n_3+\ \frac{\alpha'}{(n_1+n_2)^3}\left(\ 1-\frac{3n_1^2}{(n_1+n_2)^2}\right)\right\}^2}$$

where N = Rydberg number.

As a result of numerous investigations on the nature of the spectral series, it has been shown that for many elements the different series can be grouped according to the following schemes :

P-Series :

$$\nu = (1.5,\ s)-(m, p_1) \\ \qquad\quad -(m, p_2)\ \Big\} \qquad m = 2, 3, 4\ldots$$

II. Subordinate Series :

$$\nu = (2, p_1)-(m+\cdot 5,\ s)$$

$$\nu = (2, p_2)-(m+\cdot 5,\ s). \qquad m = 2, 3, 4\ldots$$

1st Subordinate Series :

$$\nu = (2, p_1)-(m, d).$$

Companion ;

$$\nu = (2, p_1)-(m, d')$$

$$\nu = (2, p_2)-(m, d''). \qquad m = 3, 4, 5.$$

Symbol (m, f) stands for $\frac{N}{(m+f)^2}$ according to Rydberg, and $\frac{N}{\left(m+f+\frac{\phi}{m^2}\right)^2}$ according to Ritz.

The frequency of the lines emitted thus appears as the difference of two terms, each of which is to be regarded as corresponding to the energy in a definite statical path, on the Quantum theory. Sommerfeld* has recently given reasons for assuming that in any series of statical orbits corresponding to the different kinds of s, p, and d terms the azimuthal quanta generally preserve a certain definite value, whereas the radial quanta can have all values from 0 to ∞; he thus shows that in s, p and d terms the azimuthal quanta generally have values 1, 2, and 3 respectively.

Making the above assumptions in our formula, we see that the expression of the energy comes out as $-\frac{Nh}{(n_1+n_2+n_3+A)^2}$ in the same form as required by the Rydberg formula. The constants A, however, depends only upon n_1 and n_2; it diminishes for increasing values of the azimuthal quanta; so that they decrease progressively in the s, p, and d terms. Moreover, our form shows that A depends upon n_1 and n_2 separately, so that for the same value of n_1+n_2 we may have different values of the constant. Thus, if we suppose $n_1+n_2 = 1$ we have two values corresponding to the vaues 1, 0 and 0, 1; for $n_1+n_2 = 2$ we have three values; and so on. Thus we see, even on Sommerfeld's assumption, for the constancy of the azimuthal quanta we shall have two different s, three different p, four different d terms. At least two different values of p and three different values of d seem to be required by the series formula, which is essential for the explanation of doublets and triplets of constant frequency difference**. We thus see that our model serves at least as a qualitative explanation of the following facts :

(1) The progressive decrease of the characteristic numbers in the s, p, and d terms,

(2) The existence of different sets of s, p, and d terms for the same element.

It is clear, however, that our simplified assumption will not fit in any actual case exactly. The complex nature of the internal field can in no case be properly represented by a simple term, $\frac{L\cos\theta}{r^2}$, in the Potential. Moreover, we have reason to believe that the internal arrangement of the electrons itself will be influenced, in a large measure, by the motion of the outer electron, which we have neglected in our formula. In fact, Landé† has tried in a recent paper to take account of this disturbance in the

* Sommerfeld, Verh. d. Phys. Ges. May 1919.

** If we exclude the case $n_2 = 0$—*i.e.*, if we assume that the motion in a plane containing the axis of the doublet is excluded—we get the proper number of s, p, d terms as observed in the case of the alkali metals and the doublet system of alkaline earths.

† Landé, Phys. Zeit. 1909.

comparative simple case of the helium series. But at the same time, it is hoped that the calculation, in this comparatively simple case, will serve to illustrate at least some general principles at which we have arrived by an experimental study of the spectral series.

Summary

In this paper an attempt has been made to deduce the laws of regularity in the spectral series of elements on the basis of Bohr's quantum theory of spectral emission. Starting from Sommerfeld's assumption that the ordinary line-spectra of elements are due to the vibration of one outer electron (the valency electron), it has been shown that the field of the nucleus and the remaining $(n-1)$ electrons may be represented by the Potential $V = -\frac{e^2}{r} + \frac{eL\cos\theta}{r^2}$ i.e., the field due to a single charge plus a doublet of strength L. The axis of the doublet is variable, but the emission is supposed to take place so quickly that in that short time the axis does not appreciably change.

The quanta conditions have been applied according to Sommerfeld's rule, $nh = \int p_i\, dq_i$, and the energy of the system has been reduced to the quanta numbers. The energy comes out in the form

$$W = -\frac{Nh}{(n+n_3+z)^2}, \quad n+z = y$$

in the paper, where n_3 is the radial quantum, n is the azimuthal quantum, and z is given by an equation of the sixth degree, involving only the azimuthal quantum, and is a function of n only.

It has been next shown that if, in accordance with Sommerfeld's principle, we assume $n = 1$ for the s-orbits, $n = 2$ for the p-orbits, $n = 3$ for the d-orbits, $n = 4$ for the b-orbits, then, with a very simple assumption, we obtain a single value for the energy of the s orbits, a double value for the energy in the p-orbit, a treble value for the d-orbit. Then, applying Bohr's law $h\nu = W_n - W_{n'}$, we arrive at Rydberg's laws of the regularity in spectral series, in the case of the alkali metals.

Exact calculations are not tried on account of the uncertainty of the value of L ; but it has been pointed out that the values of s, (p_1, p_2), (d_1, d_2, d_3) progressively decrease, as is actually the case.

If the value of L be supposed to vary with n_3 the radial quantum, then probably the above calculations would lead to Ritz's law.

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Plancks Gesetz und Lichtquantenhypothese.

Von **Bose** (Dacca-University, Indien).

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Der Phasenraum eines Lichtquants in bezug auf ein gegebenes Volumen wird in „Zellen“ von der Größe h^3 aufgeteilt. Die Zahl der möglichen Verteilungen der Lichtquanten einer makroskopisch definierten Strahlung unter diese Zellen liefert die Entropie und damit alle thermodynamischen Eigenschaften der Strahlung.

Plancks Formel für die Verteilung der Energie in der Strahlung des schwarzen Körpers bildet den Ausgangspunkt für die Quantentheorie, welche in den letzten 20 Jahren entwickelt worden ist und in allen Gebieten der Physik reiche Früchte getragen hat. Seit der Publikation im Jahre 1901 sind viele Arten der Ableitung dieses Gesetzes vorgeschlagen worden. Es ist anerkannt, daß die fundamentalen Voraussetzungen der Quantentheorie unvereinbar sind mit den Gesetzen der klassischen Elektrodynamik. Alle bisherigen Ableitungen machen Gebrauch von der Relation

$$\varrho_\nu d\nu = \frac{8\pi\nu^2 d\nu}{c^3} E,$$

d. h. von der Relation zwischen der Strahlungsdichte und der mittleren Energie eines Oszillators, und sie machen Annahmen über die Zahl der Freiheitsgrade des Äthers, wie sie in obige Gleichung eingeht (erster Faktor der rechten Seite). Dieser Faktor konnte jedoch nur aus der klassischen Theorie hergeleitet werden. Dies ist der unbefriedigende Punkt in allen Ableitungen, und es kann nicht wundernehmen, daß immer wieder Anstrengungen gemacht werden, eine Ableitung zu geben, die von diesem logischen Fehler frei ist.

Eine bemerkenswert elegante Ableitung ist von Einstein angegeben worden. Dieser hat den logischen Mangel aller bisherigen Ableitungen erkannt und versucht, die Formel unabhängig von der klassischen Theorie zu deduzieren. Von sehr einfachen Annahmen über den Energieaustausch zwischen Molekülen und Strahlungsfeld ausgehend, findet er die Relation

$$\varrho_\nu = \frac{\alpha_{mn}}{e^{\frac{\varepsilon_m - \varepsilon_n}{kT}} - 1}.$$

Indessen muß er, um diese Formel mit der Planckschen in Übereinstimmung zu bringen, von Wiens Verschiebungsgesetz und Bohrs Korrespondenzprinzip Gebrauch machen. Wiens Gesetz ist auf die klassische Theorie gegründet, und das Korrespondenzprinzip nimmt an, daß die Quantentheorie mit der klassischen Theorie in gewissen Grenzfällen übereinstimme.

In allen Fällen scheinen mir die Ableitungen nicht genügend logisch gerechtfertigt. Dagegen scheint mir die Lichtquantenhypothese in Verbindung mit der statistischen Mechanik (wie sie durch Planck den Bedürfnissen der Quantentheorie angepaßt worden ist) für die Ableitung des Gesetzes unabhängig von der klassischen Theorie hinreichend zu sein. Im folgenden will ich die Methode kurz skizzieren.

Die Strahlung sei in das Volumen V eingeschlossen und ihre Gesamtenergie E sei gegeben. Es seien verschiedene Sorten von Quanten von der jeweiligen Anzahl N_s und Energie $h\nu_s$ vorhanden ($s = 0$ bis $s = \infty$). Die Totalenergie E ist dann

$$E = \sum_s N_s h\nu_s = V\int \varrho_\nu d\nu. \tag{1}$$

Die Lösung des Problems verlangt dann die Bestimmung der N_s, welche ϱ_ν bestimmen. Wenn wir die Wahrscheinlichkeit für jede durch beliebige N_s charakterisierte Verteilung angeben können, dann wird die Lösung durch die Bedingung bestimmt, daß diese Wahrscheinlichkeit bei Wahrung der Nebenbedingung (1) ein Maximum sein soll. Diese Wahrscheinlichkeit wollen wir nun aufsuchen.

Das Quantum hat ein Moment vom Betrage $\frac{h\nu_s}{c}$ in der Richtung seiner Fortbewegung. Der Momentanzustand des Quantums wird charakterisiert durch seine Koordinaten x, y, z und die zugehörigen Momente $p_x p_y p_z$; diese sechs Größen können als Punktkoordinaten in einem sechsdimensionalen Raum aufgefaßt werden, wobei wir die Beziehung haben

$$p_x^2 + p_y^2 + p_z^2 = \frac{h^2\nu^2}{c^2},$$

vermöge welcher der genannte Punkt auf einer durch die Frequenz des Quants bestimmten Zylinderfläche zu bleiben gezwungen ist. Zum

Frequenzbereich $d\nu_s$ gehört in diesem Sinne der Phasenraum

$$\int dx\, dy\, dz\, dp_x\, dp_y\, dp_z = V \cdot 4\pi \left(\frac{h\nu}{c}\right)^2 \frac{h\, d\nu}{c} = 4\pi \frac{h^3 \nu^2}{c^3} V d\nu.$$

Wenn wir das gesamte Phasenvolumen in Zellen von der Größe h^3 einteilen, gehören zum Frequenzbereich $d\nu$ also $4\pi V \frac{\nu^2}{c^3} d\nu$ Zellen. In bezug auf die Art dieser Einteilung kann nichts Bestimmtes gesagt werden. Indessen muß die Gesamtzahl der Zellen als die Zahl der möglichen Anordnungen eines Quants in dem gegebenen Volumen angesehen werden. Um der Tatsache der Polarisation Rechnung zu tragen, erscheint es dagegen geboten, diese Zahl noch mit 2 zu multiplizieren, so daß wir für die Zahl der zu $d\nu$ gehörigen Zellen $8\pi V \frac{\nu^2 d\nu}{c^3}$ erhalten.

Nun ist es einfach, die thermodynamische Wahrscheinlichkeit eines (makroskopisch definierten) Zustandes zu berechnen. Es sei N^s die Zahl der zum Frequenzbereich $d\nu^s$ gehörigen Quanten. Auf wie viele Arten können diese auf die zu $d\nu^s$ gehörigen Zellen verteilt werden? Sei p_0^s die Zahl der vakanten Zellen, p_1^s die Zahl derer, die ein Quant enthalten, p_2^s die Zahl der Zellen, die zwei Quanten enthalten usf. Die Zahl der möglichen Verteilungen ist dann

$$\frac{A^s!}{p_0^s!\, p_1^s! \ldots}, \quad \text{wobei} \quad A^s = \frac{8\pi\nu^2}{c^3} d\nu^s,$$

und wobei

$$N^s = 0 \cdot p_0^s + 1 \cdot p_1^s + 2\, p_2^s \ldots$$

die Zahl der zu $d\nu^s$ gehörigen Quanten ist.

Die Wahrscheinlichkeit des durch sämtliche p_r^s definierten Zustandes ist offenbar

$$\prod_s \frac{A^s!}{p_0^s!\, p_1^s! \ldots}.$$

Mit Rücksicht darauf, daß wir die p_r^s als große Zahlen betrachten können, haben wir

$$\lg W = \sum_s A^s \lg A^s - \sum_s \sum_r p_r^s \lg p_r^s,$$

wobei

$$A^s = \sum_r p_r^s.$$

Dieser Ausdruck soll ein Maximum sein unter der Nebenbedingung

$$E = \sum_s N^s h\nu^s; \quad N^s = \sum_r r p_r^s.$$

Die Durchführung der Variation liefert die Bedingungen

$$\sum_s \sum_r \delta p_r^s (1 + \lg p_r^s) = 0, \qquad \sum_s \delta N^s h\nu^s = 0$$

$$\sum_r \delta p_r^s = 0 \qquad \delta N^s = \sum_r r\,\delta p_r^s.$$

Hieraus folgt

$$\sum_s \sum_r \delta p_r^s (1 + \lg p_r^s + \lambda^s) + \frac{1}{\beta} \sum_s h\nu^s \sum_r r\,\delta p_r^s = 0.$$

Daraus folgt zunächst

$$p_r^s = B^s e^{-\frac{r h\nu^s}{\beta}}.$$

Da aber

$$A^s = \sum_r B^s e^{-\frac{r h\nu^s}{\beta}} = B^s \left(1 - e^{-\frac{h\nu^s}{\beta}}\right)^{-1},$$

so ist

$$B_s = A^s \left(1 - e^{-\frac{h\nu^s}{\beta}}\right).$$

Ferner hat man

$$N^s = \sum_r r p_r^s = \sum_r r A^s \left(1 - e^{-\frac{h\nu^s}{\beta}}\right) e^{-\frac{r h\nu^s}{\beta}}$$

$$= \frac{A^s e^{-\frac{h\nu^s}{\beta}}}{1 - e^{-\frac{h\nu^s}{\beta}}}.$$

Mit Rücksicht auf den oben gefundenen Wert von A^s ist also

$$E = \sum_s \frac{8\pi h {\nu^s}^3 d\nu^s}{c^3} V \frac{e^{-\frac{h\nu^s}{\beta}}}{1 - e^{-\frac{h\nu^s}{\beta}}}.$$

Mit Benutzung der bisherigen Resultate findet man ferner

$$S = k\left[\frac{E}{\beta} - \sum_s A^s \lg\left(1 - e^{-\frac{h\nu^s}{\beta}}\right)\right],$$

woraus mit Rücksicht darauf, daß $\frac{\partial S}{\partial E} = \frac{1}{T}$, folgt, daß $\beta = kT$. Setzt man dies in obige Gleichung für E ein, so erhält man

$$E = \sum_s \frac{8\pi h \nu^{s3}}{c^3} V \frac{1}{e^{\frac{h\nu^s}{kT}} - 1} d\nu^s,$$

welche Gleichung Plancks Formel äquivalent ist.

(Übersetzt von A. Einstein.)

Anmerkung des Übersetzers. Boses Ableitung der Planckschen Formel bedeutet nach meiner Meinung einen wichtigen Fortschritt. Die hier benutzte Methode liefert auch die Quantentheorie des idealen Gases, wie ich an anderer Stelle ausführen will.

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Planck's Law and the Light-Quantum Hypothesis

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The phase space of a light-quantum in a given volume is divided up in 'cells' of size h^3. The number of possible distributions of the light-quanta of a macroscopically defined radiation among the cells gives the entropy and with that all the thermodynamic properties of the radiation.

Planck's formula for the distribution of energy in the radiation of a blackbody forms the starting point for the quantum theory which has been developed in the last 20 years and has been very fruitful in all parts of physics. Since its publication in 1901 many methods for the derivation of this law has been proposed. It is recognized that the fundamental assumptions of the quantum theory are incompatible with the laws of classical electrodynamics. All derivations till now use the relation

$$\varrho_\nu d\nu = \frac{8\pi\nu^2 d\nu}{c^3} E,$$

that is, the relation between the density of radiation and the average energy of an oscillator, and they make assumptions about the number of degrees of freedom of the ether, which enters the above equation (the first factor on the right hand side). This factor could, however, be derived only from the classical theory. This is an unsatisfactory feature in all derivations, and it is not surprising that efforts are made again and again to give a derivation free from this logical flaw.

Einstein has given a remarkably elegant derivation. He has recognized the logical flaw in all previous derivations and has tried to deduce the formula independently of classical theory. Starting from very simple assumptions about the energy exchange between molecules and the radiation field, he finds the relation

$$\varrho_\nu = \frac{\alpha_{mn}}{e^{\frac{\epsilon_m - \epsilon_n}{kT}} - 1}.$$

In order to make this formula agree with that of Planck he has to use Wien's displacement law and Bohr's correspondence principle. Wien's law is based on classical theory and the correspondence principle assumes that the quantum theory agrees with the classical theory in certain limiting cases.

In all cases it appears to me that the derivations are not sufficiently justified from a logical point of view. On the other hand, the light-quantum hypothesis combined with statistical mechanics (as adapted by Planck to comform to the requirements of quan-

tum theory) appears sufficient for the deduction of the law independent of classical theory. In the following I shall sketch the method briefly.

Let the radiation be enclosed in a volume V and its total energy be E. Suppose there are different types of quanta each having number N_s and energy $h\nu_s$ ($s = 0$ to $s = \infty$). The total energy is then

$$E = \sum_s N_s h\nu_s = V\int \varrho_\nu d\nu. \tag{1}$$

The solution of the problem then requires the determination of of N_s which in turn determine ρ_ν. If we can give the probability for each distribution characterized by arbitrary values of N_s, then the solution is determined by the condition that this probability is a maximum subject to the subsidiary condition (1). We now want to find this probability.

The quantum has a momentum of magnitude $\frac{h\nu_s}{c}$ in the direction of its motion. The instantaneous state of the quantum is characterised by its coordinates x, y, z and the corresponding momenta p_x, p_y, p_z. These six quantities can be considered to be the coordinates of a point in a six dimensional space, where we have the relation

$$p_x^2 + p_y^2 + p_z^2 = \frac{h^2\nu^2}{c^2},$$

by virtue of which the point is forced to lie on the surface of a cylinder determined by the frequency of the quantum. The phase space belonging to the frequency interval $d\nu_s$ is.

$$\int dx\, dy\, dz\, dp_x\, dp_y\, dp_z = V.4\pi\left(\frac{h\nu}{c}\right)^2 \frac{h\,d\nu}{c} = 4\pi\frac{h^3\nu^2}{c^3} V d\nu.$$

If we divide the total phase space volume in cells of size h^3, then $4\pi V \frac{\nu^2}{c^3} d\nu$ cells will belong to the frequency interval $d\nu$. Nothing definite can be said about the method of this division. In any case, the total number of cells must be regarded as the number of possible arrangements of a quantum in the given volume. It seems, however, appropriate to multiply this number once again by 2 in order to take into account the fact of polarization, so that we obtain $8\pi V \frac{\nu^2\, d\nu}{c^3}$ as the number of cells belonging to $d\nu$.

Now it is easy to calculate the thermodynamic probability (macroscopically defined) of a state. Let N^s be the number of quanta belonging to the frequency range $d\nu^s$. In how many ways can these be distributed among the cells belonging to $d\nu^s$? Let p_0^s be

the number of empty cells, p_1^s those which contain one quantum, p_2^s those which contain two quanta and so on. The number of possible distributions is then,

$$\frac{A^s!}{p_0^s!\, p_1^s! \ldots}, \quad \text{where} \quad A^s = \frac{8\pi\nu^2}{c^3} d\nu^s,$$

and

$$N^s = 0 \cdot p_0^s + 1 \cdot p_1^s + 2 p_2^s \ldots$$

is the number of quanta belong to the range $d\nu^s$.

The probability of the state defined by all p_r^s is clearly

$$\prod_s \frac{A^s!}{p_0^s!\, p_1^s! \ldots}.$$

Taking into account that we can consider p_r^s to be large numbers, we have

$$\lg W = \sum_s A^s \lg A^s - \sum_s \sum_r p_r^s \lg p_r^s,$$

where

$$A^s = \sum_r p_r^s.$$

This expression must be a maximum with the constraints

$$E = \sum_s N^s h\nu^s; \quad N^s = \sum_r r p_r^s.$$

Carrying out the variation, we obtain the conditions

$$\sum_s \sum_r \delta p_r^s (1 + \lg p_r^s) = 0, \qquad \sum_s \delta N^s h\nu^s = 0$$

$$\sum_r \delta p_r^s = 0 \qquad \delta N^s = \sum_r r\, \delta p_r^s.$$

From this it follows that

$$\sum_s \sum_r \delta p_r^s (1 + \lg p_r^s + \lambda^s) + \frac{1}{\beta} \sum_s h\nu^s \sum_r r\, \delta p_r^s = 0.$$

Next it follows from this that

$$p_r^s = B^s e^{-\frac{r h\nu^s}{\beta}}.$$

But since

$$A^s = \sum_r B^s e^{-\frac{r h \nu^s}{\beta}} = B^s \left(1 - e^{-\frac{h\nu^s}{\beta}}\right)^{-1},$$

therefore

$$B_s = A^s \left(1 - e^{-\frac{h\nu^s}{\beta}}\right).$$

We further have

$$N^s = \sum_r r p_r^s = \sum_r r A^s \left(1 - e^{-\frac{h\nu^s}{\beta}}\right) e^{-\frac{r h \nu^s}{\beta}}$$

$$= \frac{A^s e^{-\frac{h\nu^s}{\beta}}}{1 - e^{-\frac{h\nu^s}{\beta}}}.$$

Taking into account the value of A^s found above, we get

$$E = \sum_s \frac{8\pi h \nu^{s^3} d\nu^s}{c^3} V \frac{e^{-\frac{h\nu^s}{\beta}}}{1 - e^{-\frac{h\nu^s}{\beta}}}.$$

Using the results found so far, one further finds

$$S = k\left[\frac{E}{\beta} - \sum_s A^s \lg\left(1 - e^{\frac{h\nu^s}{\beta}}\right)\right],$$

Whence, using the relation $\frac{\partial S}{\partial E} = \frac{1}{T}$, if follows that $\beta = k T$. Substituting this in the equation for E, we get

$$E = \sum_s \frac{8\pi h \nu^{s^3}}{c^3} V \frac{1}{e^{\frac{h\nu^s}{kT}} - 1} d\nu^s,$$

which is the same as Planck's formula.

(Translated by A. Einstein)

Translator's remarks :

In my opinion Bose's derivation signifies an important advance. The method used here gives the quantum theory of an ideal gas as I will work out elsewhere.

English translation of *Z Physik* **26**, pp. 168-171, 1924 (Springer — Verlag, Heidelberg).

Wärmegleichgewicht im Strahlungsfeld bei Anwesenheit von Materie.

Von **S. N. Bose** in Ramna (Indien).

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Die Wahrscheinlichkeit eines beliebigen Zustandes eines aus Strahlung und Molekeln bestehenden Systems wird berechnet, und es werden hieraus die Bedingungen für das statistische Gleichgawicht abgeleitet. Im Anschluß hieran werden neue Ausdrücke für die statistische Wahrscheinlichkeit der Elementarvorgänge vorgeschlagen, welche der Wechselwirkung zwischen Strahlung and Materie entsprechen.

Debye[1] hat gezeigt, daß das Plancksche Gesetz mit Hilfe der statistischen Mechanik abgeleitet werden kann. Seine Ableitung ist jedoch insofern nicht völlig unbhängig von der klassischen Elektrodynamik, als er Gebrauch macht von dem Begriff der Eigenschwingungen des Äthers und annimmt, daß hinsichtlich der Energie das Spektralgebiet zwischen ν und $\nu+d\nu$ ersetzt werden kann durch $\frac{8\pi\nu^2}{c^3} V\, d\nu$ Resonatoren, deren Energie nur Vielfache von $h\nu$ betragen kann. Man kann aber zeigen, daß die Ableitung so abgeändert werden kann, daß man überhaupt keine Anleihen bei der klassischen Theorie *zu* machen braucht. $\frac{8\pi\nu^2}{c^3} V\, d\nu$ kann gedeutet werden als die Anzahl Elementargebiete des schesdimensionalen Phasenraumes für die Quanten. Die weitere Rechnung bleibt im wesentlishen dieselbe.

Einstein hat sich einer anderen Methode bedient. Er. betrachtet die Wechselwirkung zwischen den materiellen Telichen und dem Strahlungsfeld. Kennt man die Energieverteilung untr den materiellen Teilchen, so kann man das Gesetz der schwarzen Strahlung ermitteln aus der Bedingung der Stationarität dieser Verteilung beim Austausch von Energie zwischen Materie und Strahlung. Dieser Austausch sit weiterhin eine Folge gewisser Elementarprozesse. Eine geeignete Formulierung der Eigenschaften und Wahrscheinlichkeit des Elementar prozesses ermoglicht es, das Plancksche Gesetz abzuleiten, wenn man die Energieverteilung unter den Teilchen als gegeben ansieht. In seiner ersten Arbeit[2] sind die materiellen Gebilde Bohrsche Atome, die nur einer diskreten Reihe stationärer Zustände fähig sind. Energieaustausch findet statt infolge von Emissionsund Absroptionsprozessen und gleichzeitigen Übergängen der Atome von einem Zustand zu einem anderen. Er hat dargetan, daß das Plancksche Gesetz sich ergibt, wenn den Emissions- und Absorptionsprozessen gewisse Wahrscheinlichkeiten zugeschrieben werden. Neuerdings hat aber das Problem

(1) Debye, *Ann. d. Phys.* **33**, 1427, 1910.

(2) Einstein, *Phys. ZS.* **18**, 121, 1917.

des thermischen Gleichgewichts zwischen dem Strahlungsfeld und freien Elektronen Wichtigkeit erlangst durch die Arbeiten von Debye[3] Compton[4] und anderen, die sich mit der Zerstreuung der Strahlung durch Elektronen befassen, Pauli[5] hat dieses Problem in einer interessanten Arbeit diskutiert. Als Elementarprozeß betrachtet er die Streuung, durch die ein Quant, das in einer bestimmten Richtung fliegt und innerhalb eines bestimmten Spektralintervalls liegt, sich in ein anderes Quant eines anderen Spektralintervalls und von verschiedener Richtung verwandelt. Gleichzeitig andert ein Elektron, das sich mit einer gewissen Geschwindigkeit in einer bestimmten Richtung bewegt Große und Richtung seiner Geschwindigkeit. Für alle diese Prozesse gilt das Erhaltungsgesetz von Impuls und Energie. Pauli hat gezeigt, daß, wenn die Wahrscheinlichkeit dieses Elementarprozesses $(A\rho+B\rho\rho')dt$ ist, wo ρ und ρ' die Strahlungsdichten der Frequenzen ν und ν', A und B unabhängig von q and q' aber abhänging von der Art des betrachteten Stoßes sind, dieses Wahrscheilichkeitsgesetz zum Planckschen Gesetz führt, falls für das Elektronengas das Maxwellsche Verteilungsgesetz gültig ist. Einstein unde Ehrenfest haben das Paulische Ergebnis noch verallgemeinert[6]. Sie betrachten einen Prozeß, bei dem ein Molekül order ein Elektron Quanten $h\nu_1$, $h\nu_2$ usw. absorbiert und gleichzeitig $h\nu_1'$, $h\nu_2'$ ...emittiert. Es wird angenommen, daß die emittierten und absorbierten Quanten bestimmte Richtungen haben. Zugleich andert sich auch Große und Richtung der Geschwindigkeit des betreffenden Moleküls. Energie und Impuls bleiben aber erhalten. Das Wahrscheinlichkeitsgesetz von Pauli wird folgenderma β on verallgemeinert :

$$dW_1 = \Pi b_1\rho_1\Pi(a_1'+b_1'\rho_1')dt,$$

$$dW_2 = \Pi(a_1+b_1\rho_1)\Pi(b_1'\rho_1')dt.$$

für den direkten und inverson Prozeß.

Er zeigt, daß Gleichgewicht vorhanden ist, wenn gewisse Beziehungen zwischen den Koeffizienten bestehen. Dies ist sowohl eine Verallgemeinerung seines früheren Gesetzes fur Bohrsche Atom wide des Gesetzes von Pauli

Das Problem des Warmegleichgewichts in einem Strahlungsfeld in Anwesenheit von materiellen Gebilden kann aber nach den Methoden der statistischen Mechanik behandelt werden, unabhängig von jeder besonderen Hypothese über den Mechanismus der Elementarprozesse, auf denen der Energieaustausch beruht. Wir gelangen so zu einer allgemeinen Beziehung, die güetig ist für alle speziellen Hypothesen über die Elementarprozesse und ihre Wahrscheinlichkeiten. Wenn es moglich ist, die thermodynamische Wahrescheinlichkeit für irgend eine spezielle Verteilung der

(3) Debye, *Phys. ZS.* **24**, 161, 1923.
(4) Compton, *Phys. Rev.* **21**, 483, 1923.
(5) Pauli, *ZS. f. Phys.* **18**, 272, 1923.
(6) Einstein und Ehrenfest, *ZS. f. Phys.* 19, 301, 1923.

Quanten im Strahlungsfeld und die Wahrscheinlichkeit für irgend eine willkürliche Verteilung der Enerige unter den materiellen Teilchen niederzuschreiben, dann ist die thermodynamische Wahrscheinlichkeit des gröβeren Systems, das Materie unc Strahlung umfaβt, einfach das Produkt der bieden Wahrscheinlichketien. Die Gleichgewichtsbedingung lautet einfach, daβ die resultierende Wahrscheinlichkeit ein Maximum sein muβ. Das Gleichgewichts-problem kann somit in einer ganza ganz anderen Weise betrachtet werden. Statt geeignete Hypothesen uber Elementarprozesse, die das Planckschе Gesetz ergeben, aufzustellen, versuchen wir, die Relation abzuleiten, zu der alle Hypothesen fuhren mussen, wenn das Planckschе Gesetz fur das Strahlungsfeld und das Maxwellsche Gesetz für die materiellen Teilchen gelten sollen. Die Beziehung, die wir suchen, ist offenbar aquivalent mit der der zuvor aus der Bedingung, daβ die thermodynamischche Wahrscheinlichkeit für das gröβere System ein Maximum ist, aabgeleiteten. Um das Problem in der angedeuteten Weise zu lüsen, mussen wir den Ausdruck für die thermodynamische Wahrscheinlichkeit für die Strahlung und für die materiellen Teilchen haben.

Thermodynamische Wahrscheinlichkeit für das Strahlungsfeld. Sie wurde in einer fruherent Arbeit[7] abgeleitet. Wenn die Zahl der Quanten im Spektralbereich ν und $\nu+d\nu$ gleich $N\,d\nu$ ist, so ist sie :

$$W = \prod_{\nu} \frac{(A_\nu + N_\nu d\nu)!}{A_\nu!\, N_\nu d\nu\,!},$$

wo $A_\nu = \frac{8\pi\nu^2}{c^3} V\,d\nu$ bedeutet. Es ist leicht zu sehen, daβ dieses Wahrscheinlichkeitsgesetz zum Planckschen Gesetz führt, wenn wir die Bedingung aufstellen, daβ W bei gegebener Energie ein Mximum sein muβ.

Thermodynamische Wahrscheinlichkeit für materielle Teilchen. Man kann sie unschwer finden. Wir wollen unsere Voraussetzungen ein wenig verallgemeinern, um sowohl den Fall Bohrscher Atome mit diskreten Energieniveaus als auch den Fall der Verteilung der Translationsenergie bei materiellen Teilchen[8] zu umfassen. Wir nehmen an, der Phasenraum sie in Elementargebiete geteilt. Zu jeder Elementarzelle gehort eine bestimmte Zahl g, die die Wahrscheinlichkeit angibt, $da\beta$ irgend ein Teilchen sich darin befindet. Die g sind im algemeinen einander gleich, ausgenommen den Fall Bohrscher Atome. Die thermodynamische Wahrscheinlichkeit irgend einer willkurlichen Verteillung n_1, n_2 use uber die verschiedenen Zellen ist :

$$\frac{N!\, g_1^{n_1} g_2^{n_2} \cdots}{n_1! n_2! \cdots}.$$

(7) Der Verfasser, erscheint im Phil. Mag. Siche auch Debye, *Ann. d. Phys.* **33**, 1427, 1910.

(8) Bohr, *ZS. f. Phys.* **18**, 117, 1923.

Die Bedingung des Wahrscheinlichkeitsmaximums bei gegebener Gesamtenergie und Zahl der Teilchen ist :

$$n_r = Cg_r\, e^{-\frac{E}{KT}}.$$

Jetzt sind wir in der Lage, die thermodynamische Wahrscheinlichkeit für das gesamte System zu berechnen. Sie ist offenbar :

$$W = \prod_s \frac{A_s+N_s!}{A_s!N_s!} \prod_T \frac{g_T^{n_T} N!}{n_T!},$$

wo

$$\Sigma N_s h\nu_s + \Sigma n_T E_T = E$$

und

$$\Sigma n_T = N.$$

Beim Gleichgewicht ist W ein Maximum, wobei die Gesamtenergie und die Zahl der Teilchen als gegeben anzusehen ist.

Wir betrachten den Elementarprozeß :

n_r verwandelt sich in n_r-1 und n_s in n_s+1; N_{ν_1}, N_{ν_2} ... vermindert sich um 1 und $N_{\nu_1'}$, $N_{\nu_2'}$ ... wächst um 1.

Die Bedingung der Stationaritat von W verlangt, daß W sich dabei nicht ändert.

Der betrachtete Elementarprozeß kann aufgefaßt werden als der Übergang eines Teilchens vom r-ten zum s-ten Zustand. Die Veränderung, die das Strahlungsfeld dabei erleidet, kann angesehen werden als das Ergebnis einer Zerstreuung nach einem Zusammenstoß.

Die gesuchte Bedingung lautet :

$$\frac{n_r}{g_r} \prod_\nu \frac{N_{\nu_r}}{N_{\nu_r}+A_{\nu_r}} = \frac{n_s}{g_s} \prod_{\nu'} \frac{N_{\nu'}}{N_{\nu'}+A_{\nu'}} \qquad \ldots \text{ (I)}$$

wo

$$\Sigma h\nu' - \Sigma h\nu + E_s - E_r = 0.$$

Diese Gleichung ist offenbar äquivalent mit

$$\frac{n_r}{g_r} \prod \frac{b_1\rho_{\nu_1}}{a_1+b_1\rho_\nu} = \frac{n_s}{g_s} \prod \frac{b'_1\rho_{\nu'}}{a'_1+b'_1\rho'_\nu},$$

wo

$$\frac{a_1}{b_1} = 8\pi \frac{\nu^2}{c^3} h\nu, \quad \frac{a'_1}{b'_1} = 8\pi \frac{\nu'^2}{c^3} h\nu'.$$

Dies kann auch geschrieben werden :

$$n_r \Pi b_1\rho_{\nu_1} \Pi(a'_1 + b'_1 e'_\nu) = n_s \Pi b'_1 \rho'_\nu \Pi(a_1+b_1\rho_\nu),$$

wenn

$$g_r = g_s.$$

Das ist die verallegemeinerte Gleichung, die fur Elementarprozesse von der Art, wie sie Einstein und Ehrenfest[9] betrachetet haben, gilt. Nach der hier gegebenen Ableitung ergibt sich die Beziehung als eine Folgerung aus der Bedingung, daβ beim Gleichgewicht die Wahrscheinlichkeit ein Maximum sein muβ. Wenn aber der Mechanisms des Elementarprozesses uns gestattet, die Gleichung direkt niederzuchreiben fuhrt die Substitution des Wertes fur das Verhältnis $\frac{n_r}{n_s}$ zum Planckschen Gesetz.

Es is jedoch klar, daβ Gleichung (I) fundamentaler ist als die transformierte Form von Einstein. Die nachfolgende Betrachtung wird zeigen, daβ eine einfache Uberlegung den Wert des Wahrscheinlichkeitskoeffizienten in der gewünschten Form liefert.

Der Fall des Bohrschen Atoms.

Das ist der Elementarprozeβ, den Einstein[10] in seiner fruheren Arbeit betrachtet hat.

Die fundamentale Gleichung (I) reduziert sich hier auf

$$\frac{n_r}{g_r} \cdot \frac{N_\nu}{A_\nu + N_\nu} = \frac{n_s}{g_s}.$$

Einsteins ursprüngliche Gleichung lautet :

$$n_r b_1 \rho_\nu dt = n_s(a_1' + b_1' \rho_\nu) dt \qquad \ldots \quad (1)$$

oder

$$\rho_\nu = \frac{a_1' n_s}{n_r b_1 - n_s b_1'}.$$

Nimmt man ferner an, daβ

$$g_r b_1 = g_s b_1' \quad \text{und} \quad \frac{a_1'}{b_1'} = 8\pi \frac{h\nu^3}{c^3},$$

so folgt das Plancksche Gesetz.

Um seine Gleichung (I) abzuleiten, nimmt er an, daβ Atome von höheren Energieniveaus zu Zuständen niederer Energie auf zwei Wegen übergehen konnen :

1. Eine Art von spontanem Übergang, wie bein einem radioactiven Prozeβ, dessen Wahrscheinlickkeit unäbhangig vom Zustand des Strahlungsfeldes ist.

(9) Einstein und Ehrenfest, a. a. O.

(10) Einstein, *Phys. ZS.* **18**, 121, 1917.

2. Ein durch die charakteristische Strahlung induzierter Übergang, dessen Wahrscheinlichkeit vom Zustand des Feldes abhängt.

Der Übergang von einem niederen zu einem höheren Energieniveau erfolgt durch Absorption, deren Wahrscheinlichkeit vom Felde abhängt.

Weiterhin werden gewisse Benziehungen zwischen den Koeffizienten postuliert, damit das Plancksche Gesetz herauskommt. Diese Beziehungen konnen nicht auf einfache Weise begründet werden.

Das gleiche Problem kann auf die folgende Weise behandelt werden, bei der viele von den vorkommenden willkürlichen Annahmen beseitigt werden.

Wir setzen auch voraus, daβ der Übergang von niederen zu höheren Niveaus stets durch Absroption von Quanten einer charakteristischen Frequenz erfolgt. A. Der Übergang von höheren zu niederen Niveaus ist eine Art von spontaner Änderung, dessen Wahrscheinlichkeit ünabhangig vom Strahlungsfeld ist. Die zweite Hypothese von Einstein (negative Einstrahlung) ist nicht erforderlich. B. Wir wollen versuchen, den Wahrscheinlichkeitskoeffizienten für Übergang A zu berechnen.

Die Zahl der Elementargebiete, die den in Betracht kommenden Quanten entsprechen, ist A_ν. Die Gesamtzahl der vorhandenen Quanten ist $N_\nu d\nu$. Diese sind aber nicht gleichförming über den Phasenraum verteilt. Sei p_0 die Zahl der leeren Zellen, p_1 die *mit* 1 Quant, p_2 die *mit* 2 Quanten usw. besetzten. Damit die Strahlung und das materielle Teilchen in Wechselwirkung treten können, muβ is dem Elementargebiet sein, das von einem Quant besetzt ist. Das besondere Elementargebiet, in dem sich das betrachtete Molekül befindet, wird, wenn wir es lange genug beobachten, alle moglichen Zustände durchlaufen : d.h. manchmal ist es leer, manchmal wird 1 Quant vorhanden sein, manchmal 2 usw. Die Dauer dieser Ereignisse wird Schleiβlich proportional zu den Gleichgewichtswerten von $p_0, p_1 \ldots$ Wenn r Quanten und ein Teilchen zugleich vorhanden sind, dann gibt es entweder einen Energieaustausch oder gar nichts. Somit sind $r+1$ verschiedene Ereignisse möglich, nämlich Energieaustausch von 1 Quant, 2 Quanten usw. oder kein Austausch

Daher ist die Zahl der möglichen Fälle :

$$p_0+2p_1+3p_2+\ldots = A_s+N_s d\nu_s = \Sigma(r+1)p_r .$$

Die Zahl der Fälle, in denen Wechselwirkung eintritt, ist :

$$p_1+2p_2+\ldots = N_s d\nu_s = \Sigma r p_r .$$

Somit ist die Wahrscheinlichkeit einer Wechselwirkung :

$$\frac{\Sigma\, r p_r}{\Sigma(r+1)p_r} = \frac{N_s d\nu_s}{A_s+N_s d\nu_s} .$$

Nun mussen wir die sgezielle Art der Wechselwirkung in Rucksicht ziehen, d.h. die Absorption, die zu unterscheiden ist von Zerstreuung. Die Wahrscheinlichkeit hierfür sie β.

Die Wahrscheinlichkeit der Absorption ist :

$$\frac{\beta \Sigma r p_r}{\Sigma(r+1)p_r} = \beta \frac{N_\nu d\nu}{A_\nu + N_\nu d\nu} .$$

Der Wahrscheinlichkeitskoeffizient für einen Übergang vom Typus B ist offenbar irgend eine Konstante, die den Übergang von dem speziellen höheren Zustand zu dem speziellen niederen Zustand bestimmt. Diese Konstante sei α.

Die Gleichgewichtsbedingung kann dann geschrieben werden :

$$n_r \beta \frac{N_\nu d\nu}{A_\nu + N_\nu d\nu} = \alpha n_s$$

was mit der fundamentalen Gleichung (I) übereinstimmt, wenn wir annehmen, daβ

$$g_r \beta = g_s \alpha.$$

Es ist klar, daβ die Substitution von $\frac{n_r}{n_s}$ zum Planckschen Gesetz führt.

Der Fall von Pauli.

Er ist einfach zu behandeln. Die Wahrscheinlichkeit einer Wechselwirkung ist wire vorhin $\frac{\Sigma r p_r}{\Sigma(r+1)p_r} \cdot \beta^{s}_{s'}$ sei der Wahrscheinlichkeitskoeffizient für Zerstreuung von ν_s in $\nu_{s'}$ falls eine Wechselwirkung statthat. Für die inverse Transformation seien die entsprechenden Koeffizienten $\frac{\Sigma r p'_r}{\Sigma(r+1)p'_r}$ und $\beta^{s'}_{s}$.

Beim Gleichgewicht ist also :

$$n_r \beta^{s}_{s'} \frac{N_s d\nu_s}{A_s + N_s d\nu_s} = n_s \beta^{s'}_{s} \frac{N_{s'} d\nu_{s'}}{A_{s'} + N_{s'} d\nu_{s'}} .$$

Nehmen wir ferner an, daβ $\beta^{s}_{s'} = \beta^{s'}_{s}$ so erhalten wir die fundamentale Gleichung (I).

Der Fall von Einstein und Ehrenfest.

Die Wahrscheinlichkeit einer gleichzeitigen Wechselwirkung ist das Produkt der Einzelwahrscheinlichkeiten, also gleich :

$$\Pi \frac{N_s d\nu_s}{A_s + N_s d\nu_s} .$$

Analog sei $\beta_{s_1' s_2' \ldots}^{s_1 s_2 \ldots}$ die Wahrscheinlichkeit für den speziellen Akt der Zerstreuung, und ein ähnlicher Ausdruck gelte für die inverse Streuung, woraus sich das Übrige leicht ergibt.

Bemerkungen. Aus unserer Rechung geht hervor, da β die Wahrscheinlichkeit einer Wechselwirkung eines Teilchens mit einem Quantum in einem Strahlungsfeld $\frac{N_\nu d\nu}{A_\nu + N_\nu d\nu}$ nicht einfach proportional zur Zahl der vorhandenen Quanten ist, wie man auf den ersten Blick vermuten könnte. Es ist leicht zu sehen, daβ diese Annahme zum Wienschen Gesetz führen würde, wie Pauli erkannt hat[11]. Pauli sah sich deshalb genotigt, eine andere Form für den Wahrscheinlichkeitskoeffizienten nzuunehmen. Die von Pauli angenommene und von Einstein und Ehrenfest verallgemeinerte Form erscheint aber als ganz willkurliche Hypothese, da man sich kein einfaches Bild von dem Zustandekommen eines solchen Ausdrucks machen kann. Die andere hier vorgeschlagene Form ist siemlich einfach und kann auf Grund elementarer Überlegungen gerechtfertigt werden. Es wird auch die Notwendigkeit vermieden, Felationen zwischen den Koeffizienten selbst annehman zu müssen. Bei der Ableitung des Wahrscheinlichkeitskoeffizienten für die Wechselwirkung (oder Kopplung, wie Bohr sagt) wurde angenommen, daβ selbst bei einem Zusammenstoβ der Fall, daβ keine Wechselwirkung eintritt, gerade so wahrscheinlich ist, wie der Fal irgend einer spezielen Kopplung Diese Annahme ist ein fundamentaler Punkt in der hier gegebenen Ableitung. Nach der klassischen Theorie würde man erwarten, daβ irgend eine Wechselwirkung eintritt, wenn ein Quant mit einem materiellen Teilchen zusammentrifft. Eshandelt sich also um eine principielle Abweichung von der klassischen Theoire. Diese Hypothese ist aber (wie mir scheint) ganz ähnlich wie die Hypothese, die man gewohnlich über die Stabilität der stationaren Zustände macht. Die klassische Theorie lie βe uns erwarten, daβ keine stationären Zustände moglich sind, und daβ stets Wechselwirkung oder Kipplung (Bohr) zwischen Äther und erregtem Atom und damit verbundene Strahlung vorhanden ist. Auf der anderen Seite mussen wir, um die spektrale Emission zu erklaren, den Fall als moglich annehmen, daβ keine Wechselwirkung (Emission) eintritt. So kommen wir dauz, wegen der Persistenz der stationaren Zustande jedem beliebigen stationären Zustand einen Wahrscheinlickkeitskoeffizienten oder eine mittlere Lebensdaur zususchreiben.

Die oben benutzte Hypothese, daβ aich bei einem Zuszmmenstoβ keine Wechselwirkung eintreten kann, ist ganz analog der Annahme der Stabilität der stationären Zustände, die so grundlegend für die Bohrsche Theorie der Linienemission ist, und kann auf die gleiche Ursache zurückgefuhrt werden : Die den materiellen Teilchen innewohnende Wahrscheinlichkeit der Persistenz ihrer stationären Zustände. In

(11) Pauli, *Z.S f. Phys.*, a a 0).

diesem Zusammenhang ist es interessant, zu bemerken, daβ wir beim analogen Fall des Zusammenstoβes eines Elektrons und eines Atoms experimentell Fälle nachweisen konnen, wo bei Zusammenstoβen das Elektron durch das Atom hindurchgeht, ohne die intraatomare Bewegung oder seine eigene zu verandern.

Die Beziehung $g_r\beta = g_s\alpha$.

Die Koeffizienten, oder wie Bohr[12] sie nennt, Gwichte g werden eingefuhrt, wenn Valenzelektron im Atom eine entartet bedingt periodische Bewegung ausfuhrt. Diese numerischen Koeffizienten geben nach Bohr einfach an, auf wieviel verschiedene Weisen man von einer benachbarten nichtentarteten Bewegung zu der speziellen entarteten Bewegung als Grenzfall gelangen kann. Nehmen wir diese Koeffizienten einfach als proportional zur Zahl der Übergangsmöglichkeiten von einem bestimmten zu dem betrachteten Zustand an, so ist die obige Beziehung fast selbstverständlich. Es gibt g_s Wege, auf denen ein Übergang von irgend einem r-ten zu dem betrachteten s-ten Zustand durch Absorption möglich ist. Analog gibt es g_r Übergänge zu niederen Zuständen infolge von Emission. Der Wahrscheinlichkeitskoeffizient hat dann den Faktor g_s der Wahrscheinlichkeitskoeffizient der Emission den Faktor g_r. Wenn wir ferner annehmen, daβ die Wahrschelichkeit einer Transformation durch Strahlung auf einem bestimmten Wege die gleiche ist wie der Wahrscheinlichkeitskoeffizient der Emission, wenn das Atom in bestimmter Weise zu niederen Zuständen übergeht, so ergibt sich sofort die fragliche Relation[13].

Manindra Physikal Laboratorium, Dacca-Universität, 14 Juni 1924.

Ich halte Boses Hypothese uber die Wahrscheinlichkeit der Strahlungselementarvorgange aus folgenden Grunden fur nicht zutreffend.

Für das statistische Gleichgewicht zwischen einem Bohrschen Zustande zu einem anderen gilt, wie Bose dargelegthat, die Beziehung

$$\frac{n_r}{g_r}\,\frac{N_\nu}{A_\nu+N_\nu} = \frac{n_s}{g_s}$$

Daraus folgt, daβ die Wahrscheinlichkeiten für die Übergange $r\to s$ und $s\to r$ der linken bzw. rechten Seite dieser Gleichung proportional sein müssen. Die Übergangswahrscheinlichkeiten für ein Molekül müssen sich also (wenn wir der Einfachheit halber die statistichen Gewichte beider Zustände gleich 1 setzen) wie $\frac{A_\nu}{A_\nu+N_\nu}$: 1 verhalten. Mehr kann aus der Kenntnis des thermodynamichen Gleichgewichts

(12) Bohr, *Z.S f. Phys.*, a.a.O.

(13) Vgl. in diesem Zusammenhang P. Hertz, *Repert. d. Phys.*, statistische Mechanik, 1. *Bd.*, Teil 2, S. 549.

nicht entonommen werden. Nach der von mir auftestellten Hypothese sollen diese Wahrscheinlichkeiten N_ν (d.h. der Strahlungsdichte) bzw. $A_\nu+N_\nu$ proportional sein, nach der Hypothese Boses $\frac{N_\nu}{A_\nu+N_\nu}$ *bzw.* 1.

Nach der letzteren Hypothese kann die aüßere Strahlung wohl einen Übergang von Zustande Z, kleinerer Energie nach dem Zustande Z_s größerer Energie bewirken, nicht aber umgekhrt einen Übergang von Z_s nach Z_r. Dies widrespricht aber dem mit Recht allgemein anerkannten Grundsatz, daß die klassische Theorie einen Grenzfall der Quantentheorie darstellen müsse. Nach letzterer kann nämlich ein Strahlungsfeld auf einen Resonator sowohl positive als auch negative Energie übertragen (je nach der Phase), und zwar beides gleich wahrscheinlich. Die Wahrscheinlichkeiten beider Übergange müssen also von der Strahlungsdichte, d.h. von N_ν abhängen, im Gegensatz zu Boses Hypothese. Inwiefern die Qunatentheorie die klassische zum Grenzfall hat, hat Planck in der letzten Auflage seines Buches über Strahlungstheorie ausführlich erötert.

Zweitens mü te nach Boses Hypothese ein kalter Körper ein von der Strahlungsdichte abhängiges (mit ihr abnehmendes) Absorptionsvermögen besitzen. Die Körper sollten in Kaltem Zustande "nicht-Wiensche" Strahlung schwächer absorbieren als weniger intensive aus dem Gültigkeitsbereich der Wienschen Strahlungsformel. Dies wäre bei ultraroter Strahlung heißer Lichtquellen gewiß schon entdeckt worden, wenn es sich so verhielte.

A. Einsten

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Thermal Equilibrium in the Radiation Field in the Presence of Matter

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The probability of an arbitrary state of a system consisting of radiation and molecules is calculated and from it the conditions for statistical equilibrium are derived. Further, new expressions for the statistical probability of elementary processes, appropriate for the interaction between radiation and matter, are proposed.

Debye[1] has shown that Planck's law can be derived using statistical mechanics. His derivation is, however, not completely independent of classical electrodynamics, because he uses the concept of normal modes of the ether and assumes that for calculating the energy the spectral range between ν and $\nu + d\nu$ can be replaced by $\frac{8\pi\nu^2}{c^3} V d\nu$ resonators whose energy can be only multiples of $h\nu$. One can however show that the derivation can be so modified that one does not have to borrow anything from the classical theory. $\frac{8\pi\nu^2}{c^3} V d\nu$ can be interpreted as the number of elementary cells in the six dimensional phase space of the quanta. The further calculations remain essentially unchanged.

Einstein has used another method. He considers the interaction between material particles and the radiation field. If one knows the energy distribution among the particles, then one can derive the law of black-body radiation from the condition of stationarity of this distribution when there is an exchange of energy between matter and radiation. This exchange is moreover a result of certain elementary processes. With a suitable formulation of properties and the probability of these elementary processes one can derive Planck's law, if the energy distribution among the particles is assumed to be known. In his first paper[2] the material particles are Bohr atoms which can have only a series of discrete stationary states. The energy exchange occurs because of emission and absorption processes and the simultaneous transitions of atoms from one state to another. He has shown that Planck's law is obtained when the emission and absorption processes have certain probabilities. Recently the problem of thermal equilibrium between radiation and and free electrons has acquired added importance through the studies of Debye[3], Compton[4] and others, which deal with the scattering of radiation off electrons. Pauli[5] has discussed this problem in an interesting paper. As an elementary process he considers the scattering of an electron by a quantum which moves in a definite direction and has a frequency in a given spectral range and is converted into another quantum with a different frequency and moving in a different

direction. Simultaneously an electron which moves with a certain velocity in a given direction changes the magnitude and the direction of its velocity. The laws of conservation of energy and momentum hold for all these processes. Pauli has shown that if the probability of this elementary process is $(A\rho + B\rho\rho')\,d\nu$ where ρ and ρ' are radiation densities for frequencies ν and ν', A and B are independent of ρ and ρ' but depend on the nature of collision, this probability law leads to Planck's law, provided Maxwell's distribution is valid for the electron gas. Einstein and Ehrenfest[6] have further generalized Pauli's result. They consider a process in which a molecule or an electron absorbs quanta $h\nu_1$, $h\nu_2$, etc. and simultaneously emits $h\nu_1'$, $h\nu_2'$ ... It is assumed that the emitted and absorbed quanta have definite directions. The magnitude and the direction of the velocity of the molecule under consideration also change at the same time. Energy and momentum are however conserved. The probability law of Pauli is generalized as follows,

$$d\,W_1 = \Pi b_1 \varrho_1 \,\Pi(a_1' + b_1' \varrho_1')\,dt,$$
$$d\,W_2 = \Pi(a_1 + b_1 \varrho_1)\,\Pi(b_1' \varrho_1')\,dt,$$

for the direct and the inverse processes. They show that equilibrium is obtained if certain relations between the coefficients exist. This is a generalization of Einstein's earlier result for Bohr atoms and also of Pauli's result.

The problem of thermodynamic equilibrium of radiation in the presence of material particles can, however, be studied using the methods of statistical mechanics, independently of any special assumption about the mechanism of the elementary processes on which the energy exchange depends. In this way we obtain a general relation which is valid for all special assumptions about the elementary processes and their probabilities. If it is possible to write down the thermodynamic probability for any special distribution of quanta of radiation and the probability for any arbitrary energy distribution among the particles, then the thermodynamic probability for the bigger system containing matter and radiation is simply the product of the two probabilities. The condition of equilibrium is simply that the resulting probability must be a maximum. The problem of equilibrium can thus be looked at in a completely different way. We try to derive the relation to which all assumptions must lead when Planck's law for radiation and Maxwell's law for particles hold instead of making assumptions about elementary processes adopted to give Planck's law. The relation which we are looking for is clearly equivalent to the previous one which is derived from the condition that the thermodynamic probability for the bigger system is a maximum. In order to solve the problem in the way indicated, we must have the expressions for the thermodynamic probability for radiation and the material particles.

Thermodynamic probability for radiation :

This has been derived earlier[7]. If $N_\nu\,d\nu$ is the number of quanta in the frequency range ν and $\nu + d\nu$, then the probability is,

$$W = \prod_{\nu} \frac{(A_\nu + N_\nu d\nu)!}{A_\nu!\, N_\nu d\nu!},$$

where $A_\nu = \frac{8\pi\nu^2}{c^3} V d\nu$. It is easily seen that this probability law leads to Planck's law if we impose the condition that for a given energy W should be a maximum.

Thermodynamic probability for particles :

This can be easily found. We want to generalize our assumptions a little so that the case of the Bohr atom with discrete energy levels as well as the case of the distribution of translational energy of particles can be included. Let the phase space be divided in to cells. For every cell there is a probability g that a particle occupies it. The g's are in general equal except for the case of Bohr's atoms. The thermodynamic probability for any arbitrary distribution n_1 , n_2 etc. among the different cell is,

$$\frac{N!\, g_1^{n_1} g_2^{n_2} \ldots}{n_1!\, n_2!}.$$

The condition that the probability be maximum for a given total energy and number of particles is,

$$n_r = C g_r c^{-\frac{E}{kT}}.$$

Now we are in a position to calculate the thermodynamic probability for the total system. It is clearly

$$W = \prod_{s} \frac{A_s + N_s!}{A_s!\, N_s!} \prod_{T} \frac{g_T^{n_T} N!}{n_T!},$$

where

$$\sum N_s h\nu_s + \sum n_T E_T = E$$

and

$$\sum n_T = N.$$

At equilibrium W is a maximum, the total energy and number of particles being given.

We consider the following elementary process : n_r changes to $n_r - 1$ and n_s to $n_s + 1$; N_{ν_1} , N_{ν_2}, ... reduce by 1 and $N_{\nu'_1}$, $N_{\nu'_2}$ increase by 1.

The condition of stationarity of W requires that W does not change on account of this process.

The elementary process under consideration can be considered as the transition of a particle from the rth to the sth state. The change which the radiation field undergoes can be looked upon as the result of a scattering after a collision. The required condition reads

$$\frac{n_r}{g_r}\prod_{\nu}\frac{N_{\nu_r}}{N_{\nu_r}+A_{\nu_r}}=\frac{n_s}{g_s}\prod_{\nu'}\frac{N_{\nu'}}{N_{\nu'}+A_{\nu'}}. \tag{I}$$

where

$$\sum h\nu' - \sum h\nu + E_s - E_r = 0.$$

The equation (I) is evidently equivalent to

$$\frac{n_r}{g_r}\prod\frac{b_1\varrho_{\nu_1}}{a_1+b_1\varrho_\nu}=\frac{n_s}{g_s}\prod\frac{b_1'\varrho_\nu'}{a_1'+b_1'\varrho_\nu'}.$$

where

$$\frac{a_1}{b_1}=8\pi\frac{\nu^2}{c^3}h\nu, \qquad \frac{a_1'}{b_1'}=8\pi\frac{\nu'^2}{c^3}h\nu'.$$

This can also be written as

$$n_r\,\Pi\, b_1\,\varrho_{\nu_1}\,\Pi\,(a_1'+b_1'\varrho_\nu')=n_s\,\Pi\, b_1'\varrho_\nu'\,\Pi\,(a_1+b_1\varrho_\nu),$$

if

$$g_r=g_s.$$

This is the generalized equation which holds for elementary processes of the type considered by Einstein and Ehrenfest[6]. According to the derivation given here the relation follows from the condition that for equilibrium the probability must be a maximum. However, if the mechanism of the elementary process allows us to write down the equation directly, the substitution of the value for the ratio $\frac{n_r}{n_s}$ gives Planck's law.

It is however clear that the equation (I) is more fundamental than the original one of Einstein. The following discussion will show that simple considerations give the value of the probability constants in their desired form.

The case of Bohr's atoms :

This is the elementary process which Einstein had considered in his earlier paper[2]. The fundamental equation (I) here reduces to

$$\frac{n_r}{g_r}\frac{N_\nu}{A_\nu+N_\nu}=\frac{n_s}{g_s}.$$

Einstein's original equation states

$$n_r b_1\varrho_\nu dt = n_s(a_1'+b_1'\varrho_\nu)dt$$

or

$$\varrho_\nu=\frac{a_1'n_s}{n_r b_1-n_s b_1'}.$$

If we assume further that

$$g_r b_1 = g_s b_1' \quad \text{and} \quad \frac{a_1'}{b_1'}=8\pi\frac{h\nu^3}{c^3},$$

then Planck's law follows :

To derive his equation (I), Einstein assumes that atoms in higher energy levels make transitions to lower energy levels in two ways :

1. A type of spontaneous transition as in a radioactive process, whose probability is independent of the state of the radiation field.
2. A transition induced by the characteristic radiation, whose probability depends on the state of the radiation field.

The transition from a lower to a higher energy level is the result of an absorption whose probability depends on the radiation field. Further, certain relations between the coefficients are postulated so that Planck's law follows. These relations cannot be justified in a simple way.

The same problem can be treated in the following way removing many of the arbitrary assumptions.

We also presume that the transition from a lower to a higher state takes place through an absorption of a quantum of characteristic frequency.

A. The transition from higher to lower levels is a spontaneous change, whose probability is independent of the field of radiation. The second assumption of Einstein (negative radiation) is not necessary.

B. We shall try to calculate the probability coefficients for the transition A.

The number of cells corresponding to the quanta under consideration is A_ν. The total number of quanta present is $N_\nu\, d\nu$. They are however not distributed evenly in phase space. Let p_o be the number of empty calls, p_1 with 1 quantum, p_2 with two quanta etc. In order that the radiation and a particle may interact, it must be in a cell occupied by a quantum. The particular cell which the molecule under consideration occupies will pass through all possible states if we observe it long enough, that is, sometimes it is empty, sometimes occupied by 1 quantum sometimes by 2 quanta etc. The length of these events will be finally proportional to the equilibrium values of p_o , p_1. When r quanta and a particle are together, then either an exchange of energy takes place or nothing happens. Therefore $r + 1$ different events are possible, namely energy exchange of 1 quantum, 2 quanta etc. or no exchange. Therefore the number of possible cases is

$$p_0 + 2\,p_1 + 3\,p_2 + \cdots = A_s + N_s\,d\nu_s = \Sigma\,(r+1)\,p_r.$$

The number of cases in which interactions occur is,

$$p_1 + 2\,p_2 + \cdots = N_s\,d\nu_s = \Sigma\,r\,p_r.$$

Consequently the probability of an interaction is,

$$\frac{\Sigma\,r\,p_r}{\Sigma\,(r+1)\,p_r} = \frac{N_s\,d\nu_s}{A_s + N_s\,d\nu_s}.$$

Now we must take into consideration the special nature of the interaction, that is, absorption which is to be distinguished from scattering. Let the probability for this be β. The probability for absorption is

$$\frac{\beta \sum r p_r}{\sum (r+1) p_r} = \beta \frac{N_\nu d\nu}{A_\nu + N_\nu d\nu}.$$

The probability coefficient for a transition of type B is evidently any constant which determines the transition from the particular higher state to a particular lower state. Let this constant be α.

The equilibrium condition can then be written as

$$n_r \beta \frac{N_\nu d\nu}{A_\nu + N_\nu d\nu} = \alpha n_s,$$

which agrees with the fundamental equation (I) if we assume that

$$g_r \beta = g_s \alpha.$$

It is clear that substituting the value of $\frac{n_r}{n_s}$ „one gets Planck's law.

Pauli's case.

This can be easily discussed. The probability of an interaction is as before $\frac{\Sigma r p_r}{\Sigma (r+1) p_r}$. Let $\beta_{s'}^{s}$ be the probability coefficient for scattering from ν_s to $\nu_{s'}$ if there is an interaction. Let the corresponding coefficients for the inverse process $\frac{\Sigma r p_{r'}}{\Sigma (r+1) p_{r'}}$ and $\beta_{s}^{s'}$. Then at equilibrium

$$n_r \beta_{s'}^{s} \frac{N_s d\nu_s}{A_s + N_s d\nu_s} = n_s \beta_{s}^{s'} \frac{N_{s'} d\nu_{s'}}{A_{s'} + N_{s'} d\nu_{s'}}.$$

Let us assume further that $\beta_{s'}^{s} = \beta_{s}^{s'}$; then we obtain the fundamental equation (I).

Einstein and Ehrenfest's case.

The probability of a simultaneous interaction is the product of individual probabilities and therefore equal to

$$\Pi \frac{N_s d\nu_s}{A_s + N_{s'} d\nu_s}.$$

Analogously let $\beta_{s_1', s_2', \ldots}^{s_1, s_2, \ldots}$ be the probability for scattering and let a similar expression hold for the inverse scattering. The rest then easily follows.

Remarks.

Our calculations show that the probability $\frac{N_\nu d\nu}{A_\nu + N_\nu d\nu}$ of an interaction between a particle and a quantum in a radiation field is not simply proportional to the number

of the quanta present as one would presume at first sight. It is easy to see that this assumption leads to Wien's law, as realized by Pauli[5]. He found it necessary to assume another form for the probability coefficients. However the form assumed by Pauli and generalized by Einstein and Ehrenfest appears to be completely arbitrary because one cannot easily see how such an expression can be derived. The form suggested here is quite simple and can be justified on the basis of elementary considerations. The necessity of assuming relations between the coefficients themselves is also avoided. It was assumed in deriving the probability coefficients for the interaction (or coupling, as Bohr says) that even in a collision no interaction is as probable as the occurence of any special interaction. This assumption is a fundamental point in the derivation given here. From the classical theory one would expect that there is some interaction whenever a quantum and a material particle come together. Therefore here it is a question of a departure from classical theory. This hypothesis is (as it appears to me), however, very similar to the assumption one usually makes about the stability of stationary states. From classical theory one would expect that no stationary states are possible and that an interaction or coupling (Bohr) between ether and the excited atom and the radiation connected with it is always present. On the other hand, in order to explain spectral emission we must assume it to be possible that no interaction (emission) occurs. Because of the persistence of stationary states we are therefore led to assign to every stationary state a probability coefficient or a mean lifetime.

The assumption made above, that even in a collision no interaction need occur, is quite analogous to the assumption of the stability of stationary states which is so fundamental to Bohr's theory of line emission and can be traced back to the same origin—the probability of existence of the stationary states of the particles. It is interesting to remark in this connection that in the analogous case of a collision of an electron with an atom, experiments show that the electron goes through the atom without changing either the interatomic or its own motion.

The relation $g_r\ \beta = g_s\ \alpha$.

The coefficients or as Bohr calls them, the weights g, are introduced when the valence electron in the atom executes a finite degenerate periodic motion. These numerical coefficients give, according to Bohr, in how many ways from a neighbouring non-degenerate orbit one can arrive at the special degenerate orbit as the limiting case. Let us assume that these coefficients are simply proportional to the number of possibilities of transition from a definite state to the state under consideration ; then the above relation is almost obvious. There are g_s possible ways for a transition from an rth state to the given sth state through absorption. Similarly there are g_r transitions to the lower states as a result of emission. The probability coefficient β has then the factor g_s, the probability coefficient for emission the factor g_r. If we further assume that the probability of a transformation through radiation in a definite way is the same as the probability coefficient for emission when the atom makes transitions to lower states in a definite way, then the relation in question follows immediately[9].

Manindra Physical Laboratory, Dacca University, June 14, 1924.

I consider Bose's hypothesis about the probability of elementary radiation precesses as not appropriate for the following reasons.

For the statistical equilibrium between two states of a Bohr atom the following relation holds, as given by Bose :

$$\frac{n_r}{g_r}\frac{N_\nu}{A_\nu + N_\nu} = \frac{n_s}{g_s}.$$

It follows that the probabilities for the transitions $r \rightarrow s$ and $s \rightarrow r$ respectively on the left and the right hand side of the equation must be proportional to each other. The transition probabilities for a molecule must therefore (if for simplicity we put the statistical weights of both the states equal to 1) behave as $\frac{N_\nu}{A_\nu + N_\nu} : 1$. Nothing more can be learnt from the knowledge of the thermodynamic equilibrium. According to my assumptions these probabilities should be proportional to N_ν (i.e. the radiation density) and $A_\nu + N_\nu$ respectively, whereas according to Bose's assumption, $\frac{N_\nu}{A_\nu + N_\nu}$ and 1 respectively.

According to the latter assumption the external radiatian can indeed cause a transition from a state Z_r of lower energy to a state Z_s of higher energy, but not the reverse transition from Z_s to Z_r . This, however, contradicts the generally and rightly accepted fundamental principle that the classical theory should be a limiting case of the quantum theory. According to the latter a radiation field can transfer to a resonator positive as well as negative energy (depending on the phase) and indeed with equal probability. The probabilities of both the transitions must depend on the density of radiation, that is, on N_ν as oppsed to Bose's hypothesis. Planck has discussed in detail in the latest edition of his book on the theory of radiation to what extent the classical theory is the limiting case of the quantum theory.

Secondly, according to Bose's hypethesis a cold body should have an absorbing capacity dependent on the density of radiation (decreasing with it). Bodies in cold state should absorb "non- Wien" radiation to a weaker extent than less intense radiation as defined by the range of validity of Wien's radiation formula. If the behaviour had been such, then it would certainly have already been discovered in infra-red radiation from hot light sources.

A. Einstein

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(*rearranged sequentially*)

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Messungen der Zersetzungsspannung in nichtwässerigen Lösungsmitteln.

Von

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Aus dem Englischen übertragen.

(Mit 5 Figuren im Text.)

(Eingegangen am 18. 11. 26.)

Über die Zersetzungsspannungen von Salzen in nichtwässerigen Lösungsmitteln liegen nur sehr wenige Untersuchungen vor. Patten und Mott[1]) haben die Kurven der Zersetzungsspannung von Lithiumchlorid in Alkoholen, Aceton und Pyridin unter Berücksichtigung der Vorgänge an Anode und Kathode aufgenommen. Kürzlich haben Müller und Duschek[2]), ferner Müller[3]), Hölzl und andere die Zersetzungsspannung bei Lösungen von Silbernitrat in Pyridin untersucht und die Zersetzungsspannung des Lithiums in Lösungen von Lithiumchlorid und Lithiumnitrat in Pyridin. Diese Untersuchungen ergaben für Pyridin beträchtlich höhere Werte der Zersetzungsspannung als für wässerige Lösungen. Doch waren sie ohne rotierenden Kommutator im Stromkreis angestellt und die elektromotorische Gegenkraft erscheint daher durch das dem grossen Übergangswiderstand der an der Elektrodenoberfläche haftenden Schicht entsprechende Potential vergrössert. Ganz kürzlich hat Newbury in einer Veröffentlichung über „Überspannung und Übergangswiderstände“[4]) festgestellt, dass „die gesamte, dem Durchgang des Stromes von Elektrode zu Elektrolyt sich entgegenstellende Hemmung aus zwei verschiedenen Teilen besteht, wovon der eine reversibel ist (reine Überspannung) und der andere irreversibel (Übergangswiderstände). Die ohne Verwendung eines Kommutators angestellten Messungen der Überspannung pflegen mit einem Fehler von 0.5 bis etwa 2 Volt behaftet zu sein“. Pearce und France[5]), sowie

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[3]) Monatsh. f. Chemie **43**, 429 (1923).

[4]) Proc. Roy. Soc. **443** A, 486 (1925).

[5]) Journ. Phys. Chem. **18**, 729 (1914).

Mortimer und Pearce[1]) haben bei Silbernitrat Einzelpotentiale und Gesamtspannung in reinen und gemischten Lösungsmitteln untersucht, und zwar in Methylalkohol, Äthylalkohol, Pyridin und Wasser, sowie in binären Gemischen dieser Stoffe. Dabei fanden sie, dass die Eigenschaften der binären Gemische von Alkoholen mit Pyridin der Mischungs-

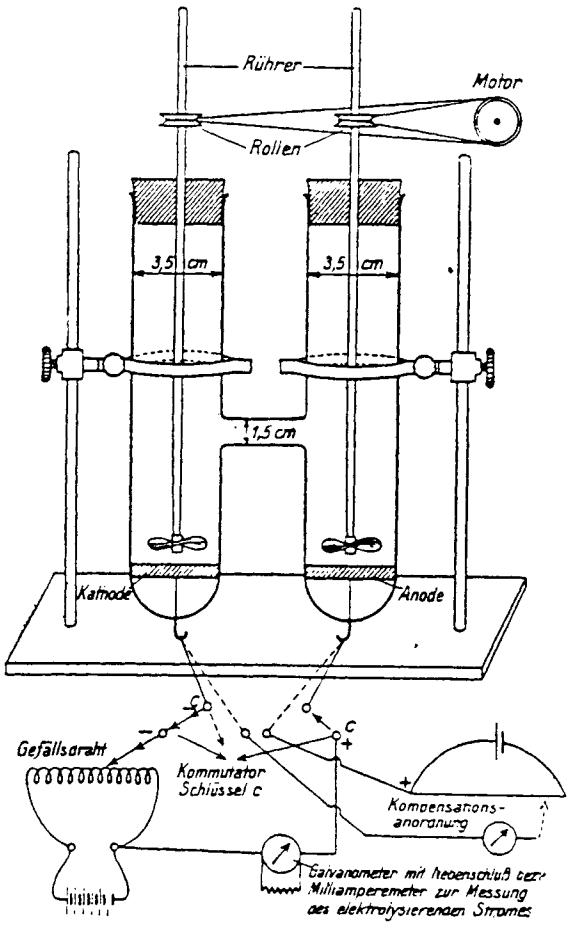


Fig. 1.

regel folgen, während binäre Gemische von Wasser mit Methylalkohol, Äthylalkohol und Pyridin meist ein ausgesprochenes Maximum oder Minimum der hier in Betracht kommenden Eigenschaften aufweisen. Dies deutet auf eine gegenseitige Beeinflussung der beiden Lösungsmittel hin.

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1) Journ. Phys. Chem. **21**, 275 (1917).

Im folgenden sollen die Ergebnisse dargelegt werden, die bei der Untersuchung der Entladungsspannung von Chlorwasserstoff und einigen Alkalihalogeniden in reinem Methylalkohol sowohl als auch in Mischungen

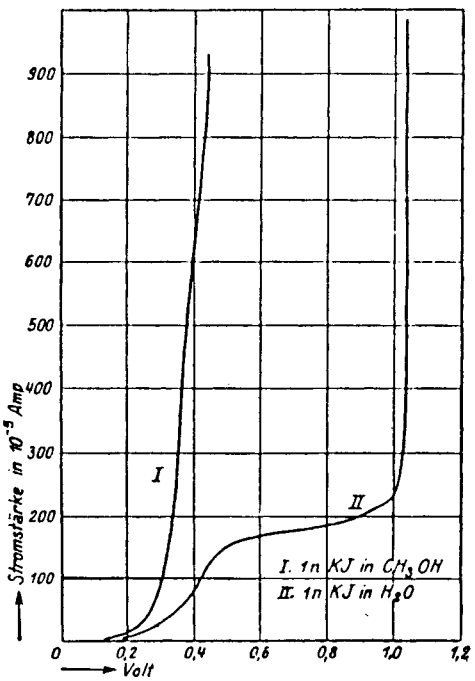


Fig. 2.

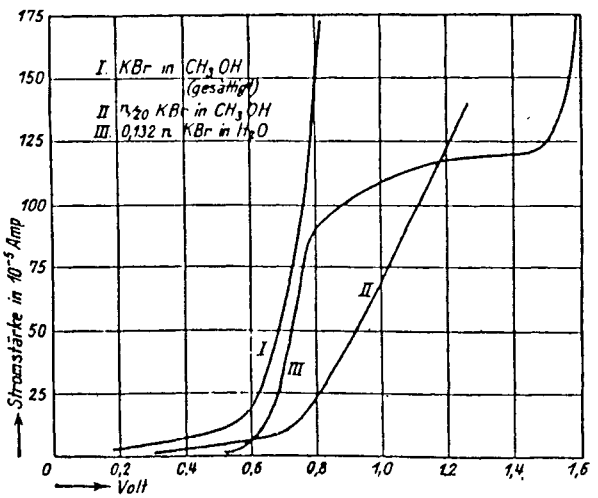


Fig. 3.

von Methylalkohol und Wasser erhalten wurden. Der Methylalkohol war von Merck acetonfrei bezogen, wurde etwa 12 Stunden über Kalk

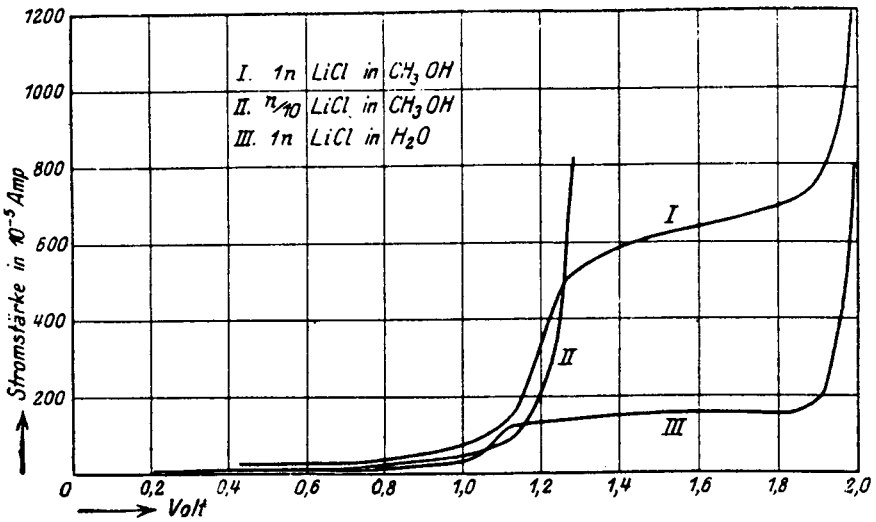


Fig. 4.

stehen gelassen und dann nochmals destilliert.

Die verwendete Versuchsanordnung erhellt aus Fig. 1. Die Elektroden waren aus dikkem Platinblech und mit Platinschwarz überzogen. Durch den rotierenden Kommutator *C* konnte der Strom fast 3300 mal in der Minute gewendet werden, so dass die Polarisationsspannung der Zelle gemessen werden konnte, nachdem der elektrolysierende Strom bei jeder Wendung nur während 0·009 Sekunden die Zelle durchflossen hatte. Die Stärke des durch die Zelle gesandten Stroms wurde durch ein sorgfältig geeichtes Galvanometer mit Nebenschluss gemessen, für die höheren

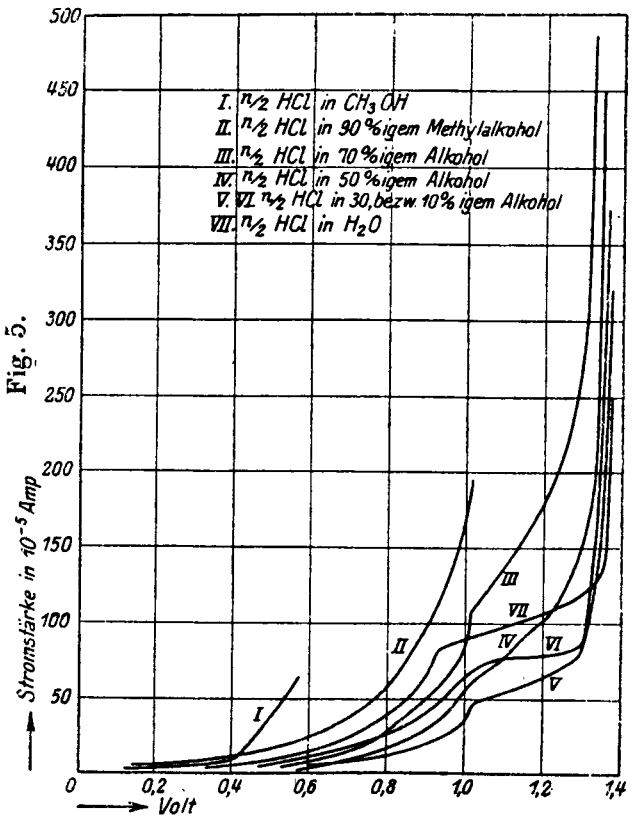


Fig. 5.

Stromstärken wurde ein Präzisions-Milliamperemeter verwendet.

Die Ergebnisse sind aus den Fig. 2, 3, 4 und 5 ersichtlich, worin sie als Stromspannungskurven niedergelegt sind. Die Kurven für die Zersetzungsspannungen in wässeriger Lösung (bei entsprechender Konzentration) sind ebenfalls eigenen Messungen entnommen und stehen in guter Übereinstimmung mit früheren Arbeiten anderer Autoren[1]).

Tabelle 1. *KJ.*

1 norm. *KJ* in $CH_3 . OH$		1 norm. *KJ* in H_2O	
Entladungsspannung in Volt	Stromstärke in 10^{-5} Amp.	Entladungsspannung in Volt	Stromstärke in 10^{-5} Amp.
0·148	5·6	0·194	1·7
0·251	27·8	0·441	110·7
0·294	67·5	0·606	164·7
0·318	92·2	0·794	195·8
0·332	141·8	1·0468	237·0
0·338	181·8	1·116	744·0
0·354	250·0	1·152	1200·0
0·369	372·0		
0·409	610·0		
0·448	970·0		

Tabelle 2. *KBr.*

0·132 norm. *KBr* in $CH_3 . OH$		0·066 norm. *KBr* in $CH_3 . OH$		0·132 norm. *KBr* in H_2O	
Entladungsspannung in Volt	Stromstärke in 10^{-5} Amp	Entladungsspannung in Volt	Stromstärke in 10^{-5} Amp.	Entladungsspannung in Volt	Stromstärke in 10^{-5} Amp.
0·194	3·69	0·3	1·12	0·254	0
0·386	7·7	0·493	4·2	0·408	1·4
0·588	15·4	0·726	9·5	0·592	5·32
0·645	32·0	0·800	23·0	0·65	16·8
0·688	58·0	0·846	37·0	0·699	37·8
0·757	90·0	0·933	54·0	0·779	87·8
0·780	104·0	1·019	68·8	0·835	94·5
0·797	135·0	1·074	87·8	0·939	108·0
0·821	162·6	1·173	114·8	1·173	118·0
0·849	206·5	1·24	135·0	1·439	120·0
				1·579	166·0
				1·592	195·8
				1·614	236·3
				1·642	296·0
				1·655	348·0
				1·665	423·0

[1]) Parkin, „Practical methods of Electrochemistry". (Da mir dies Buch nicht mehr zugänglich ist, kann ich die Seitenzahl nicht angeben.)

Tabelle 3. *LiCl.*

1 norm. *LiCl* in $CH_3 . OH$		0·1 norm. *LiCl* in $CH_3 . OH$		1 norm. *LiCl* in H_2O	
Entladungs-spannung in Volt	Stromstärke in 10^{-5} Amp.	Entladungs-spannung in Volt	Stromstärke in 10^{-5} Amp.	Entladungs-spannung in Volt	Stromstärke in 10^{-5} Amp.
0·444	9·94	0·205	1·12	0·207	0
0·768	34·4	0·586	8·4	0·649	6·72
0·917	54·0	0·794	22·4	0·884	15·26
1·089	104·6	0·925	42·7	1·051	48·5
1·163	155·0	1·024	52·7	1·108	114·8
1·188	297·6	1·113	81·2	1·812	137·8
1·221	372·0	1·138	101·25	1·916	213·0
1·254	469·7	1·16	138·25	1·946	292·0
1·318	565·0	1·177	168·75	1·966	440·0
1·504	613·8	1·18	200·0	1·992	628·0
1·672	660·0	1·19	229·56	2·015	790·0
1·864	730·0	1·212	274·0	2·046	1210·0
1·913	855·6	1·22	311·0		
1·97	967·0	1·229	356·0		
1·984	1116·0	1·236	388·8		
2·0148	1302·0	1·243	443·8		
		1·244	480·0		
		1·254	527·2		
		1·27	625·0		
		1·277	683·0		
		1·287	800·0		

Tabelle 4. *HCl.*

Zersetzungsspannung der Salzsäure in 0·5 norm. Lösungen. V = Entladungsspannung in Volt, a = Stromstärke in 10^{-5} Amp.

100% $CH_3 . OH$		90% $CH_3 . OH$ 10% H_2O		70% $CH_3 . OH$ 30% H_2O		50% $CH_3 . OH$ 50% H_2O	
V	a	V	a	V	a	V	a
0·123	2·1	0·145	2·8	0·201	1·4	0·173	0·84
0·362	5·6	0·269	4·2	0·538	4·2	0·327	1·54
0·404	9·8	0·404	9·8	0·678	12·6	0·42	2·8
0·436	16·0	0·478	14·0	0·769	20·8	0·703	9·8
0·47	21·0	0·57	22·1	0·829	36·4	0·76	13·7
0·482	27·8	0·619	26·6	0·893	50·6	0·855	23·1
0·499	35·7	0·694	36·4	0·94	62·1	0·956	40·6
0·514	42·0	0·756	48·0	0·947	82·4	1·052	63·4
0·530	49·35	0·807	62·0	1·067	107·4	1·167	94·5
0·543	55·5	0·842	74·0	1·105	143·1	1·243	112·25
		0·893	91·0	1·215	200·5	1·291	141·75
		0·937	119·0	1·26	225·8	1·32	195·8
		0·995	155·0	1·317	353·4	1·341	232·5
		1·037	195·0	1·338	478·9	1·345	274·4
				1·357	618·4	1·356	325·5

Tabelle 4 (Fortsetzung).

30% $CH_3 . OH$ 70% H_2O		10% $CH_3 . OH$ 90% H_2O		100% H_2O	
V	a	V	a	V	a
0·153	0·84	0·203	0·28	0·34	4·2
0·276	2·66	0·249	1·4	0·604	12·6
0·403	4·2	0·47	5·6	0·929	81·0
0·467	4·2	0·601	8·4	1·116	101·25
0·601	7·7	0·687	16·1	1·298	118·6
0·748	9·8	0·844	33·6	1·355	229·5
0·892	19·8	0·948	54·0	1·376	423·12
1·013	36·4	1·104	77·6	1·4	678·9
1·073	46·2	1·26	81·0	1·404	924·6
1·294	81·0	1·331	148·5	1·434	1353·2
1·332	155·3	1·366	287·7		
1·352	229·5	1·387	451·0		
1·363	372·0	1·404	650·0		
1·386	511·5				
1·393	687·0				

Die Versuchsergebnisse.

Kaliumjodid (vgl. Tabelle 1 und Fig. 2).

Kaliumjodid wurde in ungefähr normaler methylalkoholischer Lösung verwendet. Die gefundene Zersetzungsspannung liegt bei nur 0·32 Volt. Die Anodenflüssigkeit wurde durch die heftige Jodabscheidung schnell angefärbt. In wässeriger Lösung wurde die Zersetzungsspannung von normalem Kaliumjodid bei 1·12 Volt gefunden (Parkin fand 1·14 Volt).

Kaliumbromid (vgl. Tabelle 2 und Fig. 3).

Etwa 0·1 und 0·05 norm. Lösungen von Kaliumbromid in reinem Methylalkohol ergaben eine Zersetzungsspannung von 0·68 bzw. 0·71 Volt. Offensichtlich steigt hier die Zersetzungsspannung mit der Verdünnung. In wässeriger Lösung liegt der Wert für 0·1 norm. Kaliumbromid bei 1·54 Volt (nach Parkin in normaler Lösung bei 1·61 Volt).

Lithiumchlorid (vgl. Tabelle 3 und Fig. 4).

Bei 0·1 norm. Lithiumchlorid zeigt die Kurve nur einen scharfen Knickpunkt und zwar bei etwa 1·2 Volt; bei einfach norm. Lösung, ebenfalls in reinem Methylalkohol, tritt bei etwa 1·90 ein zweiter Knick auf. Nun ergibt eine einfach normale Lösung von Lithiumchlorid im Wasser bei 1·95 Volt einen Knickpunkt. So könnte man vielleicht an-

nehmen, dass in der einfach normalen Lithiumchloridlösung in reinem Methylalkohol Spuren von Wasser enthalten sind, die durch das stark hygroskopische Salz selbst hinein gebracht wurden.

Chlorwasserstoff (vgl. Tabelle 4 und Fig. 5).

Chlorwasserstoff wurde in etwa 0·5 norm. Lösung untersucht und zwar sowohl in reinem Methylalkohol und reinem Wasser, als auch in Mischungen der beiden von verschiedenem Prozentgehalt (10, 30, 50, 70 und 90 % Wasser). In reinem Methylalkohol wurde die Zersetzungsspannung bei 0·41 Volt ermittelt. Bei reinem und 90 %igem Methylalkohol zeigte sich nach Erreichung der Zersetzungsspannung bei 0·4 bzw. 1·0 Volt deutlich die Wirkung des Chlors auf die Anode. Aus Fig. 5 ist die Änderung der Kurvenform bei den Gemischen deutlich zu ersehen.

Erörterung des Einflusses von Gemischen.

Beim Chlorwasserstoff hat der Zusatz von Wasser zu Methylalkohol einen sehr deutlichen Einfluss auf die Zersetzungsspannung dieser Säure. Nun werden durch den Zusatz eines zweiten Lösungsmittels die Eigenschaften der Ionen des Elektrolyten immer dann sehr verändert, wenn zwischen ihnen und dem ersten Lösungsmittel eine engere Bindung möglich war. Dies hat Kraus[1]) an der Leitfähigkeit von Säurelösungen in Alkohol festgestellt, und zwar bei Zusatz von Wasser sowohl als von Salzen, die zur Komplexbildung neigen. Steigt der Prozentgehalt des Wassers in den methylalkoholischen Lösungen der Salze bis zu 30 % und mehr, so tritt nach dem Knickpunkt bei etwa 1·0 Volt noch ein zweiter zwischen 1·3 und 1·33 Volt auf, der der Zersetzungsspannung der Salzsäure in reinem Wasser entspricht. Für 10 %igen Wassergehalt liegt die Zersetzungsspannung bei 1·0 Volt und für reinen Methylalkohol bei 0·41 Volt, wobei die Salzsäure in allen Fällen

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[1]) „Eigenschaften von Systemen mit elektrischer Leitfähigkeit." S. 176 bis 184: Leitfähigkeit von Elektrolyten in Lösungsmittelgemischen. Zeitschr. f. physik. Chemie. CXXV.

0·5 normal war. Demnach ändert sich die Zersetzungsspannung der Salzsäure nicht fortschreitend mit steigendem Wassergehalt, sondern die Mehrzahl der Kurven zeigt zwei ausgeprägte Knickpunkte bei 1·0 und 1·33 Volt. Da in reinem Methylalkohol der Knick bei etwa 0·41 Volt liegt, muss man den bei 1·0 Volt wohl der Bildung eines Komplexes in dem Lösungsmittelgemisch zuschreiben.

Der Einfluss des Wasserzusatzes auf Lithiumchlorid in Pyridin ist von Patten und Mott[1]) untersucht worden. Ihre Resultate sind nicht massgebend, da bei ihren Versuchen die Elektroden sich mit einer isolierenden Schicht überzogen hatten. Die von ihnen erhaltenen Kurven entsprechen in ihrer Gestalt den hier wiedergegebenen. Müller, Hölzl[2]) und andere untersuchten die Entladung des Lithiums in Lösungen von Lithium in Pyridin mit 5 und 10% Wasser. Die erhaltenen Kurven waren vielfach geknickt und unregelmässig. Sie schreiben dies der Abscheidung von Lithiumhydroxyd zu. Lithiumchlorid geht mit Wasser überdies bekanntermassen Molekularverbindungen ein, demnach notwendigerweise auch mit einer Lösung von Methylalkohol und Wasser[3]).

Der Einfluss des Wasserzusatzes auf methylalkoholische Lösungen von Kaliumjodid und Kaliumbromid kann die Gestalt der Kurven nicht sehr ändern, da die Ionen dieser einfach gebauten Salze schwerlich mit dem stark assoziierten Methylalkohol zur Komplexbildung neigen werden. Die Leitfähigkeit wurde jedenfalls durch Wasserzusatz nicht stark verändert.

Erörterung der Stromspannungskurven.

Die Stromspannungskurven in alkoholischer Lösung zeigen sämtlich einen einzigen charakteristischen Knickpunkt, der ziemlich scharf ausgeprägt ist. In wässerigen Lösungen wurden zwei deutlich verschiedene Knickpunkte festgestellt, wovon der eine, und zwar der bei der kleineren Spannung, nicht mit sichtbarer Zersetzung parallel geht, was bei dem zweiten aber untrüglich der Fall ist.

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[1]) A. a. O.

[2]) A. a. O.

[3]) Kraus, a. a. O.

Die Werte der Entladungsspannung wurden in Alkohol sehr viel niedriger gefunden als in Wasser. Andere Untersuchungen an Pyridinlösungen ergaben viel höhere Werte als in wässeriger Lösung. Müller gibt für 1 norm. Lithiumchlorid in Pyridin 3·8 bis 4·0 Volt als Zersetzungsspannung an und für 0·1 norm. Lösung 4·5 Volt, Patten und Mott fanden 4·0 und 4·15 Volt als gesamte Polarisationsspannung der Zelle in Pyridin- bzw. Acetonlösung, während der von uns in Methylalkohol gefundene Wert für 0·1 norm. Lithiumchlorid nur 1·2 Volt beträgt.

Für die hier untersuchten Substanzen zeigt sich in methylalkoholischer Lösung die gleiche Reihenfolge der Zersetzungsspannung wie in wässeriger, sie nehmen in der Reihe *LiCl*, *KBr*, *HCl*, *KJ* ständig ab.

Zusammenfassung.

1. Für Kaliumjodid, Kaliumbromid, Lithiumchlorid und Chlorwasserstoff wurden in reinem Methylalkohol Stromspannungskurven aufgenommen, die Zersetzungsspannungen bestimmt und die Kurven mit denen in wässeriger Lösung verglichen

2. Bei Lithiumchlorid wurde festgestellt, dass schon die Gegenwart von Spuren Wassers die Gestalt der Zersetzungskurve ändert.

3. Es zeigte sich, dass die Werte für die Zersetzungsspannung von Lösungen in reinem Methylalkohol tiefer liegen als in wässerigen Lösungen, aber dieselbe Reihenfolge zeigen, nämlich absteigend von Lithiumchlorid über Kaliumbromid zu Chlorwasserstoff und Kaliumjodid.

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Measurements of the Decomposition Voltage in Non-aqueous Solvents

Susil Chandra Biswas and S. N. Bose

(The text contains 5 diagrams)
(Received on 18.11.26)

Very few findings are available on the decomposition voltage of salts in non-aqueous solvents. Patten and Mott[1] have recorded curves for the decomposition voltage of lithium chloride in various alcohols, acetone and pyridine, taking due consideration of the process taking place at the anode and the cathode. In the recent past Mueller and Duschek[2] and then Muller[3] and Hoelzl and others examined the decomposition voltage in the solutions of silver nitrate in pyridine as well as the decomposition voltage of lithium in solutions of lithium chloride and lithium nitrate in pyridine. These results for pyridine showed considerably higher values for the decomposition voltage than those for the aqueous solutions. They were, however, taken without the installation of a rotating commutator in the circuit; it seems therefore that the opposing electromotive force was increased by the potential corresponding to the large contact resistance of the layer sticking to the surface of the electrode. Very recently Newbury in a publication about the *Overvoltage and the Contact Resistances*[4] has established that 'the total barrier opposing the flow of current from the electrode to the electrolyte consists of two different parts : out of these one (the pure overvoltage) is reversible and the other (the contact resistance) is irreversible. The readings of overvoltage without using a commutator are usually afflicted with an error of 0.5 to 2 volts.' Pearce and France[5] as well as Mortimer and Pearce[6] investigated the individual potentials and the overall voltage for silver nitrate in pure and mixed solvents. These examinations were carried out in methyl alcohol, ethyl alcohol, pyridine and in water as well as in binary mixtures of these substances. These investigations revealed that whereas the binary mixtures of alcohols with pyridine followed the rules for the mixtures, the mixtures of water with

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Fig. 1 : *(see the German original)*

Rührer	= stirring rod	Rollen	= pulleys
Motor	= motor	Kathode	= cathode
Anode	= anode	Gefällsdraht	= step-down coil
Kommutator Schlüssel = commutator switch		Kompensations anordnung = compensator arrangement	

Galvanometer mit Nebenschulss bzw. Milliamperemeter zur Messung des elektrolysierenden Stromes	=	shunted galvanometer milliammeter for measuring the electrolysing current

methyl alcohol, ethyl alcohol and pyridine mostly showed a pronounced maximum or a minimum in the properties considered here. This fact suggests mutual influence of the two solvents. The results as obtained from the examination of the discharge voltage of hydrogen chloride and a few alkali halides in pure methyl alcohol as well as in mixtures of methyl alcohol and water are presented below.

Fig. 2 *and* **Fig. 3** : *(see the German original)*

(Stromstärke in = Current intensity in)10^{-5} Amp

Fig. 4 *and* **Fig. 5** : *(see the German original)*

(Stromstärke in = Current intensity in)10^{-5} Amp

II. N/2 HCl in 90% methyl alcohol, etc.

Acetone-free methyl alcohol was obtained from Merck. It was allowed to stand for about 12 hours over lime and was then distilled once more.

The experimental set-up used is clear from Fig. 1. The electrodes were made of thick platinum plate and were coated with platinum black. The current could be reversed almost 3300 times per minute with the help of the rotating commutator C. This enabled the measurement of the polarization voltage of the cell when the electrolysing current after every reversal had passed through the cell only for a period of 0.009 seconds. The strength of the current sent through the cell was measured by a carefully calibrated shunted galvanometer. A precision milliammeter was used for measuring stronger currents.

The results are clear from the Figs. 2,3,4 and 5, where they are set down as current-voltage curves. The curves for the decomposition voltages in aqueous solutions (with respective concentrations) were similarly obtained from appropriate measurements and they tally well with earlier works of other authors.[7].

Table 1. ***KI***

1 N *KI* in $CH_3.OH$		1 N *KI* in H_2O	
Discharge potential in volts	Current strength in 10^{-5} Amp	Discharge potential in volts	Current strength in 10^{-5} Amp

(For numerical values, see the German original.)

Table 2. *KBr*

0.132 N *KBr* in $CH_3.OH$		0.066 N *KBr* in $CH_3.OH$		0.132 N *KBr* in H_2O	
Discharge potential in volts	Current strength in 10^{-5} Amp	Discharge potential in volts	Current strength in 10^{-5} Amp	Discharge potential in volts	Current strength in 10^{-5} Amp

(For numerical values, see the German original.)

Table 3. *LiCl*

1 N *LiCl* in $CH_3.OH$		0.1 N *LiCl* in $CH_3.OH$		1 N *LiCl* in H_2O	
Discharge potential in volts	Current strength in 10^{-5} Amp	Discharge potential in volts	Current strength in 10^{-5} Amp	Discharge potential in volts	Current strength in 10^{-5} Amp

(For numerical values, see the German original.)

Table 4. *HCl*

The decomposition voltage of the hydrochloric acid in 0.5 N solutions
V = discharge potential in Volts, a = current strength in 10^{-5} Amp.

100% $CH_3.OH$		90% $CH_3.OH$ 10% H_2O		70% $CH_3.OH$ 30% H_2O		50% $CH_3.OH$ 50% H_2O	
V	a	V	a	V	a	V	a

(For numerical values, see the German original.)

The Results of the Experiments

Potassium iodide (Cf. Table 1 and Fig. 2)
A nearly 1N solution of potassium iodide in methyl alcohol was used. The reading for the decomposition voltage lies in the vicinity of only 0.32 Volt. The anode liquid became quickly tinged by the heavy iodine deposition. The decomposition voltage of 1N potassium iodide in an aqueous solution was found to be 1.12 Volt (Parkin found it to be 1.14 Volt).

Potassium bromide (Cf. Table 2 and Fig. 3)
Approximately 0.1 and 0.5 N solutions of potassium bromide in pure methyl alcohol yielded decomposition voltages of 0.68 and 0.71 Volt respectively. Apparently the

decomposition voltage increases here with dilution. In the aqueous solution it was in the vicinity of 1.54 Volt for 0.1N potassium bromide (according to Parkin it is 1.61 Volt for a 1N solution).

Lithium chloride (Cf. Table 3 and Fig. 4)
In the case of a 0.1N lithium chloride solution the curve shows a sharp bend in the neighbourhood of about 1.2 Volt. In the case of a 1N solution in pure methyl alcohol, a second bend similarly appears in the neighbourhood of 1.90 Volt. A 1N solution of lithium chloride in water shows a bend in the vicinity of 1.95 Volt. One could therefore assume that there are traces of water present (coming from the strongly hygroscopic salt) in a 1N lithium chloride solution in pure methyl alcohol. On the contrary, the majority of the curves show two marked bends in the vicinity of 1.0 Volt and 1.33 Volt. Since in the case of methyl alcohol the bend lies in the neighbourhood of 0.41 Volt, one must attribute the bend in the vicinity of 1.0 Volt to the formation of a complex in the mixture of solvents.

The influence of adding water to lithium chloride in pyridine was examined by Patten and Mott[9]. Their results are not decisive as the electrodes became coated in their experiments with an insulating layer. The shapes of the graphs produced by them correspond with the ones reproduced here. Mueller, Hoelzl[10] and others examined the discharge of lithium in solutions of lithium in pyridine with 5 and 10% water content. The graphs thus produced contained several bends and were irregular. They attribute this fact to the deposition of lithium hydroxide. Moreover, it is well known that lithium chloride enters to form a molecular compound with water and therefore necessarily also with a solution of methyl alcohol and water[11].

The addition of water to the solution of potassium iodide and potassium bromide in methyl alcohol cannot influence the curve in such a way as to change its shape very much, because the ions of these simply built salts will be scarcely inclined to form complexes with the strongly associated methyl alcohol. In any case the conductivity was not much changed because of the addition of water.

Discussion of the Current Voltage Curves

The current voltage curves in alcoholic solutions show a single sharply pronounced characteristic bend. In aqueous solutions two distinctly different bends were established of which the one with the lower voltage does not go along with a visible decomposition which is unmistakably the case with the other one.

The values for the discharge potential were found to be very much lower in alcohol than those in water. Other studies on pyridine solutions have yielded much higher values than the ones for an aqueous solution. Mueller states 3.8 to 4.0 Volt to be the decomposition voltage for a 1 N lithium chloride solution in pyridine. For a 0.1 N solution he states it to be 4.5 Volt. Patten and Mott found 4.0 Volt and 4.15 Volt to be the total polarization potential of the cell in pyridine and acetone solutions respectively. In contrast, the value for 0.1 N lithium chloride in methyl alcohol was found by us to be only 1.2 Volt.

The decomposition voltage for substances studied here show the same sequence in

a methyl alcohol solution as in an aqueous solution. They decrease steadily in the sequence of LiCl, KBr, HCl, KI.

Summary

1. Current voltage curves were obtained for potassium iodide, potassium bromide, lithium chloride and hydrogen chloride in pure methyl alcohol. The decomposition voltage was determined ; the curves were compared with those for aqueous solutions.
2. It was established for lithium chloride that even the presence of traces of water changes the shape of the decomposition voltage curve.
3. It was seen that the values for the decomposition voltage in solutions of pure methyl alcohol are lower than those in aqueous solutions but show the same sequence, namely descending from lithium chloride to potassium bromide, hydrogen chloride and potassium iodide.

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(rearranged sequentially)

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Beryllium Spectrum in the Region λ 3367-1964. *By* S.N. BOSE, *Professor of Physics, Dacca University, and* S.K. MUKHERJEE, *Assistant Lecturer in Physics, Dacca University**.

[Plates II. & III.]

THE beryllium spectrum is remarkable owing to the small number of lines that have been observed in the visible and the ultra-violet. The earlier measurements are somewhat conflicting. Exner and Haschek could not find some of the lines observed by Rowland and Tatnall †. Glaser ‡ investigated the spectrum by sparking between the metal tips, but he also could not corroborate the earlier observations.

Recently Millikan and Bowen § subjected the spectrum to a thorough analysis by their method of hot spark, and extended it up to 1943 Å.U. They classified some of the important lines, and gave the term values both of Be II and Be I.

It was with a view to study the spectrum thoroughly, under different conditions of excitation, that the present work was undertaken.

The instrument that we have used almost exclusively for the work is Hilger's Quartz Spectrograph, type E 1. In the near ultra-violet between the regions 2400-1850 it is almost an ideal instrument for spectrum analysis, combining a high dispersion with a high light-value. We have extended our observations to the visible region; but here, because of the low dispersion, the measurements were not more accurate than by ·1 Å.U., though we could very easily identify the lines and thus verify the observations of the previous workers.

Most of the observations were made on the arc spectrum, and obtained by feeding the nitrate or the metal in a carbon arc. We have used both ordinary and Hilger's extra pure carbon rods, and the presence of traces of iron has been an advantage in furnishing suitable standard lines in the different regions. In the extreme ultra-violet we have used copper arc and spark as comparison spectra, and in some cases the silicon lines almost always present in carbon spectra were helpful in making accurate measurements.

The plates used were Ilford Empress and Ordinary up to 2175. In the extreme ultra-violet we tried sensitization with machine oil, but the lines were a little broad. As our aim was to resolve very narrow doublets, we could only get the best results by using Schumann plates as supplied by Hilger.

The materials used were, at first, certain old samples of beryllium nitrate and metal from Merck's, which were found to contain lead, aluminium, and silver as impurities. Later, owing to the kindness of Professor Mark, of Badische Anilin Fabrik, we were enabled to get a different sample of the metal. But this was found to contain traces of rare earths, notably scandium and yttrium. By a comparison of the different samples the lines due to contamination were eliminated, and we give our results tabulated below (*vide* Table I.).

As may be seen, we could go almost to the same limit as Millikan and Bowen by their hot-spark method, and, incidentally, we have discovered that the following lines, 2351, 2175,2126, 2056, and 2033, given as singlets by Millikan and Bowen, are clearly doublets of approximate wave-number difference 2.6. Most probably these are triplets due to the triplet P-terms, and our spectrograph could only separate P_1 from $P_2 + P_3$. The wave-number difference thus measured is the distance of P_1 from the centre of gravity of P_2- and P_3- lines, and as such has a slightly higher value. The lines 1998 and 1964 just appeared as doublets, but owing to their hazy character the measurements of the doublet separation were not possible. We could verify the earlier measurements of Rowland and Tatnall as well as the line of Glaser, viz. λ 4672·9. Incidentally, we have discovered a few new lines whose wave-lengths are given in Table I. The line 3019 appeared as a doublet, the line 2986 as a triplet, and the lines 3110 and 2738 as singlets.

* Communicated by the Authors.

† Kayser, *Handbuch d. Spectro.* vol. v.

‡ Glaser, *Ann. d. Phys.* (4) lxvii. pp. 73-88 (1922). § Millikan and Bowen, Phys. Rev., Aug. 1926.

We have made some observations by feeding a fair amount of metal or the salt in the arc, and we could observe that the lines 3321, 2651, 2494, and 2175 with their components were reversed in some cases, thus showing that the *p*-level is probably the common origin of the lines as classified by Millikan and Bowen. In rare occasions, however, the line 3131, usually attributed to Be II, could be reversed. The line 2348 (1S-2P) is always reversed in the arc.

We have tried the spark spectra in air, where, owing to rapid oxidation, we could make very little progress. We have also tried sparking between the tips of the metal, enclosed in a partially evacuated vessel with quartz window; and even then we could not go beyond 2175. In this way,however, we have got an interesting band spectrum, which we believe to be due to beryllium and which consists of bands degraded to the red. The fine structures of the band have not yet been measured, and we give in Table II. the measurements of the edges.

There does not seem to be any line of Be between 4700 up to the extreme red. We are now continuing the work in the infra-red, and hope to be able to corroborate the solitary work of Theo Volk * in the region.

The following table gives the wave-lengths (in I.a, vac.) of the lines between λ3367-1964, with the frequency difference between the components, and also the lines (in I.a) of Rowland and Tatnalll, together with the few lines marked * observed by us as new :—

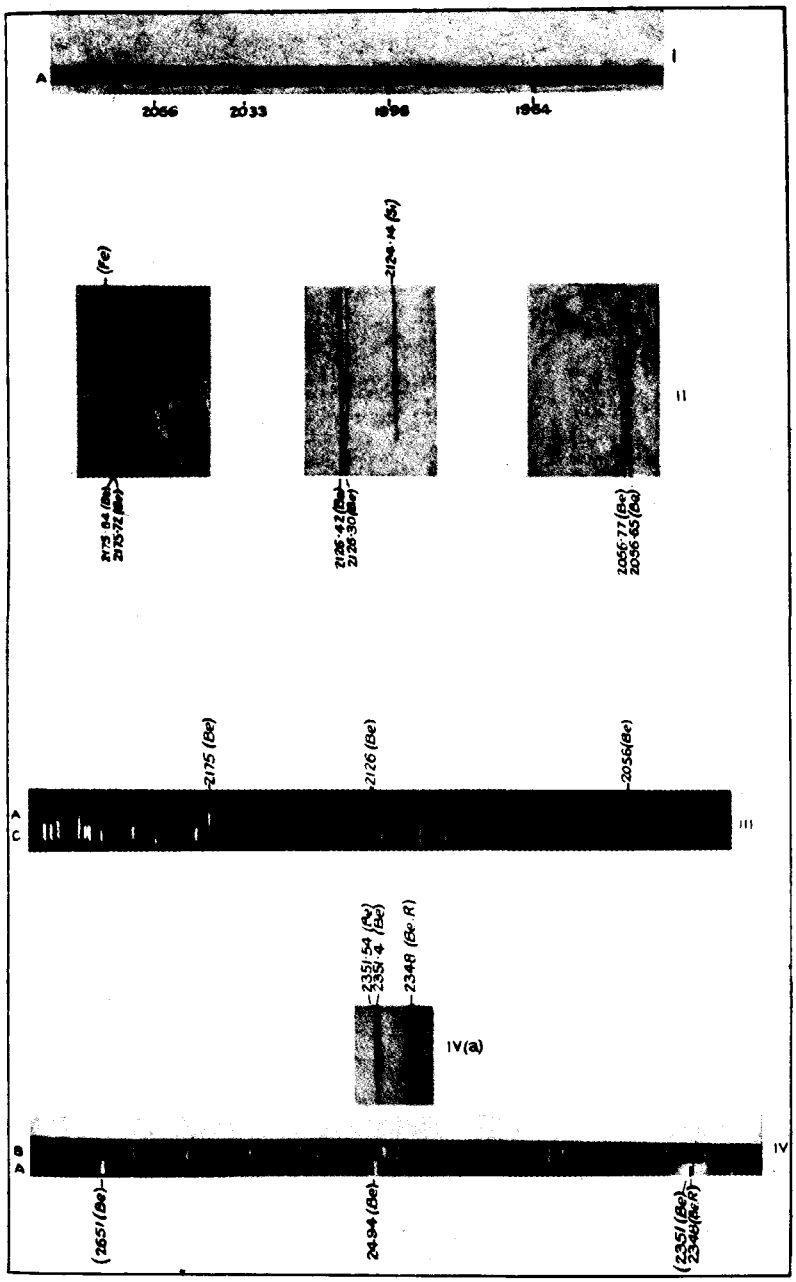
TABLE I.

Bose & Mukherjee	Rowland & Tatnall I. *a*.	Bowen & Millikan I. *a*, vac.	$\delta\nu$
3367.55	3367.579	-	-
*3110.91	-	-	-
*3019.54	-	-	-
*3019.33	-	-	-
2986.63	-	-	-
2986.44	2986.426	-	-
2986.08	2986.057	-	-
2898.26	2898.242	-	-
*2738.08	-	-	-
2351.40 2351.54	-	2351.50	2.50
2175.72 2175.84	-	2175.72	2.64
2126.30 2126.42	-	2126.57	2.66
2056.65 2056.77	-	2056.71	2.60
2033.23 2033.34	-	2033.43	2.66
1998(a)	-	1998.19	
(a)	-	1964.81	

The lines marked (a) appear in plates, but their measurements are unsatisfactory owing to the absence of standards in this region.

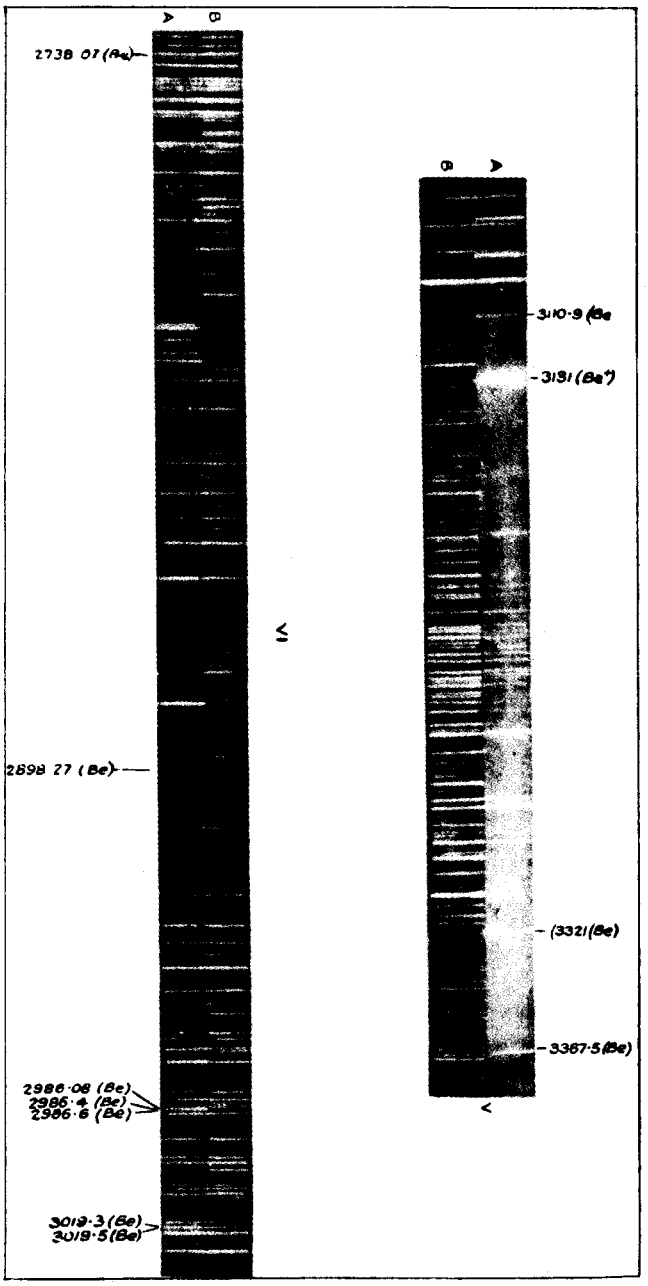
*Dissert. Tübingen, 1924 (Dresden bei Teubner). 17 pp.

Bose & Mukherjee. Phil. Mag. Ser. 7, Vol. 7, Pl. II.



A. Carbon arc fed with Be Salt.
B. Iron arc.

Bose & Mukherjee. Phil. Mag. Ser. 7, Vol. 7. Pl. III.



A. Carbon arc fed with Be Salts.
B. Iron Arc.

In the accompanying plates (Pls.II. and III.) I. and III. are contact-prints from Schumann plates; II. and IV. (a) are microphotographic enlargements for showing the fine structure; IV., contact-print from ordinary plate; V. and VI. are slightly enlarged prints to show the new lines.

The following table gives the wave-lengths (in I.a) of the edges of the band; those marked (a) are fully developed with their fine structures:—

TABLE II.

1.....	2474.2	6.....	2285.8
2 (a).....	2446.0	7.....	2260.8
3 (a)	2419.2	8.....	2238.3
4 (a).....	2325.1	9.....	2221.7
5 (a)	2299.4	10(a).....	2189.0

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Section of Mathematics and Physics.

President :—PROF. S. N. BOSE, M.Sc

Presidential Address.

TENDENCIES IN THE MODERN THEORETICAL PHYSICS.

The ultimate aim of Scientific inquiry is to arrive at a minimum number of hypotheses which will explain the maximum number of facts. The hypotheses should obviously not contradict one another. At the present moment, however, we see two contradictory theories, in the domain of Physics. On the one hand we have the classical theory based on the dynamical laws of Newton. On the other hand we have the Quantum theory first introduced by Planck, which has been differently formulated by different Scientists; while the Classical Theory explain satisfactorily all problems relating to motion and interaction of big masses as well as the problem of propagation of radiation, the Quantum Theory has succeeded with the help of a few principles in co-ordinating a large amount of experimental material accumulated in the various domains of spectroscopy, X-rays, etc. It has succeeded best in all problems dealing with the ultimate constitution of matter, or in problems dealing with the interaction of matter with radiant energy. As a result of work of the last twenty-five years, we seem to be much nearer to understanding the problem of matter. The periodic classification of Mendeljeff does not now appear as an unexplained riddle and the huge amount of spectroscopic material can now be classified and explained with the help of a few fairly simple principles. The task that faces the Physicists to-day is how best to harmonise the seeming discord of the two theories, which are at the present moment utilised to explain the physical phenomena. It is a difficult task, and the first step towards fulfilment will be to place in a clear light the differences that characterise them: for this purpose it is necessary to trace the history of the development of our physical ideas indicating the various points where fresh hypotheses had to be introduced before the next move towards progress could be made.

Theoretical Physics may be said to have begun its career as a Science with the formulation of the famous laws of motion by Newton. The mathematical formulation of this principle leads to a series of differential equations in which we equate certain quantities which depend upon the state and nature of the body under investigation with certain other quantities which we interpret as forces arising out of the

interaction of other bodies. The solution of these differential equations introduces a certain number of arbitrary constants which depend upon the initial state of the system investigated. These constants once known determine completely the subsequent history of the system under observation. Newton's original equations involved the use of the cartesian co-ordinate system, but very soon these equations were transformed into a form in which the arbitrary character of the co-ordinates was removed, and finally Hamilton and Jacobi introduced the characteristic function, which reduced the problem of solving the equations of dynamics to the solution of a single partial differential equation. Simultaneously, with the help of the concept of the Variational Calculus, we arrived at the celebrated Hamilton's Principle, by which the whole problem of dynamics was reduced to the problem of making a certain integral an extremum, within certain suitable limits.

The writing down of the set of differential equations, or of the single partial differential equation of Jacobi requires however the knowledge of the laws of interaction of various bodies on one another. The exact formulation of these laws seem therefore to be the immediate aim of the earlier classical Physicists. These laws once known allow us to apply to any problem the general dynamical methods, whereby the problems of Physics are reduced to problems of pure mathematics. In the laws of gravitation, in the law of Coloumb and of Ampere, we have some classical examples ; in every case it is endeavoured to express the force in terms of the relative co-ordinates, and the velocities of the interacting masses.

The laws of dynamics were originally formulated to explain the motion of observable bodies. When with the progress of knowledge the discrete nature of the constitution of matter was evident, the natural endeavour has then been to extend to those ultimate particles (the atoms, protons, electrons), the same dynamical laws which have been so successfully applied to the study of big masses. As the aim of Physics is to explain the observed physical phenomena, in terms of the motion of the ultimate particles, a consistent application of the dynamical methods has been responsible for the whole structure of classical Physics.

Before the methods of classical dynamics could be applied important developments in two directions had to be made. The problem of specifying the law of interaction of the different particles, when we have to take account of the immense number of the constituent particles raised difficulties which were resolved and the problems made more amenable to analysis by the introduction of the conception of the Field. It was discovered that instead of basing the description of interaction on the various laws, (formulated on the action-at-a-distance basis), a much simpler and a better treatment could be obtained by the

introduction of certain auxiliary magnitudes—The scalar or the vector potential, or The Electric or Magnetic intensities. These quantities vary with the position in space, as well with time. The characteristic functions determining the field were found in the case of electro-magnetism to satisfy a certain linear set of partial differential equations, whereas the relation between the magnitudes introduced, and the reacting masses may be expressed by a certain type of equation of the second order. These auxiliary magnitudes thus came to be looked upon as having their seat in a hypothetical medium which was postulated to pervade all space, and the partial differential equations which connect them came to be looked upon as related in some way to the physical properties of the hypothetical medium, the Ether. By the introduction of the concept of energy and momentum density, the abstract conception of ether seemed to gain in substantial reality. This was further strengthened when it was shown that it was possible to bring the partial differential equations in line with the usual dynamical equations, in as much as they appeared to be deducible from the Hamilton's principle if certain quantities were identified with the potential and the kinetic energies of the medium. Thus we come to the classical conception of Ether, and the electro-magnetic equations of Maxwell which served as the starting point of the modern Electron-Physics. Gravitation, however, did not immediately fall in line with the other field theory, until the formulation of the generalised Relativity-Principle by Einstein. Of this we shall have occasion to speak later on.

There was however another difficulty which stood in the way of immediate application of the classical methods to the problems of Physics. The dynamical laws seem at first applicable to the motion of the ultimate particles, which themselves however always escape direct observation. What we observe is to be looked upon as the resultant of a large number of elementary events; and the measurable microscopically sensible magnitudes cannot also be regarded as determining uniquely the component microscopic elements. Thus, from the classical standpoint, the necessary magnitudes for uniquely characterising the state of the system remain largely undetermined, and the Physical laws connecting observable things cannot be regarded as immediately deducible from the dynamical laws, at any rate without the introduction of further hypotheses. The method of statistical mechanics was developed to tackle this fundamental difficulty. The earlier method consisted in regarding the ultimate particles as practically independent systems. Each system is characterised by certain values of co-ordinates and momenta. Under their mutual interaction, Space and momenta cordinates of the individual systems are supposed to vary within certain limits. The actual physically observable magnitudes are determined by the distribution of the systems among the

different physically possible states. Mathematically this distribution is visualised by the distribution of the representative points of the individual systems in a phase space, which is supposed to be decomposed into a certain number of elementary phase-cells. The actual position of any particular representative point inside the phase-cell, is supposed to have no effect on the value of the phenomenologically observable magnitudes. The equilibrium-state of a physical system can thus be related to a particular distribution of the representative points in the phase cells which occurs most frequently, or remain longest during the period of observation. By this way the important physical idea of entropy can be connected wtih the probability of distribution in the Phase-Space, and important Thermodynamical laws can be looked upon as certain statistical laws which are necessary consequences of the very large number of individual components. When the results of classical dynamics are combined with this concept of statistics, we get certain very general laws, regarding the partition of energy. Though at first, these results seemed to agree with the facts observed, subsequent and more exact experiments have shown them to be erroneous.

I have already referred to the equations of classical mechanics which determine the motion of a material particle, as well as the field equations, which connect the vector magnitudes serving to describe the electro-dynamical field. Though it was attempted to bring these equations in line with the dynamical laws by showing that they also can be deduced from Hamilton's Principle, a fundamental difference between the two sets of equations was clearly brought out during the early part of this century.. The dynamical equations of Newton possess an invariant group of transformation, the Galilean Group, which expresses the equivalence of all inertial systems as frames of reference for the description of motion. The field equations of Maxwell however, have a different invariant group, the Lorentzian group. The space and time co-ordinates enter symmetrically into these equations, or rather the space-time symmetry can be brought out by introducing after Minkowski an imaginary time ict., as the fourth co-ordinate. The Lorentzian group of transformation can be represented by a rotation of axes in a four dimensional orthogonal space, which leaves the invariant distance unaltered. By extending the conception of vector quantities of the usual three dimensional space to four dimensions, the field equations can be given an invariant form. The recognition of this formal equivalence of the time and space co-ordinates ushers in the ideas of relativity. This equivalence now-a-days is regarded as axiomatic, and furnishes a test which all physical laws have to satisfy in order to be exact. By generalising this conception further, and by giving up the condition of orthogonality Einstein was able to present

a field theory of gravitation. The two different field theories have however remained essentially distinct though attempts have repeatedly been made to fuse them into one single unitary theory.

I have now indicated to you the principal steps by which the classical theory arrived from its first elementary stage to the present developed state. I shall now tell you of the difficulties that arose successively and which led to the formulation of the quantum theory. The conceptions of energy and momentum have been early deduced from the dynamical laws and have played very important roles in the subsequent development of the Science. For example the Hamilton-Jacobi differential equation requires for the mathematical expression the writing down of total energy of the system in terms of the momenta, whereas the integrand in the Hamilton's Principle is the difference between the potential and the kinetic energy of the system. The introduction of the concept of energy seems necessary for the applicability of the general dynamical laws, and we have seen how the field equations can be made compatible with the dynamical laws by interpreting a certain magnitude as the energy-density of Ether. As a necessary corollary to the equations however it follows that there will be a continuous interchange of energy between the ultimate particles composing matter and the surrounding radiation field. The principle of equipartition of energy which follows as a necessary consequence of the idea, leads us however to entirely wrong results when applied to the problem of equilibrium of radiation-field with matter. In order to explain the distribution of energy in the black-body radiation spectrum, as well as to explain the problem of generation of radiation we have been compelled to make certain assumptions regarding the constitution of the radiation-field, and about the equilibrium states of material particles, which directly contradict our classical ideas. This has led to the introduction of the Quantum theory in Physics. The energy in the monochromatic radiation field, has to be assumed to exist in definite quanta. The study of the constitution of matter has led us to the conception of the atom as a comparatively stable structure built out of the nucleus and electrons. These electrons according to the earlier formulation of the quantain theory are to be supposed as rotating in certain characteristic orbits ; whereas such a constitution on the classical theory will necessarily continually radiate and lose energy, we have to assume here that such a structure keeps generally intact its energy-content and also that there are a series of discrete radiation-free states characterised by a discrete series of values of the energy content. An interchange of energy can only take place, when the atomic system passes from one distinct state to another resulting in an emission or an absorption of radiation. The passage of the atom from a higher energy state to a lower is

associated with an emission of a monochromatic radiation whose frequency is given by the relation

$$W,—W'=hn.$$

where W, and W' are the characteristic energies of the two states.

Modern development of spectroscopy seems to indicate that the state of a system may be defined by the introduction of certain sets of quanta members, each set being associated with a definite constituent electron of the atomic system. No two electrons of the atomic system may have the same group of the quanta-numbers, and the successive building up of the different atoms may be looked upon as a gradual increase in the number of electrons in the outer *orbit*, each electron coming to occupy a place characterised by a different group of quanta numbers.

These few principles which are so contradictory to the ideas of classical dynamics have proved to be invaluable in the development of modern Physics. To them we owe the scientific classification of the spectroscopic terms, explanation of the Periodic classification, etc., etc. The direct and simple way in which the different problems, which have long baffled the attempt of classical physics, find their explanations in the new theory shows that it contains a large element of truth, and the most important problem of the present time is to find a way of reconciling the two theories. It has become evident also that no one theory will be able to entirely replace the other. For example the conception of the field, which has been developed originally from the classical standpoint, seems to contain also a great element of truth. The large mass of experimental material connected with the problem of propagation of radiation, seems to find in the field theory a very simple and direct explanation, which appear so difficult to explain from the Quantum-theoretical standpoint.

The solution of the dilemma seems to lie in suitably uniting the field and the quanta-theories, which will ultimately form a more general theory of which the two theories may then be regarded as limiting cases. We have in recent years already some indications of tentative attempts in the same direction. One may attempt to visualise the radiation-field as simply determining the interaction of the different particles and give up the classical concept of energy and momentum-density in Ether. Consistent with our ideas of the quantum theory we may regard that the energy in the field is only discretely distributed throughout the space, (somewhat after the ideas of J. J. Thomson). The motion of these quantas of energy may be supposed to be guided as it were by the field, and it may be supposed to take place along the direction of the Poynting

Vector. The same field which guides the motion of the quanta may be also supposed to control the motion of the electrons in the different orbits. The nature of the field itself should be determined by some equations depending upon the distribution of the charges. Such a field theory will have as its task the explanation not only of the fundamental relation $E=h\nu$, but also of the occurrence of the discrete quanta-states and the different quanta-members.

That the idea of discrete energy-states is not inconsistent with a continuous field-theory seems to follow from the recent developments of the wave-mechanics (Schrodinger, Dirac). In the classical theory the impossibility of the existence of stationary electron-orbits follows from the result that an accelerated electron gives rise according to it to an electro-magnetic field such that the surface integral of the Poynting Vector at a great distance does not vanish. The result is interpreted as indicating a continuous loss of energy from the moving electron which makes stationary orbits impossible. One can think however, that if there are other electrons present the resultant field may be such that the electron may in the average gain as much energy from the surrounding field as it loses by radiation, or in other words the resultant electro-magnetic field due to all the moving electrons may be such that the integral of the Poynting-Vector vanishes. Such a hypothetical solution of the field-equations which make the field vanish at infinity may be thus likened to a stationary vibration rather than to a divergent wave-train. It is clear, however, that the distribution of the charges will have to be suitably made and their motions definitely related in phase with one another in order that such a thing may be possible. The electro-magnetic field may then be supposed to be in resonance after de Broglie. What Schrodinger succeeds in establishing may be likened to finding exactly such a law of distribution of charges. The quanta conditions then may be fittingly compared with the resonance conditions. Dirac seems to have succeeded in deducing a set of four linear partial differential equations which correspond to facts more closely. The four functions or rather eight of them give us the distribution of the hypothetical charges and currents in the electro-magnetic field necessary to establish resonance-conditions. The field itself in these two theories determines also the distribution or rather the statistically equivalent distribution (in space as well as in velocity) of the charges in the field.

One may hope that the final solution of problem may be found and the proper set of equations which will determine the radiation-less field may be determined with the help of the ideas of the generalised relativity theory. If an unitary field theory which could explain gravitation and electro-magnetism could be found, one would hope to obtain thereby not only the solution of the quanta-problem, but perhaps also of a more fundamental problem, the relation of charge with gravitational mass, and explanation of the difference in mass of the proton and the electron.

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ON THE COMPLETE MOMENT-COEFFICIENTS OF THE D^2-STATISTIC.

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The D^2-statistic was defined by Mahalanobis[1] to measure the divergence between two statistical populations, and the moment-coefficients were also calculated by him by approximative methods. Rajchandra Bose[2] has found the exact distribution of the D^2-statistic* which can be expressed in terms of Bessel functions. He has used the actual distribution function to obtain the moment-coefficients, and has shown that the results previously obtained by Mahalanobis are exact. In the present note a recurrence-formula for the D^2-statistic has been obtained directly without assuming the distribution, and certain properties of these moment-coefficients have been investigated.

2. Let $(a_1, a_2, \ldots a_p)$ and $(a_1', a_2', \ldots a_p')$ be the observed mean values in two samples Σ_1 and Σ_2 of size n_1 and n_2 respectively drawn from two normal p-variate correlated populations with mean values $(\alpha_1, \alpha_2, \ldots \alpha_p)$ and $(\alpha_1', \alpha_2', \ldots \alpha_p')$ respectively. The distribution of the differences in observed mean values $(a_1-a_1', a_2-a_2', \ldots a_p-a_p')$ can be written in the form :—

$$\text{Const.} \times e^{-\frac{n}{p\beta}\left[\beta_{11}\{(a_1-a_1')-(\alpha_1-\alpha_1')\}^2+\ldots 2\beta_{12}\{(a_1-a_1')-(\alpha_1-\alpha_1')\}\{(a_2-a_2')-(\alpha_2-\alpha_2')\}+\ldots\right]} \times d(a_1-a_1'), d(a_2-a_2')\ldots d(a_p-a_p') \quad \ldots (2.1)$$

where β and β_{ij} $(i, j = 1, 2, \ldots p)$ are certain functions of the population variances and co-variances whose values have been given by Rajchandra Bose[3] and

$$\frac{2}{n} = \frac{1}{n_1} + \frac{1}{n_2} \quad \ldots (2.2)$$

Then the D^2-statistic is defined by

$$D^2 = D_1^2 - \frac{2}{n} \quad \ldots (2.31)$$

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* In this paper p and n have been used in the place of P and $\bar{n}$ in the earlier papers of P. C. Mahalanobis and Raj Chandra Bose. The D^2-statistic was intended to be and was defined as a quantity determined entirely by the sample values of the variates. Raj Chandra Bose has investigated the exact distribution and S. N. Bose the moment-coefficients of a modified form of the D^2-statistic in which the population values of the variances and co-variances have been substituted for the corresponding sample estimates.—*Editor, Sankhyā.*

and $$D_1^2 = \frac{1}{p\beta} \sum_{i,j=1}^{i,j=p} [\beta_{ij}(a_i - a_i')(a_j - a_j')] \quad \ldots \quad (2.32)$$

The population value of D^2 is Δ^2 which is defined by

$$\Delta^2 = \frac{1}{p\beta} \sum_{i,j=1}^{i,j=P} [\beta_{ij}(\alpha_i - \alpha_i')(\alpha_j - \alpha_j')] \quad \ldots \quad (2.4)$$

3. If we put

$$(a_i - a_i') = x_i, \text{ and } (\alpha_i - \alpha_i') = \xi_i$$

we can write

$$D_1^2 = \frac{1}{p\beta} [\beta_{11}x_1^2 + \beta_{22}x_2^2 + \ldots 2\beta_{12}x_1x_2 + \ldots] \quad \ldots \quad (3.1)$$

$$\Delta^2 = \frac{1}{p\beta} [\beta_{11}\xi_1^2 + \beta_{22}\xi_2^2 + \ldots 2\beta_{12}\xi_1\xi_2 + \ldots] \quad \ldots \quad (3.2)$$

If we now use a linear transformation which changes the quadratic form

$$\frac{1}{2\beta} \sum_{i,j=1}^{i,j=p} (\beta_{ij}x_ix_j) \quad \text{to} \quad (y_1^2 + y_2^2 + \ldots y_p^2) \quad \ldots \quad (3.3)$$

then the same transformation will change

$$\frac{1}{2\beta} \sum_{i,j=1}^{i,j=p} (\beta_{ij}\xi_i\xi_j) \quad \text{to} \quad (\eta_1^2 + \eta_2^2 + \ldots \eta_p^2) \quad \ldots \quad (3.4)$$

where $(\eta_1, \eta_2, \ldots \eta_p)$ are connected with $(\xi_1, \xi_2, \ldots \xi_p)$ by the same linear transformation which connects $(y_1, y_2, \ldots y_p)$ and $(x_1, x_2, \ldots x_p)$

Also $\frac{1}{2} \sum_{i,j=1}^{i,j=p} (\beta_{ij}x_i\xi_j)$ will be transformed to $(y_1\eta_1 + y_2\eta_2 + \ldots y_p\eta_p)$ $\quad \ldots \quad (3.5)$

We can then write

$$\tfrac{1}{2}p \cdot D_1^2 = y_1^2 + y_2^2 + \ldots y_p^2 \quad \ldots \quad (3.6)$$

$$\tfrac{1}{2}p \cdot \Delta^2 = \eta_1^2 + \eta_2^2 + \ldots \eta_p^2 \quad \ldots \quad (3.7)$$

Equation (2.1) can then be written in the form :—

$$\text{Const.} \times e^{-\frac{np}{4}[D_1^2 + \Delta^2 - 2D_1 \cdot \Delta \cos\theta]} \times dV \quad \ldots \quad (3.8)$$

where $dV = dy_1, dy_2 \ldots dy_p$

4. Let $\mu_k'(D_1^2)$ be the k-th raw moment of D_1^2, and let

$$M_k = \int_0^\infty D_1^{2k} \cdot e^{-\frac{np}{4}[D_1^2+\Delta^2-2D_1.\Delta\cos\theta]} .dV \quad \ldots \quad (4.1)$$

where the single sign of integration stands for the p-fold integration.

Then $$\mu_k'(D_1^2) = C.M_k \quad \ldots \quad (4.2)$$

we also write $$np/4 = t \quad \ldots \quad (4.3)$$

and $$M_k = \int_0^\infty D_1^{2k} \cdot e^{-t(D_1^2+\Delta^2-2D_1.\Delta\cos\theta)} \cdot dV \quad \ldots \quad (4.11)$$

Then

$$\frac{\partial M_k}{\partial \Delta} = -2t \int D_1^{2k}(\Delta - D_1\cos\theta)e^{-(tD_1^2+\Delta^2-2D_1\Delta\cos\theta)} \cdot dV \quad \ldots \quad (4.4)$$

$$= -2t\cdot\Delta\cdot M_k + 2t \int D_1^{2k+1} \cdot \cos\theta \cdot e^{-t(D_1^2+\Delta^2-2D_1\Delta\cos\theta)} \cdot dV \quad \ldots \quad (4.41)$$

Again

$$\frac{\partial M_k}{\partial t} = -\int D_1^{2k}(D_1^2+\Delta^2-2D_1\cdot\Delta\cos\theta)e^{-t(D^2+\Delta^2-2D_1\Delta\cos\theta)} \cdot dV \quad \ldots \quad (4.5)$$

$$= -M_{k+1} - \Delta^2 \cdot M_k + \frac{\Delta}{t}\left[\frac{\partial M_k}{\partial \Delta} + 2t\cdot\Delta\cdot M_k\right] \quad \ldots \quad (4.51)$$

Therefore

$$M_{k+1} = \Delta^2 \cdot M_k + \frac{\Delta}{t}\cdot\frac{\partial M_k}{\partial \Delta} - \frac{\partial M_k}{\partial t} \quad \ldots \quad (4.6)$$

5. Now actually

$$M_0 = \int_0^\infty e^{-t[(y_1-\eta_1)^2+(y_2-\eta_2)^2+\ldots(y_p-\eta_p)^2]} \cdot dy_1\, dy_2 \ldots dy_p$$

$$= \left(\frac{\pi}{t}\right)^{p/2} \quad \ldots \quad (5.1)$$

Therefore

$$\frac{\partial M_0}{\partial t} = -\frac{p(\pi)^{p/2}}{2(t)^{(p+2)/2}} = \frac{p\cdot(\pi)^{p/2}}{2(t)^{(p+2)/2}} = \frac{p\cdot M_0}{2t} \quad \ldots \quad (5.2)$$

$$\frac{\partial M_0}{\partial \Delta} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5.3)$$

Also $\mu_0'(D_1^2) = c \cdot M_0 = 1$ and therefore

$$c = \left(\frac{t}{\pi}\right)^{p/2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5.4)$$

We can use the above initial values to calculate $M_1, M_2, \ldots M_k$ etc., and hence with the help of equation (4.2) obtain the raw moments (about any arbitrary origin) of D^2 which can be written in the form :—

$$\mu_k'(D_1^2) = \left(\frac{t}{\pi}\right)^{p/2} M_k \quad \ldots \quad \ldots \quad (5.5)$$

6. It will be convenient to use new variables

$$u = \Delta^2, \quad \text{and} \quad s = 1/t \quad \ldots \quad \ldots \quad (6.1)$$

Equation (4.6) can now be written as

$$M_{k+1} = u \cdot M_k + 4\, u.s. \frac{\partial M_k}{\partial u} + 2s^2 \cdot \frac{\partial M_k}{\partial s} \quad \ldots \quad (6.2)$$

Remembering that $M_0 = (\pi \cdot s)^{p/2}$, we note from the form of equation (6.2) that M_k is a homogeneous function of order $(k + \frac{1}{2}p)$ in u and s.

Therefore

$$M_{k+1} = u.M_k + 2u.s. \frac{\partial M_k}{\partial u} + 2s\left[u\frac{\partial M_k}{\partial u} + s\frac{\partial M_k}{\partial s}\right]$$

$$= u \cdot M_k + 2u \cdot s \cdot \frac{\partial M_k}{\partial u} + 2s(k+\tfrac{1}{2}p).\, M_k, \text{ by Euler's theorem} \quad \ldots \quad (6.3)$$

Thus

$$M_{k+1} = [s(2k+p)+u]M_k + 2u.s. \frac{\partial M_k}{\partial u} \quad \ldots \quad (6.4)$$

Writing briefly μ_k' for $\mu_k'(D_1^2)$, we have

$$\mu_k' = \frac{M_k}{(2s\pi)^{p/2}} \quad \ldots \quad (6.5)$$

Equation (6.4) may now be written in the form :—

$$\mu_{k+1}' = [s(2k+p)+u]\mu_k' + 2u \cdot s \cdot \frac{\partial \mu_k'}{\partial u} \quad \ldots \quad (6.6)$$

This is the fundamental recurrence formula. We notice that μ_k' is a homogeneous function of the k-th degree in s and u.

The general expression for μ_k' can be easily obtained.

$$\mu_k' = c_k \cdot u^k + c_{k-1} \cdot u^{k-1} \cdot s + \ldots c_0 \cdot s^k \qquad \ldots \quad (7.1)$$

Then from equation (6.6) we get

$$\mu_{k+1}' = C_k . u^{k+1} + [c_{k-1} + (p+4k)c_k] u^k \cdot s + \ldots + [c_{k-2} + (p+4k-2)c_{k-1}] u^{k-1} \cdot s^2 + \ldots$$
$$+ [c_0 + (p+2k+2)c_1] u . s^k + (p+2k)c_0 . s^{k+1} \qquad \ldots \quad (7.2)$$

From this equation we can immediately write down the coefficients occurring in μ_k' :—

μ_0'.....1

μ_1'.....1, p

μ_2'.....1, $2(p+2)$, $p(p+2)$,

μ_3'.....1, $3(p+4)$, $3(p+2)(p+4)$, $p(p+2)(p+4)$ etc.

The general coefficients can be easily proved by induction to be of the form :—

$$1^k c_1[p+2(k-1)],\ {}^k c_2[\{p+2(k-1)\}\{p+2(k-2)\}],\ldots\ldots$$
$$\ldots\ldots {}^k c_r[\{p+2(k-1)\}\{p+2(k-2)\}\ldots\ldots\{p+2(k-r)\}],$$
$$\ldots {}^k c_k[\{p+2(k-1)\}\{p+2(k-2)\}\ldots p]\ldots\ldots \qquad \ldots \quad (7.4)$$

Remembering that $u = \Delta^2$, and $s = 1/2t = 2/(np)$ we finally obtain the general formula for the k-th raw moment-coefficient :—

$$\mu_k'(D_1^2) = \Delta^{2k} + \frac{2 . {}^k c_1 (p+2k-2)}{np} \cdot \Delta^{2k-2} + \frac{2^2 \cdot {}^k c_2 (p+2k-2)(p+2k-4)}{n^2 p^2} \cdot \Delta^{2k}$$
$$+ \ldots + \frac{2^k . (p+2k-2)(p+2k-4) \ldots (p+2)p}{n^k \cdot p^k} \qquad \ldots \quad (7.5)$$

$$= \underset{\alpha \to \infty}{\text{Lt.}} \Delta^{2k} \cdot F\left(-k; -k-\tfrac{1}{2}p+1; \alpha; \frac{4\alpha}{np\Delta^2}\right) \qquad \ldots \quad (7.6)$$

The two formulae (7.5) and (7.6) are in agreement with the results obtained by Rajchandra Bose[2] in equations (8.9) and (8.91) on p. 152 of *Sankhyā*, Vol. 2, Part 2.

8. It is clear from the preceding considerations that $\mu_k'(D^2)$'s, whose recurrence formulae have been calculated, belong to a Gaussian distribution in p-variables; it may be therefore interesting to deduce certain mathematical properties of these functions.

We start from the recurrence formula

$$\mu'_{k+1} = [s(2k+p)+u]\mu'_k + 2us\frac{\partial \mu'_k}{\partial s} \quad \ldots \quad (6.7)$$

Putting $\mu'_k = (2s)^k \cdot \psi_k(u/2s)$ and using $\chi = u/2s$ as the new variables, we have

$$\psi_{k+1}(x) = [(k+\tfrac{1}{2}p)+x]\psi_k(x) + x\psi'_k(x) \quad \ldots \quad (8.1)$$

Multiplying by $e^x . x^{k+\frac{1}{2}p-1}$, we have

$$e^x \cdot x^{k+\frac{1}{2}.-1}\psi_{k+1}(x) = \frac{d}{dx}[e^x \cdot x^{k+\frac{1}{2}p}\psi_k(x)],$$

$$\text{or} \quad e^x \cdot x^{k+\frac{1}{2}p+1}\psi_{k+1}(x) = x^2 \cdot \frac{d}{dx}[e^x \cdot x^{k+\frac{1}{2}p}\psi_k(x)] \quad \ldots \quad (8.2)$$

By regular descent, since $\psi_0(x) = 1$, it follows easily that

$$e^x \cdot x^{k+\frac{1}{2}p}\psi_k(x) = \left(x^2 \frac{d}{dx}\right)^k [e^x \cdot x^{\frac{1}{2}p}] \quad \ldots \quad (8.3)$$

which may be regarded as a solution of the recurrence formula.

If we put $t = 1/x = 2s/u$, we get,

$$\mu'_k = (2s)^k . \psi_k(x) = (2st)^k . e^{-1/t} . t^{\frac{1}{2}p}\left(-\frac{d}{dt}\right)^k [e^{1/t} \cdot t^{-\frac{1}{2}p}] \quad \ldots \quad (8.4)$$

which is identical with (8.6) in Rajchandra Bose's paper.[3]

Starting from the equation (8.3) and denoting by y, the expression $e^x x^{\frac{1}{2}p}$, and by D, the operator $x^2 \frac{d}{dx}$, we easily find that y satisfies the differential equation

$$\frac{1}{x^2} Dy = \left(\tfrac{1}{2}p \cdot \frac{1}{x} + 1\right) y.$$

Operating on both sides of this equation by D^{k+1} and remembering that $D^k y = \psi_k(x)$ one easily finds that $\psi_k(x)$ satisfies the differential equation

$$x\frac{d^2\psi_k}{dx^2} + (\tfrac{1}{2}p+x)\frac{d\psi_k}{dx} - k\psi_k = 0 \quad \ldots \quad (8.5)$$

If $$\psi_k(-x) = (-1)^k \cdot f_k(x),$$

$f_k(x)$ satisfies the differential equation

$$x\frac{d^2 f_k}{dx^2} + (\tfrac{1}{2}p - x)\frac{df_k}{dx} + k \cdot f_k = 0 \quad \ldots \quad (8.6)$$

The solutions are easily seen to be polynomials of order k, when k is an integer, and these in case of (8.6) have been studied by Sonine, Gegenbauer, and others. The differential equations have also been studied by Whittaker and others, and give rise to solutions of the confluent hypergeometric types.

9. There is an alternative method of arriving at the differential equation for μ', which is here appended as the method is capable of wide generalisation.

The Gaussian distribution function

$$F = \frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot e^{-\Sigma(x_r..y_r)^2/2t}, \text{ where } r = 1, 2, 3, \ldots p$$

can be easily seen to satisfy the differential equation

$$\frac{\partial^2 F}{\partial x_1^2} + \frac{\partial^2 F}{\partial x_2^2} + \ldots + \frac{\partial^2 F}{\partial x_p^2} = 2\frac{\partial F}{\partial t} \qquad \ldots \quad (9.1)$$

where instead of $(x_1, x_2, \ldots x_p)$, we may also take $(y_1, y_2 \ldots y_p)$ as variables.

The expression $\frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot e^{-\Sigma_r(x_r - y_r)^2/2t}$ denoted by F also satisfies the differential equation

$$\frac{\partial^2 F}{\partial y_1^2} + \frac{\partial^2 F}{\partial y_2^2} + \ldots\ldots + \frac{\partial^2 F}{\partial y_p} = 2\frac{\partial F}{\partial t} \qquad \ldots \quad (9.11)$$

Now, since this differential equation is linear which means that if $\psi_1, \psi_2, \ldots$ etc. are each a solution of the differential equation then $\psi_1 + \psi_2 + \ldots$ etc. is a solution of the same equation, and since an integral is nothing but a sum, therefore,

$$\psi(y_1, y_2, \ldots y_p, t) \equiv \int \phi(x_1, x_2, \ldots x_p) \cdot \frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot e^{-\Sigma(x_r - y_r)^2/2t}\, dx_1, dx_2 \ldots dx_p) \quad \ldots \quad (9.2)$$

which consists of a sum of terms like $\frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot e^{-\Sigma(x_r - y_r)^2/2t}$ each satisfying the differential equation and each multiplied by a term like $\phi(x_1, x_2 \ldots x_p) \cdot dx_1\, dx_2 \ldots dx_p$ not depending upon the independent variables the y's, is easily seen to satisfy the equation (9.11).

Therefore

$$\mu_k'(y_1, y_2, \ldots y_p, t) = \int (x_1^2 + x_2^2 + \ldots x_p^2)^k \cdot \frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot e^{-\Sigma(x_r - y_r)^2/2t} \times dx_1.\, dx_2 \ldots dx_p \qquad (9.3)$$

also satisfies the same differential equation.

An orthogonal transformation of $(x's)$ and $(y's)$ will not change the integrand; also making a transformation

$$\left.\begin{aligned} x_1 &= \lambda x_1', \; x_2 = \lambda x_2', \quad \ldots\ldots\ldots\ldots t = \lambda^2 t' \\ y_1 &= \lambda y_1', \; y_2 = \lambda y_2', \quad \ldots\ldots\ldots\ldots\ldots\end{aligned}\right\} \qquad \ldots \quad (9.4)$$

multiplies the integral by λ^{2k}. Hence

$$\mu_k'(y_1, y_2, \ldots y_p, t) = \mu_k'[y_1^2 + y_2^2 + \ldots y_p^2, t] \qquad \ldots \quad (9.41)$$

Putting
$$y_1^2 + y_2^2 + \ldots y_p^2 = R^2 \qquad \ldots \quad (9.42)$$

it follows that
$$\mu_k'(y_1, y_2, \ldots y_p, t) = \mu_k'(R^2, t) \qquad \ldots \quad (9.5)$$

also
$$\mu_k'(\lambda R^2, \lambda t) = \lambda^k \mu_k'(R^2, t) \qquad \ldots \quad (9.51)$$

or μ_k' is a homogeneous function of (R^2, t) of the kth degree. Therefore μ_k' is a homogeneous solution of the kth degree of (R^2, t) alone of the equation (9.1). Making a change of variables to R and angle-coordinates in a p-dimensional space, μ_k' is seen to satisfy the differential equation

$$\frac{1}{R^{p-1}} \cdot \frac{\partial}{\partial R}\left[R^{p-1} \frac{\partial \mu_k'}{\partial R}\right] = 2\frac{\partial \mu_k'}{\partial t} \qquad \ldots \quad (9.6)$$

This transformation is seen easily as $\square_p^2 \cdot F =$ Div. grad F, and when F is a function of R alone, the expression follows as the hyper-spherical element perpendicular to R is proportional to R^{p-1}. Thus

$$\square_p^2 \cdot F = \frac{1}{R^{p-1}} \cdot \frac{\partial}{\partial R}\left[R^{p-1} \cdot \frac{\partial F}{\partial R}\right] = \frac{\partial^2 \mu_k'}{\partial R^2} + \frac{p-1}{R} \cdot \frac{\partial \mu_k'}{\partial R} = 2\frac{\partial \mu_k'}{\partial t} \qquad \ldots \quad (9.7)$$

Finally choosing $R^2 = 2t \cdot x$, $\mu_k' = (2t)^k \cdot \psi_k(x)$ $\qquad \ldots \quad (9.8)$

the differential equation of $\psi_k(x)$ is easily seen to be

$$x\frac{d^2\psi_k}{dx^2} + (\tfrac{1}{2}p + x)\frac{d\psi_k}{dx} - k\psi_k = 0 \qquad \ldots \quad (9.9)$$

which is identical with the equation (8.5)

10. The method sketched above enables one to calculate the moment functions even when k is not an integer; actually however, unless k's are integers, μ_k''s will not be expressed by a polynomial but by an infinite series. The differential equation itself determines all the constants except one, which can be determined from simple considerations.

I will conclude my remarks by deducing certain properties of the general differential equation, which will be useful for determining the form of the general distribution function about which I wish to make some remarks in future.

Consider the equation

$$\square_p^2 . F = \frac{\partial^2 F}{\partial x_1^2} + \frac{\partial^2 F}{\partial x_2^2} + \ldots \frac{\partial^2 F}{\partial x_p^2} = 2 \frac{\partial F}{\partial t} \qquad \ldots \quad (10.1)$$

Suppose ψ is a solution of the differential equation, we then seek a solution of the equation of the form $F = \psi . \chi$.
Then

$$\psi . (\square_p^2 . \chi) = \chi . (\square_p^2 . \psi) + 2(D\psi, D\chi) = 2 \left[\psi \frac{\partial \chi}{\partial t} + \chi . \frac{\partial \psi}{\partial t} \right] \qquad \ldots \quad (10.2)$$

or $$\psi(\square_p^2 . \chi) + 2(D\psi, D\chi) = 2\psi \frac{\partial \chi}{\partial t} \qquad \ldots \quad (10.3)$$

Now if $$\psi = \frac{1}{(2\pi t)^{\frac{1}{2}p}} . e^{-\Sigma x_r^2/2t}, \text{ then } \frac{\partial \psi}{\partial x_r} = -\frac{x_r}{t} . \psi \qquad \ldots \quad (10.4)$$

Therefore χ satisfies the differential equation

$$\square_p^2 . \chi = \frac{2}{t} \left[x_1 \frac{\partial \chi}{\partial x_1} + x_2 \frac{\partial \chi}{\partial x_2} + \ldots + t \frac{\partial \chi}{\partial t} \right] \qquad \ldots \quad (10.5)$$

Suppose $F(x_1, x_2, \ldots x_p, t)$ is a solution of the equation

$$\square_p^2 . F = 2 \frac{\partial F}{\partial t} \qquad \ldots \quad (10.6)$$

Then $$\psi(x_1, x_2, \ldots x_p, t) = F(x_1/t, x_2/t, \ldots x_p/t, -1/t) \qquad \ldots \quad (10.7)$$

will satisfy the differential equation (10.5), for putting $x_r' = x_r/t$, and $t' = -1/t$, it follows that

$$\frac{\partial \psi}{\partial x_r} = -t' \frac{\partial F}{\partial x_r'}, \qquad \ldots \quad (10.8)$$

$$t \frac{\partial \psi}{\partial t} + \Sigma x_r \frac{\partial \psi}{\partial x_r} = -t' \frac{\partial F}{\partial t'} \qquad \ldots \quad (10.81)$$

$$\square_p^2 . \psi - \frac{2}{t}\left[t \frac{\partial \psi}{\partial t} + \ldots \right] = t'^2 \left[\square_p'^2 . F - 2 \frac{\partial F}{\partial t'} \right] = 0 \qquad \ldots \quad (10.82)$$

11. This theorem enables us to construct solutions of the fundamental differential equation of the form

$$\frac{1}{(2\pi t)^{\frac{1}{2}p}} . e^{-\Sigma x_r^2/2t} . H\left(\frac{x_1}{t}, \frac{x_2}{t}, \ldots \frac{x_p}{t}, -\frac{1}{t} \right) \qquad \ldots \quad (11.1)$$

where $H(x_1, x_2, \ldots x_p, t)$ is a solution of the same differential equation (9.1).

We have seen that $\mu_k'(R^2, t)$ the moment functions, are solutions of the differential equation (9.1)

$$\mu_k' = (2t)^k . \psi_k(R^2/2t) = 2t^k . \psi_k(y) \qquad \ldots \quad (11.2)$$

where $\psi_k(y)$ satisfies the differential equation (8.4). Therefore,

$$\mu_k'\left(\frac{x_1}{t}, \frac{x_2}{t}, \ldots \frac{x}{t}, -\frac{1}{t} \right) = (-1)^k . \frac{2^k}{t^k} . \psi_k\left[\frac{x_1^2 + x_2^2 + \ldots x^2}{2t^2(-1/t)} \right]$$

$$= (-1)^k . (2/t)^k . \psi_k(-y) \qquad \ldots \quad (11.3)$$

where $(-1)^k . \psi_k(-y) = f(y)$ which satisfies the equation (8.5).

If $y = \Sigma x_1^2/2t$, and $(-1)^k . \psi_k(-y)$ are the polynomials of Sonine-Gegenbauer, it follows that

$$\frac{1}{(2\pi t)^{\frac{1}{2}p}} . e^{-y} . (-1)^k . (2/t)^k . \psi_k(-y) \qquad \text{are solutions of (9.1).}$$

12. A distribution function which satisfies the fundamental equation, and is of a certain type can be written in the form

$$\frac{1}{(2\pi t)^{\frac{1}{2}p}} . e^{-y} . \left[1 - C_1 . \frac{\psi_1(-y)}{t} + \ldots (-1)^k . C_k . \frac{\psi_k(-y)}{t^k} + \ldots \right] \text{where } y = \Sigma x_r^2/2t$$

Certain orthogonal properties of the Sonine polynomials will be useful to recall here. The functions $S_k(x) = (-1)^k . C_k . \psi_k(-x)$ satisfies, we have seen the differential equation

$$x \frac{d^2 S_k}{dx^2} + (\tfrac{1}{2}p - x) \frac{dS_k}{dx} + k . S_k = 0 \qquad \ldots \quad (8.5)$$

which can be written in the self-adjoint form,

$$\frac{d}{dx}\left(x^{\frac{1}{2}p}.e^{-x}.\ \frac{dS_k}{dx} \right) + k.x^{\frac{1}{2}p-1}.\ e^{-x}S_k = 0 \qquad \ldots \quad (12.2)$$

from which the orthogonal properties follow at once.

Taking two polynomials of orders k and k', we have

$$(k-k') \int_0^\infty e^{-x}.\ x^{\frac{1}{2}p-1},\ S_k.\ S_{k'}\, dx = 0 \qquad \ldots \quad (12.3)$$

Hence if $k \neq k'$, $\int_0^\infty e^{-x}.\ x^{\frac{1}{2}p-1}.\ S_k.\ S_{k'}.\ dx = 0 \qquad \ldots \quad (12.31)$

also $$\int_0^\infty e^{-x}\ .\ x^{\frac{1}{2}p-1}.\ S_k^2(x)dx \neq 0 = A = 1 \qquad \ldots \quad (12.32)$$

since the arbitrary constant in the definition of the polynomial may be so adjusted as to make $A = 1$, which we may call the normalised polynomials.

Suppose the distribution functions to be $\phi(x_1, x_2 \ldots x_p, t)$ then the distribution function in R^2 is obviously

$$\int_R^{R+dR} \phi(x_1, x_2 \ldots x_p, t)dx_1 \ldots dx_2 \ldots dx_p \qquad \ldots \quad (12.4)$$

taken between the hyper-spheres of radii R and $(R+dR)$.

It will take the form

$$A \cdot \psi(R^2, t) \ldots R^{p-1} \ldots dR = \psi'(D, t) \cdot D^{\frac{1}{2}p-1} \ldots dD \text{ when } D = R^2. \qquad \ldots \quad (12.41)$$

We shall call $\psi'(D, t)$ the distribution function, when the number of variables is p.

Then $$\int_0^\infty \psi'(D, t) \ldots D^{\frac{1}{2}p-1} \ldots dD = 1 \qquad \ldots \quad (12.5)$$

$$\int_0^\infty D^k\ .\ \psi'(D, t)\ .\ D^{\frac{1}{2}p-1} \cdot dD = \mu_k(t) \qquad \ldots \quad (12.51)$$

It is also clear that if $y = D/2t$, then substituting the expansion of $S_k\,(y)$ which can be easily obtained from the differential equation or from the expression for μ_k' found before, we get

$$\int_0^\infty (2t)^k \cdot S_k(y) \cdot \psi'(D, t) \cdot D^{\frac{1}{2}p-1} \cdot dD$$

$$= C_k \int_0^\infty \left[D^k - k(\tfrac{1}{2}p+k-1) \cdot D^{k-1} \cdot (2t) + \frac{k(k-1)}{2!} (\tfrac{1}{2}p+k-1)(\tfrac{1}{2}p+k-2) \cdot D^{k-2} \cdot (2t)^2 \right.$$

$$\left. + \ldots \right] \times \psi(D, t) \cdot D^{\frac{1}{2}p-1} \cdot dD \qquad \ldots \quad (12.6)$$

$$= C_k \left[\mu'_k - \frac{k(\frac{1}{2}p+k-1)}{1!} \mu'_{k-1} \cdot (2t) + \text{etc} \ldots \right] \qquad \ldots \quad (12.61)$$

This expression is thus calculable in terms of the moments of the different orders up to k.

Assuming now

$$\psi(D, t) = \frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot e^{-y} \left[1 + \frac{A_1}{t} S_1(y) + \frac{A_2}{t^2} S_2(y) + \ldots + \frac{A_k}{t^k} \cdot S_k(y) + \ldots \right] \qquad \ldots \quad (12.7)$$

and remembering that the above series is so chosen that each term, and therefore the whole series, if convergent, satisfies the differential equation $\square_p^2 F = 2\partial F/\partial t$, we have:

$$\int \psi(D, t) \cdot D^{\frac{1}{2}p-1} \cdot (2t)^k S_k(y) \cdot dD$$

$$= \int \frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot e^{-y} \left[1 + \frac{A}{t} S_1(y) + \ldots . \frac{A_k}{t_k} \cdot S_k(y) + \ldots \right] (2t)^k S_k(y) (2t)^p \cdot y^{\frac{1}{2}p-1} \cdot dy$$

$$= \frac{1}{(2\pi t)^{\frac{1}{2}p}} \cdot 2^k A_k$$

$$= C_k \left[\mu'_k - \frac{k(\frac{1}{2}p+k-1)}{1!} \mu'_{k-1}(2t) + \frac{k(k-1)}{2!} (\tfrac{1}{2}p+k-1)(\tfrac{1}{2}p+k-2) \mu'_{k-2}(2t)^2 + \ldots \right] \quad (12.8)$$

From (12.6) since $y = D/2t$, we get finally

$$A_k = \frac{(2\pi)^{\frac{1}{2}p}}{2^k} \cdot C_k \cdot \left[\mu'_k - \frac{k(\frac{1}{2}p+k-1)}{1!} \mu'_{k-1} + \ldots \right] \qquad \ldots \quad (12.9)$$

which determines the constants A_k in the distribution formula in terms of the moments. Hence the distribution function can be calculated in terms of the moments. Rajchandra Bose's distribution function obviously can be put in the form indicated above, the actual calculation would give interesting integral relations involving the Bessel functions.

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ON THE MOMENT-COEFFICIENTS OF THE D^2-STATISTIC AND CERTAIN INTEGRAL AND DIFFERENTIAL EQUATIONS CONNECTED WITH THE MULTIVARIATE NORMAL POPULATION

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INTRODUCTION.

In a previous paper[1] I have considered certain recursion formulæ and relations between the moment-coefficients of the D^2-statistic. Starting with certain further algebraic identities between the moment-coefficients of different orders of the D^2-statistic, I reach a number of fundamental integral and differential equations connected with the multivariate normal distribution. A number of solutions of these equations are discussed in the present paper.

1 "On the Complete Moment-Coefficients of the D^2-statistic." *Sankhyā*, Vol. 2 (4), 1936, 385-396.

Algebraic Identities deduced from the Polynomial Form of the Moment-Coefficients of the D^2-statistic.

1. I start with the identities that exist between the moment-functions of the different orders of the D^2-statistic, which can be regarded as the square modulus statistic for a Gaussian distribution in p-variates.

It has been already proved[2] that

$$\mu_k'(l^2) = \int (x_1^2+x_2^2+\ldots+x_p^2)^k \cdot \frac{e^{-\Sigma_r(x_r-y_r)^2/2t}}{(2\pi t)^{p/2}} \cdot dV \qquad \ldots \quad (1.1)$$

where
$$x_1^2+x_2^2+\ldots x_p^2 = l^2 \qquad \ldots \quad (1.11)$$

and
$$y_1^2+y_2^2+\ldots y_p^2 = \Delta^2 \quad (\text{or } u) \qquad \ldots \quad (1.12)$$

and μ_k is a homogeneous function of $(\Delta^2,\ 2t)$ of the k-th degree of the form[3] :

$$\mu_k(l^2) = \beta^k + k(k+p/2-1)\cdot\beta^{k-1}(2t)+\ldots$$
$$+\frac{\Gamma(k+1)\cdot\Gamma(k+p/2)\cdot\beta^{k-r}(2t)^r}{\Gamma(r+1)\cdot\Gamma(k+1-r)\cdot\Gamma(k+p/2-r)}+\ldots+\frac{\Gamma(k+p/2)}{\Gamma(p/2)}\cdot(2t)^k \qquad \ldots \quad (1.2)$$

It should be noted here that in (1·2) and in subsequent equations we are writing μ_k for μ_k' and β for Δ^2 for convenience of printing.

It is easily seen by direct algebraical methods that

$$\beta^k = \mu_k - k(k+p/2-1)\mu_{k-1}\cdot(2t)+\ldots+(-1)^r\cdot\mu_{k-r}\cdot(2t)^r\frac{\Gamma(k+1)\Gamma(k+p/2)}{\Gamma(r+1)\,\Gamma(k+1-r)\,\Gamma(k+p/2-r)}$$
$$+\ldots+(-1)^k\cdot\frac{\Gamma(k+p/2)}{\Gamma(p/2)}\cdot(2t)^k \equiv B_k \qquad \ldots \quad (1.3)$$

So that the various identical relations between μ's involving p and t, which will be called the identities of the first type, can be written in the form $(B_k)^{k'} = (B_{k'})^k$, or symbolically, $(k\ k') = 0$. These identities can be written down by taking any two expressions B_k and $B_{k'}$ and their existence is self-evident from their form.

Also since B_k's are polynomials in p or in $(2t)$, by taking any three expressions $\beta^a = B_a$, $\beta^b = B_b$, and $\beta^c = B_c$, we can eliminate (β, p) or (β, t) and thus arrive at identities involving alternatively either μ's and t's or μ's and p's. These identities can be written as

$$[abc]_p = 0, \quad [abc]_t = 0 \text{ etc.} \qquad \ldots \quad (1.4)$$

where the suffix denotes the variable which has been eliminated in the process.

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[2] *Sankhyā*, 2(4), 1936, p. 392, equation (9).

[3] *Sankhyā*, 2(4), 1936, p. 389, equation (7.5).

Finally by taking any four of the expressions we can eliminate β, p and t, arrive at relations between μ's alone, which can be written as

$$\{a\ b\ c\ d\} = 0 \qquad \ldots \quad (1.5)$$

These idential relations are interesting, and may be regarded as characteristic of the modulus distribution of the Gaussian type.

Direct algebraical computation will easily verify the following typical relations :—

$$2pt^2 - 4t\mu_1 + (\mu_2 - \mu_1^2) = 0 \qquad \ldots \quad (1.51)$$

$$[123]_p = 8\mu_1 t + 8t(\mu_1^2 - \mu_2) + (\mu_3 - 3\mu_1\mu_2 + 2\mu_1^3) \qquad \ldots \quad (1.52)$$

The expression $(1234) = 0$ is more complicated; by taking B_1, B_2, B_3, B_4, and putting

$$\left.\begin{aligned} A_2 &= \mu_1^2 - \mu_2, \quad A_3 = \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3 \\ \text{and} \qquad A_4 &= \mu_4 - 4\mu_2\mu_3 - 3\mu_2^2 + 12\mu_1^2\mu_2 - 6\mu_1^4 \end{aligned}\right\} \qquad \ldots \quad (1.61)$$

the existence of the following two quadratic equations can be proved,

$$\left.\begin{aligned} &8\mu_1 t^2 + 8A_2 t + A_3 = 0 \\ \text{and} \qquad &A_2 t^2 + A_3 t - A_4/12 = 0 \end{aligned}\right\} \qquad \ldots \quad (1.62)$$

from which the bracket $(1234) = 0$ follows as the t-eliminant of the equations.

Algebraic identities Deduced from the Differential Form of the Moment-coefficient.

2. It is satisfactory to note here that the identities of the above type, which can be arrived at by laborious algebraical methods, can be deduced much more simply by the following analytical procedure, which furnishes the *raison d'être* of identities of the above types, and especially of the existence of the quadratic equations. This method also allows us to investigate the inverse problem, namely, the nature of the distribution function for the modulus when the identities of the above kind are postulated for the moment-functions.

We start from the differential form of the moment-coefficient deduced before[4] for the D^2-statistic :—

$$\mu_k = (2st)^k \cdot e^{-1/t} \cdot t^{p/2} \cdot \left(-\frac{d}{dt}\right)^k (e^{1/t} \cdot t^{-p/2}) \qquad \ldots \quad (2.1)$$

[4] *Sankhyā*, 2(4), 1936, p. 390, equation (8.4).

where $t = 2s/u$ according to the present notation.

Then it follows from the fact that μ_k is a homogeneous function of $(2s, u)$ of the k-th degree, that

$$\frac{\mu_k}{(2st)^k} = \frac{\mu_k}{(ut^2)^k} = \frac{\mu_k}{u^k t^{2k}} = \psi_k(t) = e^{-1/t} \cdot t^{p/2} \cdot \left(\frac{-d}{dt}\right)^k (e^{1/t} \cdot t^{-p/2}) \quad \ldots \quad (2.2)$$

So that $\psi(t)$ is a polynomial of $-k$-th degree in t alone. Also the differential form of $\psi(t)$ can be transformed into an integral form. Assuming that

$$\frac{e^{1/t}}{t^{p/2}} = Z(t) = \int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx \quad \ldots \quad (2.3)$$

where $\phi(x)$ is a function whose properties we shall investigate later, it follows that

$$\psi_k(t) = \frac{1}{Z(t)} \cdot \left(-\frac{d}{dt}\right)^k \int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx \quad \ldots \quad (2.31)$$

which can be written in the form :—

$$\psi_k = \frac{\int_0^\infty x^k \cdot e^{-xt} \cdot \phi(x) \cdot dx}{\int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx} \quad \ldots \quad (2.32)$$

From this it follows that

$$\psi_k(t) \cdot \int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx = \int_0^\infty x^k \cdot e^{-xt} \cdot \phi(x) \cdot dx \quad \ldots \quad (2.41)$$

Hence by differentiation with regard to t, we have

$$-\frac{d\psi_k}{dt} \cdot \int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx - \psi_k \cdot \int_0^\infty x e^{-xt} \cdot \phi(x) \cdot dx = -\int_0^\infty x^{k+1} \cdot e^{-xt} \cdot \phi(x) \cdot dx \quad \ldots \quad (2.42)$$

Remembering the general form of ψ_k in (2.31) it follows that

$$\frac{d\psi_k}{dt} = \psi_1 \cdot \psi_k - \psi_{k+1} \quad \ldots \quad (2.43)$$

This is fundamental relation which allows us to calculate the various differential coefficients of ψ's in terms of ψ's of different orders. Thus

$$\left.\begin{aligned} \frac{d\psi_1}{dt} &= \psi_1^2 - \psi_2, \quad \frac{d^2\psi_1}{dt^2} = 2\psi_1^3 - 3\psi_1\psi_2 + \psi_3 \\ \frac{d^3\psi_1}{dt^3} &= 6\psi_1^4 - 12\psi_1^2\psi_2 + 3\psi_2^2 + 4\psi_1\psi_3 - \psi_4 \end{aligned}\right\} \quad \ldots \quad (2.51)$$

Since $(\psi_1 t^2)$ is a linear function of t it follows that the following differential relations are true.

$$\frac{d^2(\psi_1 t^2)}{dt^2} = 0, \quad \frac{d^3(\psi_1 t^2)}{dt^3} = 0, \quad \frac{d^4(\psi_1 t^2)}{dt^4} = 0 \quad \ldots \quad (2.52)$$

$$\left.\begin{aligned} t^2 \cdot \frac{d^2\psi_1}{dt^2} + 4t \cdot \frac{d\psi_1}{dt} + 2\psi_1 &= 0 \\ t^2 \cdot \frac{d^3\psi_1}{dt^3} + 6t \cdot \frac{d^2\psi_1}{dt^2} + 3\frac{d\psi_1}{dt} &= 0 \end{aligned}\right\} \quad \ldots \quad (2.61)$$

and generally $t^2 \cdot \dfrac{d^k\psi_1}{dt^k} + 2kt \cdot \dfrac{d^{k-1}\psi_1}{dt^{k-1}} + k(k-1) \cdot \dfrac{d^{k-2}\psi_1}{dt^{k-2}} = 0 \quad \ldots \quad (2.62)$

If we now remember that the differential coefficients of ψ_1's are expressible in ψ^k's of different orders, and that ψ's are only μ's divided by powers of t, the existence of the quadratic equations of the type found by algebraical method is evident. This at the same time allows us to write down the equations with much less labour.

Using equations (2.51) and equation (2.2), we have

$$\left.\begin{aligned} t^2(2\psi_1^3 - 3\psi_1\psi_2 + \psi_3) + 4t(\psi_1^2 - \psi_2) + 2\psi_1 &= 0 \\ t^2(6\psi_1^4 - 12\psi_1^2\psi_2 + 3\psi_2^2 + 4\psi_1\psi_3 - \psi_4) + 6t(2\psi_1^2 - 3\psi_1\psi_2 + \psi_3) + 3(\psi_2^2 - \psi_2) &= 0 \end{aligned}\right\} \quad \ldots \quad (2.63)$$

Now substituting the values of ψ's in terms of μ's the above two equations reduce respectively to (2.65) and (2.66) given below

$$\frac{t^2(2\mu_1^3 - 3\mu_1\mu_2 + \mu_3)}{u^3t^6} + \frac{4t(\mu_1^2 - \mu_2)}{u^2t^4} + \frac{2\mu_1}{ut^2} = 0 \quad \ldots \quad (2.64)$$

which can be written, by substituting $2s/u$ for t, in the form

$$8\mu_1 s^2+8(\mu_1^2-\mu_2)s+(2\mu_1^2-3\mu_1\mu_2+\mu_3) = 0 \quad \ldots \quad (2.65)$$

and $(\mu_1^2-\mu_2^2)s^2+(2\mu_1^3-3\mu_1\mu_2+\mu_3)s+(1/12)(6\mu_1^4-12\mu_1^2\mu_3+3\mu_2^2+4\mu_1\mu_3-\mu_4)$ .. (2.66)

These are the quadratic equations arrived at by direct elimination and referred to in (1.62)

The fundamental { } expression alluded to before in (1.5) can also be written in a form involving the differential co-efficients of ψ_1 's of different orders.

Thus writing $$\psi_1^k \equiv \frac{d^k \psi_1}{dt^k} \quad \ldots \quad (2.7)$$

we have also two relations like

$$\frac{d^k (\psi_1 t^2)}{dt} = 0, \quad \frac{d^\sigma (\psi_1 t^2)}{dt^\sigma} = 0 \quad \ldots \quad (2.81)$$

which can be written in the form

$$\left.\begin{aligned} \psi_1^k \cdot t^2+2k \cdot \psi_1^{k-1} \cdot t+k(k-1)\psi_1^{k-1} &= 0 \\ \psi_1^\sigma \cdot t^2+2\sigma \cdot \psi_1^{\sigma-1} \cdot t+\sigma(\sigma-1)\psi_1^{\sigma-2} &= 0 \end{aligned}\right\} \quad \ldots \quad (2.82)$$

Hence the following relation which is equivalent to (1.5) can be arrived at as the t-eliminant of the above two quadratic equations :—

$$[k(k-1)\psi_1^\sigma \cdot \psi_1^{k-2}-\sigma(\sigma-1)\psi_1^{\sigma-2} \cdot \psi_1^k]^2$$
$$= 4[k\sigma(\sigma-1)\psi_1^{k-1} \cdot \psi_1^{\sigma-2}-\sigma k(\sigma-1)\psi_1^{\sigma-1} \cdot \psi_1^{k-2}][\sigma \cdot \psi_1^k \psi_1^{\sigma-1}-k\psi_1^\sigma \cdot \psi_1^{k-1}] \quad \ldots \quad (2.9)$$

There is however one fundamental relation. The other relations deduced should therefore reduce to the fundamental one calculated above, where the different relations between μ's of different orders are taken into account.

The Integral Equation connected with the Moment-coefficients and its Solutions.

3. I shall now take up the integral form of $\psi_k(t)$

$$\psi_k(t) = \frac{\int_0^\infty x^k \cdot e^{-xt} \cdot \phi(x) \cdot dx}{\int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx} \quad \ldots \quad (2.32)$$

The form shows that $\psi_k(t)$'s are moment-functions of a certain x-distribution and

a knowledge of $\phi(x)$ will determine the distribution function to which $\psi(t)$'s can be related. In other words $\phi(x)$ will determine the μ_k's with which $\psi_k(t)$'s are simply connected. The existence of the algebraic identities between the μ's follows from two separate hypotheses—(1) the existence of the above integral relation, and (2) the linear form of $\psi_1(t) \cdot (t)^2$. We can therefore see what form of $\phi(x)$ (which for the present discussion can be regarded as arbitrary at first) is determined by the hypothesis that $\psi_1(t) \cdot t^2$ is a linear function of t.

Since

$$\psi_1(t) = \frac{\int_0^\infty x \cdot e^{-xt} \cdot \phi(x) \cdot dx}{\int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx} \qquad \ldots \quad (3.10)$$

That is,

$$\psi_1(t) \cdot \int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx = \int_0^\infty x \cdot e^{-xt} \cdot \phi(x) \cdot dx \qquad \ldots \quad (3.11)$$

The assumption of the linear form of $\psi_1(t) \cdot t^2$ implies that

$$(At+B) \int_0^\infty e^{-xt} \cdot \phi(x) \cdot dx = \int_0^\infty x \cdot t^2 \cdot e^{-tx} \cdot \phi(x) \cdot dx \qquad \ldots \quad (3.12)$$

or

$$\int_0^\infty e^{-xt} [x \cdot \phi(x) \cdot t^2 - A \cdot t \cdot \phi(x) - B \cdot \phi(x)] \cdot dx = 0 \qquad \ldots \quad (3.13)$$

The above relation can be transformed by integration by parts. Thus

$$\int_0^\infty e^{-xt} \cdot x \cdot \phi(x) \cdot t^2 \cdot dx = \left[-t \cdot x \cdot \phi(x) \cdot e^{-xt} - \frac{d}{dx}\{x\phi(x)\} \cdot e^{-xt}\right]_0^\infty + \int \frac{d^2}{dx^2}\{x\phi(x)\} \cdot e^{-xt} \cdot dx \qquad \ldots \quad (3.21)$$

Similarly

$$\int_0^\infty t \cdot e^{-xt} \cdot \phi(x) \cdot dx = \left[-\phi(x) \cdot e^{-xt}\right]_0^\infty + \int_0^\infty e^{-xt} \cdot \frac{d\phi}{dx} \cdot dx \qquad \ldots \quad (3.22)$$

The relation (3.13) implies

$$\left[-t \cdot x \cdot \phi(x) \cdot e^{-xt} \cdot -\frac{d}{dx}\{x\phi(x)\} \cdot e^{-xt} + A \cdot \phi(x) \cdot e^{-xt}\right]_0^\infty + \int_0^\infty e^{-xt}\left\{\frac{d^2}{dx^2}\{x\phi(x)\} - A \cdot \frac{d\phi(x)}{dx} - B \cdot \phi(x)\right\} dx = 0 \qquad \ldots \quad (3.23)$$

Thus

$$\left\{e^{-xt}\{-x\frac{d\phi}{dx}-(tx+1-A)\phi(x)\}\right\}_0^\infty+\int_0^\infty e^{-xt}\left\{\frac{d^2}{dx^2}\{x\phi(x)\}-A\cdot\frac{d\phi}{dx}-B\phi\right\}=0 \quad \ldots (3.24)$$

This shows that $\phi(x)$ is a regular solution existing up to infinity of the following differential equation :

$$\frac{d^2}{dx^2}[x\cdot\phi(x)]-A\cdot\frac{d\phi(x)}{dx}-B\cdot\phi(x)=0 \quad \ldots (3.25)$$

if $\left(\frac{d\phi}{dx}\right)$ is finite and either $A=1$, or $\phi(x)=0$, when $x=0$.

The differential equation

$$\frac{d^2}{dx^2}(x\cdot\phi)-A\cdot\frac{d\phi}{dx}-B\phi\cdot(x)=0 \quad \ldots (3.25)$$

can be transformed by the substitution

$$x=R^2/\rho \quad \ldots (3.26)$$

So that

$$dx=\frac{2R\cdot dR}{\rho}, \text{ and } \frac{d}{dx}=\frac{\rho}{2R}\cdot\frac{d}{dR} \quad \ldots (3.27)$$

Hence the equation becomes

$$\frac{d^2\phi}{dR^2}-\frac{(2A-3)}{R}\cdot\frac{d\phi}{dR}-\frac{4B\cdot\phi}{\rho}=0 \quad \ldots (3.28)$$

The different cases for different positive and negative values of A and B can be easily seen to be solvable in terms of the functions of the type $J_\lambda(r)$ or $J_\lambda(ir)$.

When $A=p/2$, $B=1$, the case reduces to that of the D^2-statistic. The differential equation then becomes

$$\frac{d^2\phi}{dR^2}-\frac{(p-3)}{R}\cdot\frac{d\phi}{dR}-\phi=0, \text{ when } \rho=4 \quad \ldots (3.31)$$

The solution can be easily seen to be of the form $A\cdot R^{(p-2)/2}\cdot I_{\frac{1}{2}(p-2)}(R)$

or
$$\phi(x)=A\cdot(2\sqrt{x})^{(p-2)/2}\cdot I_{\frac{1}{2}(p-2)}(2\sqrt{x}) \quad \ldots (3.32)$$

Therefore if A is so chosen that

$$\frac{e^{1/t}}{t^{p/2}} = A \cdot \int_0^\infty (2\sqrt{x})^{(p-2)/2} \cdot I_{\frac{1}{2}(p-2)}(2\sqrt{x}) \cdot e^{-xt} \cdot dx \quad \ldots \quad (3.33)$$

then

$$\psi_k(t) = \frac{A \cdot \int_0^\infty x^{\frac{k}{2}} \cdot (2\sqrt{x})^{(p-2)/2} \cdot I_{\frac{1}{2}(p-2)}(2\sqrt{x}) \cdot e^{-xt} \cdot dx}{e^{1/t}\ t^{-p/2}} \quad \ldots \quad (3.34)$$

or

$$\psi_k(t) = A \cdot \int_0^\infty x^k \cdot e^{(-1/t-x/t)} \cdot t^{p/2} \cdot (2\sqrt{x})^{(p-2)/2} \cdot I_{\frac{1}{2}(p-2)}(2\sqrt{x}) \cdot dx \quad \ldots \quad (3.35)$$

Remembering that $\psi_k(t) = \frac{\mu_k}{(2st)^k}$ ... (3.36)

We have $\psi_k = A \cdot \int_0^\infty (2stx)^k \cdot e^{(-1/t-x/t)} \cdot t^{p/2} (2\sqrt{x})^{(p-2)/2} \cdot I_{\frac{1}{2}(p-2)}(2\sqrt{x}) \cdot dx$ ... (3.37)

Assuming $(2stx) = r^2$ as a new variable, then

$$\psi_k = A \cdot \int_0^\infty r^{2k} \cdot e^{-(1/t+r^2/2s)}\ t^{p/2} \cdot \{2\sqrt{(r^2/2st)}\}^{(p-2)/2} \cdot I_{\frac{1}{2}(p-2)}\{2\sqrt{(r^2/2st)}\} \cdot \frac{d(r^2)}{2st} \quad (3.38)$$

Remerbering that

$$t = \frac{2s}{\Delta^2}, \quad \frac{r^2}{2st} = \frac{r^2\Delta^2}{4s^2}, \quad \text{or} \quad 2\sqrt{\frac{r^2}{2st}} = \frac{r\Delta}{s} \quad \ldots \quad (3.39)$$

we have $\mu_k = A \cdot \int_0^\infty \frac{r^{2k}}{s} \cdot e^{-(\Delta^2+r^2)/2s} \cdot 2^{(p-1)/2} \cdot \frac{r^{p/2}dr}{\Delta^{(p-2)/2}} \cdot I_{\frac{1}{2}(p-2)}\left(\frac{r\Delta}{s}\right)$ ... (3.41)

$$= 2^{(p-1)/2} \cdot A \cdot \int_0^\infty \frac{r^{2k} \cdot e^{-(\Delta^2+r^2)2s}}{s \cdot \Delta^{(p-2)/2}} \cdot r^{p/2}dr \cdot I_{\frac{1}{2}(p-2)}\left(\frac{r\Delta}{s}\right) \quad \ldots \quad (3.42)$$

Now putting $s = 1/n$, we have

$$\mu_k = 2^{(p-1)/2} \cdot A \cdot \int_0^\infty \frac{r^{2k} \cdot n}{\Delta^{(p-2)/2}} \cdot e^{-\frac{1}{2}n(\Delta^2+r^2)} \cdot r^{p/2} \cdot I_{\frac{1}{2}(p-2)}(nr\Delta) \cdot dr \quad \ldots \quad (3.43)$$

Thus the μ's can be regarded as the moment functions for the distribution

$$dF = \text{const.}\ n \cdot \frac{e^{-\frac{1}{2}n(\Delta^2+r^2)}}{\Delta^{(p-2/2)}} \cdot r^{p/2} \cdot I_{\frac{1}{2}(p-2)}(nr\Delta) \cdot dr \quad \ldots \quad (3.44)$$

which agrees with R. C. Bose's formula.[5]

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[5] R. C. Bose : "On the Exact Distribution and Moment co-efficients of the D^2-Statistic". *Sankhyā*, 2(2), 1936, p. 149, equation (6.4).

Spherical Form of the Fundamental Differential Equation Connected with the Multivariate Normal Population.

4. I shall now discuss the differential equation

$$\frac{\partial^2 F}{\partial x_1^2}+\frac{\partial^2 F}{\partial x_2^2}+\ldots\ldots+\frac{\partial^2 F}{\partial R_p^2}=2\cdot\frac{\partial F}{\partial t} \qquad \ldots \quad (4.1)$$

which may be regarded as the fundamental equation for the distribution function of the multivariate normal population in p-variates.

If F is a function of (R, t) where

$$R^2=x_1^2+x_2^2+x_3^2+\ldots+x_p^2 \qquad \ldots \quad (4.2)$$

then F satisfies the differential equation

$$\frac{\partial^2 F}{\partial R^2}+\frac{p-1}{R}\cdot\frac{\partial F}{\partial R}=2\cdot\frac{\partial F}{\partial t} \qquad \ldots \quad (4.3)$$

The Differential Equation in Parabolic Form.

5. When F is a function of the form

$$F(x_1^2+x_2^2+\ldots+x_p^2,\ a_1x_1+\ldots+a_px_p,\ t)$$

by suitably choosing one axis say y_1, perpendicular to the hyperplane

$$a_1x_1+a_2x_2+\ldots\ a_px_p=0 \qquad \ldots \quad (5.1)$$

and similar orthogonal axis-system as before, F takes the form

$$F(R, y_1, t) \text{ or } F(y_1, y_1^2+\ldots+y_p^2, t)$$

The equation (4.1) can be transformed to the form

$$\frac{\partial^2 F}{\partial y_1^2}+\frac{1}{r^{p-2}}\cdot\frac{\partial}{\partial r}\left(r^{p-2}\cdot\frac{\partial F}{\partial r}\right)=2\cdot\frac{\partial F}{\partial t} \qquad \ldots \quad (5.2)$$

where
$$r^2=y_2^2+y_3^2+\ldots+y_p^2 \qquad \ldots \quad (5.2)$$

Further decomposition can be carried on in the same way.

If F is of the form $F(x_1, x_2, R, t)$; the differential equation can be written in the form

$$\frac{\partial^2 F}{\partial x_1^2}+\frac{\partial^2 F}{\partial x_2^2}+\frac{1}{\omega^{p-3}}\cdot\frac{\partial}{\partial\omega}\left(\omega^{p-3}\cdot\frac{\partial F}{\partial\omega}\right)=2\cdot\frac{\partial F}{\partial t} \qquad \ldots \quad (5.4)$$

where
$$\omega^2=x_2^2+x_3^2+\ldots+x_p^2, \text{ and so on.} \qquad \ldots \quad (5.5)$$

THE DIFFERENTIAL EQUATION IN THE PRODUCT FROM.

6. If $F(x_1, x_2, \ldots x_r, t)$ can be written as the product of two functions

$$\phi_1(x_1, x_2, \ldots x_r, t) \cdot \phi_2(x_{r+1}, x_{r+2}, \ldots x_p, t) \qquad \ldots \quad (6.1)$$

then the fundamental differential equation (4.1) breaks up at once to the form

$$\frac{1}{\phi_1}\left[\Box_r^2 \phi_1 - 2\cdot \frac{\partial \phi_1}{\partial t}\right] + \frac{1}{\phi_2}\left[\Box_{r+1\cdots p}\phi_2 - 2\cdot \frac{\partial \phi_2}{\partial t}\right] = 0 \qquad \ldots \quad (6.2)$$

so that the functions ϕ_1 and ϕ_2 satisfy respectively equations of the form

$$\left.\begin{aligned} \Box_{1\cdot 2\cdots r}^2 \phi_1 - 2\cdot \frac{\partial \phi_1}{\partial t} &= F(t, a)\phi_1 \\ \text{and} \qquad \Box_{r+1\cdots p}^2 \phi_2 - 2\cdot \frac{\partial \phi_2}{\partial t} &= -F(t, a)\phi_2 \end{aligned}\right\} \qquad \ldots \quad (6.3)$$

Now these equations are transformable to the forms

$$\left.\begin{aligned} \Box_r^2 \phi_1 \cdot e^{-\frac{1}{2}\int F(t, a)\, dt} &= 2\frac{d}{dt}\left[\phi_1 \cdot e^{-\frac{1}{2}\int F(t, a)\, dt}\right] \\ \Box_{r+1\cdots p}^2 \varphi_2 \cdot e^{\frac{1}{2}\int F(t, a)\, td} &= 2\frac{d}{dt}\left[\phi_2 \cdot e^{\frac{1}{2}\int F(t, a)\, dt}\right] \end{aligned}\right\} \qquad \ldots \quad (6.4)$$

Therefore if L_1 and L_2 are solutions of the equations (6.4), their product $L_1 \cdot L_2$ is a solution of the original equation (4.1). It is easily seen that

$$\frac{e^{-x^2/2t}}{t^{1/2}}$$

satisfies the differential equation

$$\frac{\partial^2 F}{\partial x_1^2} = 2\cdot \frac{\partial F}{\partial t} \qquad \ldots \quad (6.5)$$

Hence

$$\frac{e^{-\Sigma_r(x_r^2/2t)}}{t^{p/2}}$$

is a solution of the equation in p-variables *i.e.* of the equation

$$\Box_p^2 F = 2\cdot \frac{\partial F}{\partial t} \qquad \ldots \quad (6.6)$$

Further Reduction of the Parabolic Form to Polar Form

7. The equation (5.2) can be written in the form

$$\frac{\partial^2 F}{\partial x_1^2} + \frac{\partial^2 F}{\partial r^2} + \frac{p-2}{r} \cdot \frac{\partial F}{\partial r} = 2 \cdot \frac{\partial F}{\partial t} \qquad \ldots \quad (7.1)$$

Since

$$R^2 = r^2 + x_1^2 \qquad \ldots \quad (7.11)$$

we can introduce in place of r and x_1, variables R and θ defined by the equations

$$r = R \sin\theta \quad \text{and} \quad x_1 = R \cos\theta \qquad \ldots \quad (7.2)$$

Then

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial p^2} = \frac{\partial^2 F}{\partial R^2} + \frac{1}{R} \cdot \frac{\partial F}{\partial R} + \frac{1}{R^2} \cdot \frac{\partial^2 F}{\partial \theta^2} \qquad \ldots \quad (7.31)$$

and

$$\frac{\partial F}{\partial r} = \sin\theta \cdot \frac{\partial F}{\partial R} + \frac{\cos\theta}{R} \cdot \frac{\partial F}{\partial \theta} \qquad \ldots \quad (7.32)$$

The transformed equation in R, θ becomes

$$\frac{\partial^2 F}{\partial R^2} + \frac{1}{R} \cdot \frac{\partial F}{\partial R} + \frac{1}{R^2} \cdot \frac{\partial^2 F}{\partial \theta^2} + \frac{(p-2)}{R \sin\theta} \cdot \left[\sin\theta \cdot \frac{\partial F}{\partial R} + \frac{\cos\theta}{R} \cdot \frac{\partial F}{\partial R} \right] = 2 \cdot \frac{\partial F}{\partial t} \qquad \ldots \quad (7.33)$$

or

$$\frac{\partial^2 F}{\partial R^2} + \frac{(p-1)}{R} \cdot \frac{\partial F}{\partial R} + \frac{(p-2)}{R} \cdot \cot\theta \cdot \frac{\partial F}{\partial \theta} + \frac{1}{R^2} \cdot \frac{\partial^2 F}{\partial \theta^2} = 2 \cdot \frac{\partial F}{\partial t} \qquad \ldots \quad (7.34)$$

Since

$$x_1^2 + x_2^2 + \omega^2 = R^2 \qquad \ldots \quad (7.41)$$

the equation

$$\frac{\partial^2 F}{\partial x_1^2} + \frac{\partial^2 F}{\partial x_2^2} + \frac{\partial^2 F}{\partial \omega^2} + \frac{(p-3)}{\omega} \cdot \frac{\partial F}{\partial \omega} = 2 \cdot \frac{\partial F}{\partial t} \qquad \ldots \quad (7.42)$$

can be transformed by the substitutions

$$\omega = R \cos\theta, \quad x_1 = R \sin\theta \cos\phi, \quad x_2 = R \sin\theta \sin\phi \qquad \ldots \quad (7.51)$$

to

$$\frac{1}{R^2 \partial R} \left(R^2 \cdot \frac{\partial F}{\partial R} \right) + \frac{1}{R^2 \sin\theta} \cdot \frac{\partial}{\partial \theta} \left(\sin\theta \cdot \frac{\partial F}{\partial \theta} \right) + \frac{1}{R^2 \sin^2\theta} \cdot \frac{\partial^2 F}{\partial \phi^2}$$

$$+ \frac{(p-3)}{R \sin\theta} \left[\frac{\partial F}{\partial R} \cdot \cos\theta - \frac{\sin\theta}{R} \cdot \frac{\partial F}{\partial R} \right] = 2 \frac{\partial F}{\partial t} \qquad \ldots \quad (7.52)$$

Equation (5.2) could be investigated by assuming a solution of the form

$$F = f((R, t) \cdot P(\theta) \qquad \ldots \quad (7.61)$$

so that the equation breaks up in the form

$$\frac{\partial^2 f}{\partial R^2} + \frac{(p-1)}{R} \cdot \frac{\partial f}{\partial R} - \frac{\lambda f}{R^2} = 2\frac{\partial f}{\partial t} \text{ and } \frac{\partial^2 P}{\partial \theta^2} + (p-2) \cdot \cot \theta \cdot \frac{\partial P}{\partial \theta} + \lambda P = 0 \quad \ldots \quad (7.62)$$

The equation (5.4) could be similarly investigated by assuming a solution of the form

$$F = f(R, t) \cdot P(\theta) \cdot Q(\phi) \qquad \ldots \quad (7.7)$$

so that the equation (7.42) breaks up into the three equations

$$\left.\begin{aligned} \frac{\partial^2 f}{\partial R^2} + \frac{(p-1)}{R} \cdot \frac{\partial f}{\partial R} - \frac{\lambda}{R^2} \cdot f &= 2 \cdot \frac{\partial f}{\partial t} \\ \frac{\partial^2 Q}{\partial \phi^2} + n^2 \cdot Q &= 0 \\ \frac{1}{\sin \theta} \cdot \frac{\partial}{\partial \theta}\left(\sin \theta \cdot \frac{\partial Q}{\partial \theta} \right) + \frac{n^2}{\sin^2 \theta} \cdot \frac{\partial Q}{\partial \theta} - (p-3) \cdot \tan \theta \cdot \frac{\partial Q}{\partial \theta} + \lambda Q &= 0 \end{aligned}\right\} \quad \ldots \quad (7.8)$$

SOLUTIONS OF THE DIFFERENTIAL EQUATION IN DIFFERENT CASES.

8. We shall first consider the equation of the type (4.1). The existence of a simple solution of the type

$$\frac{e^{-\Sigma_r (x_r^2/2t)}}{(2\pi t)^{p/2}} = F_1 \qquad \ldots \quad (8.11)$$

is verified by direct substitution.

We shall have occasion to make use of the theorem of Appell and Brill, which has been proved in the case of p-variables earlier[6], that if $H(x_1, x_2, \ldots x, t)$ is a solution of the differential equation (4.1), then

$$H'(x_1, x_2, \ldots \cdot \ldots x_p, t) = \frac{e^{-\Sigma_r (x_r^2/2t)}}{(2\pi)^{p/2}} \cdot H\left(\frac{x_1}{t}, \frac{x_2}{t}, \ldots \frac{x_p}{t}, -\frac{1}{t} \right) \quad \ldots \quad (8.12)$$

is also a solution.

Therefore in the case of a spherically symmetric solution of the type

$$H(R^2, t) = H(x_1, x_2, \ldots x_p, t) \qquad \ldots \quad (8.13)$$

[6] *Sankhyā*, 2(4), 1936, p. 393, equation (11.1).

$$H'(x_1, x_2, \ldots, x_p, t) = \frac{e^{-\Sigma_r(x_r^2/2t)}}{(2\pi\ t)^{p/2}} \cdot H\left(\frac{R^2}{t^2}, -\frac{1}{t}\right) \quad \ldots \quad (8.14)$$

is also a solution.

We therefore transform the equation

$$\frac{\partial^2 F}{\partial R^2} + \frac{p-1}{R} \cdot \frac{\partial F}{\partial R} = 2 \cdot \frac{\partial F}{\partial t} \quad \ldots \quad (4.3)$$

by the substitution $R^2 = y$, and the equation becomes

$$2y \cdot \frac{\partial^2 F}{\partial y^2} + p \cdot \frac{\partial F}{\partial y} = \frac{\partial F}{\partial t} \quad \ldots \quad (8.16)$$

Further putting $2t = T$, it becomes ... (8.17)

$$y \cdot \frac{\partial^2 F}{\partial y^2} + \frac{p}{2} \cdot \frac{\partial F}{\partial y} = \frac{\partial F}{\partial T} \quad \ldots \quad (8.18)$$

First consider the equation (4.3). If we assume

$$F = e^{\lambda t} \cdot \psi(R) \quad \ldots \quad (8.21)$$

$\psi(R)$ satisfies the differential equation

$$\frac{\partial^2 \psi}{\partial R^2} + \frac{p-1}{R} \frac{\partial \psi}{\partial R} = 2\lambda\psi \quad \ldots \quad (8.22)$$

whose solution can be written in either of the forms

$$\frac{e^{\lambda t}}{R^{(p-2)/2}} \cdot I_{\frac{1}{2}(p-2)}(R\sqrt{2\lambda}) \quad \ldots \quad (8.23)$$

$$\frac{e^{-\lambda t}}{R^{(p-2)/2}} \cdot J_{\frac{1}{2}(p-2)}(R\sqrt{2\lambda}) \quad \ldots \quad (8.24)$$

Therefore, from (8.13) and (8.14) we get two further types of solutions. Carrying out the transformation suggested it is seen that

$$\frac{e^{-\Sigma(x_r^2/2t)}}{(2\pi t)^{p/2}} \cdot \frac{e^{-\lambda/t}}{R^{(p-2)/2}} \cdot t^{(p-2)/2} \cdot I_{\frac{1}{2}(p-2)}\{\sqrt{(2\lambda)} \cdot R/t\} \quad \ldots \quad (8.25)$$

and

$$\frac{e^{-\Sigma(x_r^2/2t)}}{(2\pi t)^{p/2}} \cdot \frac{e^{\lambda/t}}{R^{(p-2)/2}} \cdot t^{(p-2)/2} \cdot J_{\frac{1}{2}(p-2)}\{\sqrt{(2\lambda)} \cdot R/t\} \quad \ldots \quad (8.26)$$

are also solutions of the equation (4.3).

Hence writing $\lambda = \xi^2/2$, we have as solutions

$$\frac{e^{-(\xi^2+R^2)/2t}}{t \cdot (2\pi)^{p/2} \cdot R^{(p-2)/2}} I_{\frac{1}{2}(p-2)}(R\xi/t) \quad \ldots \quad (8.27)$$

and

$$\frac{e^{-(R^2-\xi^2)/2t}}{t \cdot (2\pi)^{p/2} \cdot R^{(p-2)/2}} J_{\frac{1}{2}(p-2)}(R\xi/t) \quad \ldots \quad (8.28)$$

We shall now seek polynomial types of solution. Stating from the formula (4.3) and putting

$$y = R^2, T = 2t \quad \ldots \quad (8.31)$$

we get

$$y \cdot \frac{\partial^2 F}{\partial y^2} + \frac{p}{2} \cdot \frac{\partial F}{\partial y} = \frac{\partial F}{\partial T} \quad \ldots \quad (8.32)$$

Let us put

$$F = T^k \cdot \phi(y/T) \quad \ldots \quad (8.33)$$

Since

$$\frac{\partial F}{\partial y} = T^{k-1} \cdot \phi', \frac{\partial^2 F}{\partial y^2} = T^{k-2} \cdot \phi'', \frac{\partial^2 \phi}{\partial T^2} = k \cdot T^{k-1} \cdot \phi - y \cdot T^{k-2} \cdot \phi \quad \ldots \quad (8.34)$$

the equation (8.32) becomes

$$y \cdot T^{k-2}\phi'' + (p/2) \cdot T^{k-1} \cdot \phi' = kT^{k-1} \cdot \phi - y \cdot T^{k-2} \cdot \phi' \quad \ldots \quad (8.35)$$

or dividing by T^{k-1} and putting $y/T = s$ $\ldots \quad (8.36)$

$$s \cdot \phi_k'' + \{(p/2) + s\} \cdot \phi_k' - k \cdot \phi_k' = 0 \quad \ldots \quad (8.37)$$

This equation has been already discussed in a previous paper.[7]

It is easily seen that with integer values of k, it gives polynomial solutions. These solutions are denoted by $\mu_k(y, T)$ here. They are identical with the moment-functions discussed at some length in the earlier paper.[8] The solutions can be written in the form

$$T^k \cdot \mu_k(y/T) \quad \ldots \quad (8.41)$$

The application now of (8.12), allows us to deduce solutions of the type

$$\frac{e^{-R^2/2t}}{(t)^{p/2}} \cdot \frac{1}{T^k} \cdot S(s), \text{ where } s = y/T = R^2/2t \quad \ldots \quad (8.42)$$

[7] *Sankhyā*, 2(4), 1936, p. 390, equation (8.5).

[8] *Sankhyā*, 2(4), 1936 pp. 387—390.

and $$s \cdot S_k'' + \{(p/2) - s\} \cdot S_k' + k \cdot S_k = 0 \quad \ldots \quad (8.43)$$

We therefore see that

$$\frac{e^{-R^2/2t}}{(2\pi t)^{p/2}} \quad \frac{1}{T^{2k}} \cdot S_k(y, T) \quad \ldots \quad (8.44)$$

is also a solution where $S_k(y, T)$ is a homogeneous polynomial in (y, T).

We shall now seek relations between solutions of type (4.3) and (5.3) with type (8.44). We note that since (8.44) is a solution of the fundamental equation (4.3), writing

$$R^2 = y, \; 2t = T \quad \ldots \quad (8.45)$$

we see that

$$L = \frac{e^{-y/T}}{T^{p/2+k}} \cdot S_k(y/T) \text{ satisfies the equation (8.32)} \quad \ldots \quad (8.46)$$

that is, $$y \cdot \frac{\partial^2 L}{\partial y^2} + \frac{p}{2} \cdot \frac{\partial L}{\partial y} = \frac{\partial L}{\partial T} \quad \ldots \quad (8.47)$$

This result will be useful later on.

Consider the fundamental equation (8.32) in $(y \; T)$.

$$y \cdot \frac{\partial^2 \psi}{\partial y^2} + \frac{p}{2} \cdot \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial T} \quad \ldots \quad (8.32)$$

If we write $$\psi = e^{\lambda T} \cdot F(y) \quad \ldots \quad (8.51)$$

we can see that

$$y \cdot \frac{\partial^2 F}{\partial y^2} + \frac{p}{2} \cdot \frac{\partial F}{\partial y} = \lambda \cdot F \quad \ldots \quad (8.52)$$

The form of the equation shows that

$$\psi = e^{\lambda T} \cdot F(\lambda y) \quad \ldots \quad (8.53)$$

is a solution of (8.52) where $F(y)$ satisfies the equation

$$\frac{\partial^2 F}{\partial y^2} + \frac{p}{2y} \cdot \frac{\partial F}{\partial y} = \frac{F}{y} \quad \ldots \quad (8.54)$$

Appell's theorem applied to this form gives us a solution of the form

$$t^{-p/2} \cdot e^{-R^2/2t} \cdot e^{-\lambda/T} \cdot F(\lambda y/T) \quad \ldots \quad (8.55)$$

or $$t^{-p/2} \cdot e^{-R^2/t^2} \cdot F(\lambda R^2/T) \quad \ldots \quad (8.56)$$

Remembering that $$T = 2t, \quad \text{and putting} \quad \lambda = \Delta^2 \quad \ldots \quad (8.57)$$

the solution reduces to

$$t^{-p/2} \cdot e^{-(R^2+\Delta^2)/2t} \cdot F \cdot (\Delta^2 R^2/4t) \quad \ldots \quad (8.54)$$

Remembering again that $T = 2t$, we see that the solution of (8.54) is of the form

$$t^{-p/2} \cdot e^{-R^2/2t} \cdot e^{-\Delta^2/2t} \cdot \psi(\Delta R/t) \quad \ldots \quad (8.61)$$

Now a solution of the above type gives rise to a distribution

$$dF = H \cdot dx_1 \cdot dx_2 \ldots dx_p \quad \ldots \quad (8.62)$$

where H is a solution of the equation. Therefore

$$dF = A \cdot e^{-(R^2+\Delta^2)/2t} \cdot t^{-p/2} \cdot \psi(\Delta R/t) \cdot R^{(p-2)/2} \quad \ldots \quad (8.63)$$

If we compare the result with that in the earlier paper, it is easily seen that the function

$$\psi(\Delta R/t) = A \cdot (n\lambda l)^{-(p-2)/2} \cdot I_{\frac{1}{2}(p-2)} \cdot (nl\lambda) \quad \ldots \quad (8.64)$$

using the notation of R. C. Bose's paper[9].

If we remember that $n = 1/t$, $l = R$, $\lambda = \Delta$ the identity of the solution can also be easily seen.

By transforming the equation

$$Z \cdot \frac{\partial^2 F}{\partial Z} + \frac{p}{2} \cdot \frac{\partial F}{\partial Z} = F \quad \ldots \quad (8.72)$$

by the substitution $Z = x^2/4$, we have

$$\frac{\partial^2 F}{\partial x^2} + \frac{(p-1)}{x} \cdot \frac{\partial F}{\partial x} = F \quad \ldots \quad (8.73)$$

Since I_n/x satisfies the equation

$$\frac{\partial^2 F}{\partial x^2} + \frac{(2n+1)}{x} \cdot \frac{\partial F}{\partial x} = F \quad \ldots \quad (8.74)$$

the solution of the above is easily seen to be

$$A \cdot x^{-(p-2)/2} \cdot I_{\frac{1}{2}(p-2)}(x) \quad \ldots \quad (8.75)$$

On the Series Solution of the Differential Equation.

9. I have assumed in an earlier paper the possibility of expansion of solutions or the equation (4.1) as series of the type[10]

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[9] Raj Chandra Bose: "On the Exact Distribution and Moment-coefficients of the D^2-statistic" *Sankhyā*, 2(2), 1936, p. 148, equation (6.4). [10] *Sankhyā*, 2(4), 1936, p. 396, equation (12.7).

$$\Sigma_k A_k \cdot \frac{e^{-R^2/2t}}{t^{p/2}} \cdot \frac{1}{t^k} \cdot S_k(R^2/2t)$$

where Σ_k denotes a summation over all values of k from 1 to ∞, and S_k denotes a Sonine polynomial of the k-th order

The justification for the statement will be given here, and *I* shall, by actually developing the distribution function, arrive at the same result as before. A distribution function of the type

$$t^{-p/2} \cdot e^{-(\xi^2+R^2)/2t} \cdot F(\xi^2 R^2/4T^2)$$

where $R^2 = x_1^2 + x_2^2 + \ldots + x_p^2$, necessarily implies that

$$\int t^{-p/2} \cdot e^{-(\xi^2+R^2)/2t} \cdot F(\xi^2 R^2/4t^2) \cdot R^{p-1} \cdot dR = 1 \qquad \ldots \quad (9.12)$$

Assume now that $$R^2/2t = Z, \quad \text{and} \quad \xi^2/2t = a \qquad \ldots \quad (9.13)$$

The integral then takes the form

$$2^{(p-2)/2} \cdot \int_0^\infty t^{-p/2} \cdot e^{-(a+z)} \cdot F(az) \cdot z^{(p-2)/2} \cdot t^{p/2} \cdot dZ = 1 \qquad \ldots \quad (9.14)$$

or $$2^{(p-2)/2} \cdot e^{-a} \int_0^\infty e^{-z} \cdot F(az) \cdot z^{(p-2)/2} \cdot dz = 1 \qquad \ldots \quad (9.15)$$

Therefore so long as a remains finite

$$\int_0^\infty e^{-z} \cdot z^{(p-2)/2} \cdot F(az) \cdot dz = 2^{-(p-2)/2} \cdot e^a = \text{finite} \qquad \ldots \quad (9.16)$$

Therefore a constant c can be found such that

$$\int_0^\infty e^{-z} \cdot z^{(p-2)/2} \cdot [F(az) - c] \cdot dz = 0 \qquad \ldots \quad (9.17)$$

so that if we consider the function $$F(az) - c \equiv F_1(z) \qquad \ldots \quad (9.18)$$

we have $$\int_0^\infty e^{-z} \cdot z^{(p-2)/2} \cdot F_1(z) \cdot dz = 0 \qquad \ldots \quad (9.19)$$

Now the Sonine polynomials $S_k(z)$ satisfy the differential equation

$$z \cdot \frac{d^2 S_k}{dz^2} + \left(\frac{p}{2} - z\right) \cdot \frac{dS_k}{dz} + k \cdot S_k = 0 \qquad \ldots \quad (9.21)$$

or written in the self-adjoint form

$$\frac{d}{dz}\cdot\left(z^{p/2}\cdot e^{-z}\cdot\frac{dS_k}{dz}\cdot\right)+k\cdot S_k\cdot e^{-z}\cdot z^{(p-2)/2}=0 \quad \ldots \quad (9.22)$$

We see that $e^{-z}\cdot z^{(p-2)/2}$ is the density function for the orthogonal set of polynomials. Hence follows the possibility of expansion of a function $F_1(z)$ which satisfies the equation

$$\int_0^\infty e^{-z}\cdot z^{(p-2)/2}\cdot F_1(z)\cdot dz=0 \quad \ldots \quad (9.23)$$

Assuming therefore that $F(az)=\Sigma_k\, A_k\cdot S_k(z)$ ... (9.24)

the co-efficients A_k will be (because of the orthogonality of the polynomial functions S_k mentioned earlier) found to be given by

$$A_k=\frac{\int_0^\infty F(az)\cdot S_k(z)\cdot e^{-z}\cdot z^{(p-2)/2}\cdot dz}{\int_0^\infty e^{-z}\cdot z^{(p-2)/2}\cdot S_k(z)\cdot dz}$$

Let us suppose that the functions are normalized such that

$$\int_0^\infty e^{-k}\cdot z^{(p-2)/2}\cdot S_k^2(z)\cdot dz=1 \quad \ldots \quad (9.26)$$

Then $$A_k=\int_0^\infty F(az)\cdot S_k(z)\cdot e^{-z}\cdot z^{(p-2)/2}\cdot dz \quad \ldots \quad (9.27)$$

To calculate A_k we at first write $az=x$; then ... (9.28)

$$A_k=\int_0^\infty F(x)\cdot S_k(z/a)\cdot a^{-p/2}\cdot e^{-x/a}\cdot x^{(p-2)/2}\cdot dx \quad \ldots \quad (9.31)$$

As noted in *Sankhyā*, 2(4), 1836, pp. 394, $L=a^{-(p+2k)/2}\cdot e^{-x/a}\cdot S_k(x/a)$ ... (9.32) satisfies the differential equation

$$x\cdot\frac{\partial^2 L}{\partial x^2}+\frac{p}{2}\cdot\frac{\partial L}{\partial x}=\frac{\partial L}{\partial a} \quad \ldots \quad (9.33)$$

$$A_k/a^k=\int_0^\infty F(x)\cdot x^{(p-2)/2}\cdot L\cdot dx \quad \ldots \quad (9.41)$$

or $$\frac{\partial}{\partial a}\left(\frac{A_k}{a_k}\right) = \int_0^\infty x^{(p-2)/2} \cdot F(x) \cdot \frac{\partial L}{\partial a} \cdot dx \quad \ldots \quad (9.42)$$

$$= \int_0^\infty x^{(p-2)/2} \cdot F(x) \cdot \left[x \cdot \frac{\partial^2 L}{\partial x^2} + \frac{p}{2} \cdot \frac{\partial L}{\partial x} \right] \cdot dx \quad \ldots \quad (9.43)$$

Now by successive integration by parts, because the quantities outside the sign fo integration vanish at both limits, it easily follows that

$$\frac{\partial}{\partial a}\left(\frac{A_k}{a_k}\right) = \int_0^\infty L \cdot \left[\frac{\partial^2}{\partial x^2} \cdot (x^{p/2} \cdot F) - \frac{p}{2} \cdot \frac{\partial}{\partial x} (x^{\frac{1}{2}p-1} \cdot F) \right] \cdot dx \ldots \quad (9.44)$$

$$= \int_0^\infty L \cdot x^{p/2} \cdot \left[\frac{\partial^2 F}{\partial x^2} + \frac{p}{2x} \cdot \frac{\partial F}{\partial x} \right] \cdot dx \quad \ldots \quad (9.45)$$

As $$\frac{\partial^2 F}{\partial x^2} + \frac{p}{2x} \cdot \frac{\partial F}{\partial x} = \frac{F(2x)}{x} \quad \ldots \quad (9.46)$$

we have now

$$\frac{\partial}{\partial \alpha}\left(\frac{A_k}{a_k}\right) = \int_0^\infty L \cdot x^{(p-2)/2} \cdot F_{(x)} \cdot dx = \int_0^\infty a^{-(p+2k)/2} \cdot e^{-x/a} \cdot S_k(x/a) \cdot x^{(p-2)/2} F_{(x)} \cdot dx$$

$$\ldots \quad (9.51)$$

Writing $x = az$ as before, we have

$$\frac{\partial}{\partial a}\left(\frac{A_k}{a_k}\right)\frac{1}{a^k} = \frac{1}{a^k} \int_0^\infty e^{-z} \cdot S_k(z) \cdot z^{(p-2)/2} \cdot F(az) \cdot dx \quad \ldots \quad (9.52)$$

Therefore we have $$\frac{\partial}{\partial a} \cdot \left(\frac{A_k}{a^k}\right) = \frac{A_k}{a^k} \quad \ldots \quad (9.61)$$

or $$A_k = C \cdot a^k \cdot e^a \quad \ldots \quad (9\ 62)$$

To detrmine the constant C in the expression for A_k we observe that

$$\int_0^\infty F \cdot (az) \cdot e^{-z} \cdot z^{(p-2)/2} \cdot S_k(z) \cdot dz = C \cdot a^k \cdot e^a \quad \ldots \quad (9.63)$$

But when a is very small $C \cdot a^k \cdot e^a = C \cdot a^k \cdot (1+a+a^2/2+\ldots) = C \cdot a^k$ $\ldots$ (9.64)

Therefore $$C \cdot a^k = \int_0^\infty B_k \cdot a^k \cdot z^k \cdot e^{-z} \cdot z^{(p-2)/2} \cdot S_k(z) \cdot dz \quad \ldots \quad (9.65)$$

where B_k is the coefficient of z^k in the expansion of $F(z)$ as a power series, as terms of lower order in $F(a\,z)$ below z^k, will contribute zero values because of the orthogonal properties of $s_k(z)$'s

$$\text{Now as } \int_0^\infty z^k \cdot e^{-z} \cdot z^{(p-2)/2} \cdot S_k(z) \cdot dz = \frac{1}{\gamma_k} \int_0^\infty e^{-z} \cdot z^{(p-2)/2} \cdot S_k^2 \cdot (z) \cdot dz = \frac{1}{\gamma_k} \quad \ldots \quad (9.66)$$

where γ_k is the coefficient of the highest term in the series of normalised Sonnine polynomials, we have

$$C \cdot a^k = \frac{B_k \cdot a^k}{r_k}, \text{ or } \quad C = \frac{B^k}{\gamma^k} \quad \ldots \quad (9.71)$$

To calculate therefore the coefficients of B_k and γ_k we consider the differential equation which $F(z)$ satisfies, that is,

$$z \cdot \frac{d^2F}{\partial z^2} + \frac{p}{2} \cdot \frac{\partial F}{\partial z} = F(z) \quad \ldots \quad (9.81)$$

Assuming a series solution $$F \equiv \Sigma B_k \cdot z^k \quad \ldots \quad (9.82)$$

is easily seen that $$k \cdot (k-1) \cdot B_k + \tfrac{1}{2}p \cdot k \cdot B_k = B_{k-1} \quad \ldots \quad (9.83)$$

or $$k \cdot (k+\tfrac{1}{2}p-1) \cdot B_k = B_{k-1} \quad \ldots \quad (9.84)$$

or $$B_k = \frac{B\Gamma\ (p/2)}{\Gamma(k+1) \cdot \Gamma(k+\tfrac{1}{2}p)} \quad \ldots \quad (9.85)$$

Therefore from (9.24), (9.62), (9.66) and (9.85) we have

$$F(az) = B_0 \Sigma \frac{\Gamma(p/2)}{\Gamma(k+1)\ \Gamma(k+\tfrac{1}{2}p)} \cdot a^k\, e^a \cdot S_k'(z) \quad \ldots \quad (9.86)$$

where $$S'(z) = S_k(z)/\gamma_k \quad \ldots \quad (9.87)$$

Putting now $$a = \xi^2/2t, \quad \text{and} \quad z = R^2/2t \quad \ldots \quad (9.88)$$

we see that

$$F\left(\frac{\xi^2 R^2}{4t^2}\right) = F(0)\,\Sigma \frac{\Gamma(p/2)}{\Gamma(k+1)\,.\,\Gamma(k+\tfrac{1}{2}p)} \cdot e^{\xi^2/2t} \cdot (\xi^2/2t)^k \cdot S_k(R^2/2t) \quad \ldots \quad (9.89)$$

or $$F(x) = F(0)\Sigma \frac{\Gamma(p/2)}{\Gamma(k+1) \cdot \Gamma(k+\tfrac{1}{2}p)} \cdot e^{\xi^2/2t} \cdot (\xi^2/2t)^k \cdot S_k \cdot \left(\frac{\xi^2}{2t} \cdot x\right) \quad \ldots \quad (9.91)$$

Therefore the type of solution is of the form

$$\frac{e^{-(\xi^2+R^2/2t}}{t^{p/2}} \; F(\xi^2 R^2/4t^2)$$

$$= F(0)\Sigma \frac{\Gamma(p/2)}{\Gamma(k+1)\cdot\Gamma(k+\frac{1}{2}p)} \cdot \frac{e^{-R^2/2t}}{t^{p/2}} \cdot \frac{\xi^{2k}}{(2t)^k} \cdot S_k \cdot (R^2/2t) \qquad \ldots \quad (9.92)$$

which agrees with the previous result[11], if we remember the expression

$$\Delta^k = \xi^{2k} = \mu_k - k(k + \tfrac{1}{2}p - 1) \cdot \mu_{k-1}(2t) + \ldots \qquad \ldots \quad (9.93)$$

It follows from the above analysis that the general form of the distribution function in (R, t) can be assumed to be of the form

$$\Sigma \frac{e^{-R^2/2t}}{(2\pi t)^{p/2}} \left\{ A_0 + \frac{A_1}{t^2} \cdot S_1 (R^2/2t) + \frac{A_2}{t^4} S_2(R^2/2t) + \ldots \right\} \qquad \ldots \quad (9.94)$$

where $S_1, S_2, \ldots S_k$ are what can be called Sonine polynomials. The coefficients $A_0, A_1, A_2 \ldots$ etc. can be calculated in any particular case, when the moments of the modulus distributions are known. The distribution-function in *Sankhya*[12] may thus be seen to fall in the general class, as I have shown by a direct expansion of the expression, and also by identification of the two results.

It is interesting to stydy the general types of solutions of the equation $\square_p^2 F = 2 \frac{\partial F}{dt}$ for different types of complexity; certain results already arrived at will be published in due course.

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[11] *Sankhyā*, 2(4), pp. 395—396.

[12] *Sankhyā*, 2(4), 1936, pp. 395-396.

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Recent Progress in Nuclear Physics

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PHYSICAL science is at present passing through a remarkable phase of radical and revolutionary changes. Time-honoured concepts are crumbling down under the shock of remarkable discoveries. The familiar ideas of force, mass, and energy have undergone remarkable transformations and the dynamical laws which seemed at one time to provide the ultimate basis for the mechanical explanation of the material universe have now been replaced to a large extent by quantum-mechanical rules and probability-calculations. Controversy now rages round the validity of the very principle of causality without which science would have seemed impossible a few years ago. It is therefore not surprising that the old concept of the atom as the ultimate *indivisible* unit of a chemically simple stuff has been replaced by the modern idea of a complicated structure consisting of a positively charged material core, the nucleus, which itself in ultimate analysis appears to be heterogeneous, enveloped in a cloud of negative electrons, which possess more or less definite amounts of energy and momentum, determined by quantum-mechanical rules.

The Electron and the Periodic Table

I shall endeavour in this brief discourse to present before you a brief history of the various experiments which have compelled the modern scientists to adopt this structure for the chemical atom in place of the familiar and simple concepts of Lucretius or of Dalton. The beginnings of the change in our point of view may be said to have been initiated about 40 years ago by the discovery of the electron in 1895. The experiments which led to its discovery established at the same time its presence as a universal constituent in all kinds of atoms. The intimate connexion between electricity and matter being thus established once for all, the subsequent endeavours of the physicists and the chemists have been directed towards explaining the divergent properties of the chemical substances in terms of electricity and electrical forces.

It will be obviously impossible in this brief compass to give an adequate account of all the results so far achieved in this region where physicists and chemists have worked side by side, and I shall therefore confine myself to a bare mention of the principle facts which have led the way to the modern conceptions about the atom. The ancient atomic theory was formulated mainly on the basis of chemical evidence. The analysis of innumerable substances, which either occur as such in nature or are artificially produced in the laboratory, has established the existence of about 92 simple substances, the so-called elements, whose atoms by combining and re-combining among themselves in various proportions have given rise to all the various substances we see around us. Without losing their individual distinctive features, many of these elements show among themselves remarkable similarities in their chemical properties. These have been intensively studied by the chemists, and the main results can be conveniently represented by arranging all the elements in a series of horizontal and vertical rows, in the so-called periodic table of Mendeljieff.

The remarkable feature of this arrangement is that whereas the atomic weights of elements increase steadily as we go down the series, marked similarities in the chemical properties recur at more or less regular intervals, *i.e.*, as soon as we come to elements which lie in the same vertical column of the rectangular array. The atomic number in the

scheme plays as important or rather a more important role than the atomic weight of elements. No explanation of this regularity can obviously be found in the simple Daltonian theory. Nor was there any prospect from the chemical side alone of arriving at the explanation of the mysterious regularity in the chemical behaviour of elements.

The discovery of the electron however inaugurated novel methods of attack for the solution of the puzzle. The presence of electrons carrying negative charges, as constituent of all atoms, as well as the electrical neutrality of the atom as a whole, had brought the interesting question of the electrical structure of the atom to the fore-front. After the discovery of the electron the atom for the physicist could no longer continue to be a simple substance. The problem of its composition, *i.e.*, the distribution of mass and charge inside it, demanded an urgent solution, and speculations were at work regarding its structure even before adequate data were obtained for drawing probable conclusions.

Closely following on the discovery of the electrons, came the discovery of radium and the radioactive elements. Along with the familiar and stable elements which seem to persist unchanged through geological periods of time, the discovery of the phenomenon of radioactivity established the existence of the so-called unstable elements. Though these behave as ordinary elements in all chemical reactions, they are found to disintegrate spontaneously and give birth to new elements of smaller atomic weights, which in turn explode, regenerate fresh elements, and the series of elements of decreasing atomic weights is continued till the whole process comes to a stop with an element of ordinary stability at the end. During this process of successive disintegration the radioactive elements emit swiftly moving charged particles (the so-called α– or β-radiation). They also emit in general penetrating γ-radiations of the type of X-rays. The α-particles were early recognized as the nuclei of helium atoms which carry two units of positive charge whereas the β-rays were found to be swift electrons which move with velocities approaching the velocity of light. Though it was found impossible to control the phenomena of spontaneous disintegration of the elements, their very existence revealed the composite nature of at least the heavy atoms, and made the hypothesis of a structure for all atoms out of comparatively simpler substances a very probable one.

The swiftly moving positively charged α-particles, emitted during the process of radioactive decay, furnished the physicists with a very convenient weapon for attacking the problem of the constitution of atoms. Lord Rutherford conceived the brilliant idea of sending these swiftly moving charged particles as probing agents inside the atoms, where their mass and enormous kinetic energy would enable them to penetrate far into the mysterious interior, before they would be deviated out of their straight course by the intense Coulombian field of force.

The determination of the distribution of the scattered particles in various azimuths would, he hoped, enable the physicists to obtain a fairly correct picture of the distribution of mass and charge inside the atom. The first experiments in this direction were carried out in Rutherford's Laboratory at Manchester by Geiger and Marsden, and they at once afforded valuable information regarding the probable constitution of the atoms.

The physicists had already arrived at an estimate of the atomic size from various considerations. The kinetic theory indicated the radius to be about 10^{-8}cm. The experiments of Geiger and Marsden now revealed that the mass of the atom must be regarded as concentrated within a sphere of a much smaller radius, say (10^{-12} to 10^{-13}cm.). This central core was also found to be positively charged, and its magnitude was approximately estimated at half the atomic weight of the element. This experiment of Geiger and Marsden enabled the physicists to form a fairly correct idea about atomic exterior. The planetary atomic model suggested by Rutherford, where a positively charged nucleus controls electrons revolving in orbits controlled by Coulombian forces, gained thus a universal acceptance among the scientists and proved a valuable and fruitful hypothesis. Detailed discussion of the subsequent developments will lead us too far away from our main theme. I shall therefore mention in the briefest of terms the principle achievements which we owe to this model. In 1913 Bohr showed that the optical

spectra of elements could be explained on the basis of the above model if the electronic constituents of the atoms were supposed to move in definite orbits—determined mainly by electrostatic forces and by subsidiary quantum-laws. The study of the X-ray spectra of elements enabled Moseley at about the same time to settle with certainty the magnitude of the charge of the nucleus and also the number of electrons in outside orbits. The fundamental nature of the atomic number in the Mendeljieff's table received thereby a rational interpretation, and no interpolation, or change of arrangement in the series, could be conceived of at any subsequent period, as the sequence of atoms in the table followed the integral sequences of increase of atomic charges. The idea of the spinning electron gave a rational explanation of the periodicities in the Mendeljieff's table. The electrons were revealed to be grouped in different closed shells round the central core and the chemical properties of the elements could be definitely correlated to the number of the electrons in the outermost incomplete shells. The elements of the same vertical column were found thus to have more or less identical external structure, which explained the similarities observed in their chemical properties.

Structure of the Nucleus

In 1925 came important theoretical developments. The modern wave-mechanics was formulated which enabled the physicists to replace the former tentative calculations by exact mathematical analysis and the different empirical procedures were unified into a single consistent analytical discipline. Though all the questions which have been raised by the study of the physical and chemical behaviour of elements may thus be said to have obtained more or less satisfactory solutions, the problem of the structure of nucleus had however been scarcely touched till the beginning of the present decade.

It is not difficult to find reason for this delay in the development of nuclear physics. The nuclei of elements lie hidden behind a protective cloud of electrons. The intensity of the Coulombian field also increases very rapidly by about 10^{10} times as we approach from the outside to the neighbourhood of the nucleus. The bombardment of atoms by swiftly moving electrons had not produced any fruitful results so far, and in order that positively charged particles could overcome the enormous repulsive force and approach the nucleus within a reasonable distance sufficient to produce significant perturbations in the nuclear regions and produce sensible results, swift particles with tremendous velocities appeared at fisrt sight necessary, which were only available in small amounts from radioactive processes that continue yet to be beyond our control. The smallness of the nuclear size makes also the chances of close collision very very remote so that the percentage yield of any definite result by the bombardment of α-particles is very small indeed. Nevertheless since only the chemical properties of elements appeared to be governed by the nuclear charge, an artificial transmutation of elements could be hoped for, if one were able either to push a charged particle in the interior of the nucleus or to bring a charged particle in its immediate proximity, so that the disturbance thus set up might possibly induce spontaneous transmutation of the atom.

In order to ensure stability against the disruptive influences of the Coulombian forces, the presence inside the atom of attractive forces of unknown origin appeared also necessary. These attractive forces are most probably sensible at distances comparable with the linear dimensions of the nucleus, so that only a careful study of the large deflections of α-particles from single and close encounters with light atoms might be expected to give some information about the nature of these attractive forces. A successful carrying out of the above programme required the development of a special technique for the study of such atomic encounters. The early method of directly counting the scintillations to estimate the large angle scattering of α-particles as followed by Geiger and Marsden was further improved upon by Chadwick and Bieler in the laboratory of Rutherford at Cambridge. A direct measurement of the nuclear charge and an estimate of the nature of the electric forces acting on α-particles in the immediate neighbourhood of light nuclei were rendered possible by the study of scintillations produced by the scattered particle.

The expansion apparatus of Wilson supplied another valuable method of attack. Under suitable circumstances, the tracks of the colliding particles, the nucleus and the α-projectiles, before and after collision, could be directly photographed, and the interchange of momentum and energy between the colliding particles could be directly estimated from the measurement of stereoscopic pictures of such encounters. In case such collisions brought about artificial disintegration, the record of the explosion in the chamber photograph enabled us in many cases to follow the details of the process unequivocally.

The method of directly counting the scattered particles by the scintillations produced on screens has been replaced in recent times by simple, reliable, and automatic methods of counting the swift particles like α-particles or protons with the help of the Geiger Counter and proportionate amplifiers. The rapidity of advance during the last few years has been in large part due to the great improvements in the technical methods of attack.

Nuclear Disintegration; the Neutron

Already however in 1919 Rutherford had obtained by the simple scintillation method the first evidences of artificial disintegration. Nitrogen nucleus bombarded by swiftly moving α-particles appeared to give rise to streams of swiftly moving protons or nuclei of hydrogen, and subsequent work along the same direction had revealed the occurrence of this phenomena of artifical disintegrations during the bombardment of about twelve of the lightest elements.

By studying the phenomena of disintegration of nitrogen in a Wilson Chamber, Blackett was able in 1925 to give a satisfactory account of the details of the process. This nuclear reaction seems to arise out of a capture of the α–particle by the nucleus of nitrogen, whereby an H-particle escapes out of the nucleus and a new atom, an isotope of oxygen, is produced, according to the scheme

$$N_7^{14} + He_2^4 \rightarrow H_1^1 + O_8^{17} .$$

Two capital discoveries in recent years have revolutionized the subject and accelerated the progress of nuclear physics. Bothe in 1930 observed that beryllium when bombarded by α-rays from polonium gave rise to a markedly penetrating radiation which appeared to be of the γ-ray type. In a subsequent examination of this effect by M and Mme Curie Joliot in Paris and Chadwick in Cambridge in 1932, an important part of this radiation was found to consist of a stream of swift, uncharged particles, called neutrons, which have about the same mass as the proton. As it is uncharged the neutron does not directly ionize the gas in its path, but only reveals its presence indirectly by the recoil of the nucleus with which it collides. And since the transfer of momentum is largest when the mass of the colliding nucleus is about the same as the mass of the neutron, the secondary ionization by recoil, as produced in hydrogen or in paraffin-lined ionization-chambers, is much larger than in an ionization-chamber filled with a heavier gas. This peculiarity of behaviour has served to differentiate the neutron radiation from wave radiations of the γ-ray type, which also usually accompany the phenomena of neutron emission. Most of the nuclei of the light atoms also emit neutrons when bombarded by swiftly moving α-particles. This strange type of particles is in itself an important agent for effecting artificial transformation of other nuclei, mainly because it is uncharged and as such is not handicapped by the presence of the intense Coulombian forces. It can thus approach and penetrate into the nuclei of even the heavy atoms, and thereby bring out interesting transformations in novel ways about which I shall speak later on.

Potential Barrier

The perfection of wave-mechanical methods has induced various theoretical workers to apply the new ideas in problems of nuclear physics, especially in investigating the collision of charged particles with the atomic nuclei. The simultaneous presence of a Coulombian field of repulsion and an attractive force of unknown origin at short distances inside the nucleus gives rise to what is technically called the potential barrier. The intense attractive forces inside the barrier serve to keep the

charged particles inside the nucleus. Beyond the distance where the height of the barrier rises to a maximum, the attractive forces cease to be sensible, and the field may be ragarded as repelling charged particles according to the law of inverse squares. Any charged particle wanting to penetrate inside the nucleus will have to surmount the barrier, and according to the classical theory, it will have to have a kinetic energy sufficient to come over the top of the barrier.

Based on arguments similar to the above, one estimated (before the advent of the wave-mechanics) that a minimum acceleration-potential of a few million volts would be necessary to produce sufficient acceleration in particles, before they are able to cross the barrier. The estimate however proved erroneous and excessive.

A noteworthy contribution of the theoretical physicists *(recent wave-mechanics)* in this subject has been to predict a small but appreciable probability for penetration of protons across the barrier of light elements, even when they have energies of the order of a few hundred thousand e.volts. The prediction seemed to bring artificial transmutation on a large scale within the range of possibilities, and this theoretical conclusion was tested experimentally by Cockroft and Walton in 1932 who had been able to generate a proton stream of about 100,000 volts, in a vacuum tube, by step-wise acceleration. Their experiments at once met with complete success and the evidence of disintegration of the lithium nucleus subsequent to the capture of a proton was obtained by bright scintillations produced on the screen by the α-particles that were generated by disintegration. This initial success of the Cambridge physicists has been followed up by other workers in Germany, France, and America, and it has become clear that considerable progress in artificial disintegration can be achieved by bombarding elements with swiftly moving protons and other nuclei accelerated suitably by application of high voltages. Investigators of nuclear physics have concentrated their energies on the production of suitable high voltages, and notable success in this direction has been achieved in recent years. Three different methods have been mainly followed in producing the necessary high voltage : firstly, the original method of Cockroft and Walton, which consists in accelerating the particles in the vacuum tubes in stages with suitably insulated transformers; secondly, the method of Lawrence, who has developed a peculiar method of multiple acceleration of ions in a synchronized magnetic and electric alternating field; and thirdly, the electrostatic method of Van de Graff. Considerable progress has been achieved by the application of all the three methods and a lot of interesting results have ben obtained by the bombardment of atomic nuclei, by suitably accelerated projectiles. I have already mentioned before the production of neutrons from the light elements by the bombardment of α-particles. Results of great significance were obtained as soon as these particles were used by workers in nuclear physics. In curious contrast with the swiftly moving α-particles and the artificially accelerated proton streams, and other corpuscular rays, the capacity of the neutron to produce artifical transformations increases, in most cases, with the diminution of its velocity. Doubtless this is connected with the fact that its small velocity enables the particle to stay longer in the immediate neighbourhood of the nucleus and thus to bring about more far-reaching changes.

Induced Radioactivity

It has been established by the work of Fermi and other members of the Italian school that a few collisions of neutrons with hydrogen nuclei (present in either a free or a combined state) are sufficient to establish a sort of thermal equilibrium, so that the neutrons on the average attain, after a few collisions, the averave velocity of hydrogen particles at room-temperature. Whereas the transmutation of elements to stable varieties of other elements had been previously noticed or conjectured, during the bombardment by α-particles, or protons, Curie and Joliot discovered in 1933, that unstable and hitherto unknown isotopes of light elements are produced by the bombardment of light nuclei with α-particles. These unstable nuclei subsequently break up in the same way as ordinary radioactive elements, and phenomena of induced radioactivity are found to obey the same laws, and decay in the same characteristic way as the natural activities of the well-

known radio-elements. The identity of these artificial bodies can also be often established by purely chemical methods. In their pioneer work Curie and Joliot discovered that unstable isotopes of nitrogen, silicon, and phosphorus are formed by the bombardment of Be, Al, and Mg: these however emit, curiously enough, positive electrons though the phenomena of disintegration proceeds like the familiar β- ray disintegration of radioactive elements. The discovery that new radioactive elements can be produced by artificial transmutation has given a tremendous impetus to the study of the nuclear reactions. The task of following such nuclear changes is a very much easier one, as sensitive physical apparatus like the Geiger Counter can be utilized to detect and measure such changes. This pioneer work of Curie and Joliot has been greatly extended by Fermi and other workers in Italy. They have observed that slow-moving neutrons are in most cases quite effective in producing similar changes and of generating radioactive isotopes. Extensive work in this line has been done and a large number of new radioactive atoms have been discovered. These however almost always emit β -particles, *i.e.*, ordinary negative electrons instead of positrons.

Deuteron—the Isotope of Hydrogen

With the discovery of deuterons, the isotope of hydrogen, a new kind of corpuscle has been utilized for bombardment of atomic nuclei. Acclerated deuteron streams have been utilized both in England and America and they have proved very much more effective as agents for transmutation than the original proton rays of Cockcroft and Walton. Bombardment by deuterons also produces radioactive bodies and this method has been increasingly used in recent times to study the phenomena of induced radioactivity. The earlier methods of production of new radioactive bodies had to utilize a natural radioactive source for the supply of the suitable bombarding agents; this had necessarily limited the amount of the yield even when the process of canalization was utilized for diminishing the velocity of the neutrons and thereby to bring about an enormous increase of output. The yield of radioactive stuff by the bombardment of deuterons has been naturally very much greater. For example, in a day's exposure a yield of radioactive isotope of sodium has been reported to have been obtained by Livingstone whose activity equals that of 1 gramme of radium. Deuteron bombardment has therefore a great future both in the therapeutical application of radioactivity to medical research as well as in the investigation of nuclear problems.

Whereas bombardment of corpuscles has been almost always utlized for producing nuclear changes, results similar to photo-ionization have been obtained by Chadwick and Goldhaber. By utilizing hard γ-radiation from ThC these investigators have been able to decompose the hydrogen isotope into neutron and proton. The liberated neutron can be detected by its ability to produce induced radioactivity in suitable elements, or by a properly conducted ionization measurement. By hard X-rays emitted in tubes run at more that 1.5 million-volts pressures, the workers in Berlin have been able to eject neutron streams from beryllium, whose presence has been similarly demonstrated by the generation of radio-iodine in ethyl iodide. These preliminary results have great theoretical significance; Chadwick has been able from a tentative determination of the thresh-hold value of the frequency of the γ-rays necessary to decompose deuterons, determine the mass of the neutron as well as the strength of the binding of the two fundamental particles.

Conclusion

In this lecture I have attempted to give a rapid review of the principle results obtained in the field of nuclear physics in recent times. I shall conclude my remarks by mentioning two remarkable results that have been obtained by the bombardment of thorium and uranium, the two heaviest of the elements with neutrons. I have already observed that the production of new radioactive bodies could, in many cases, be proved by chemical methods; this serves at the same time to determine the chemical properties of the new substance, and its position in the periodic table. By the bombardment of uranium, Fermi originally reported the production of new β-ray emitting elements whose chemical behaviour seems to point to their positions beyond uranium

itself in the Mendeljieff's table. This has been corroborated by Meitner. According to these workers the table of elements is artificially extended by this process beyond uranium itself—according to the scheme

$$U_{92}^{238} + n_0^1 \longrightarrow U_{92}^{239} \xrightarrow{\beta} \text{Eka Rhenium } ER_{93}^{239} \xrightarrow{\beta} \text{Eka Os}_{94}^{239} \xrightarrow{\beta} \text{Eka Ir}_{95}^{239}$$

a truly remarkable result.

By a study of the product of the disintegration of thorium by neutron bombardment Curie has established the existence of a new series of radioactive elements whose mass numbers are in the form 4n + 1. This analogous series of radioactive bodies is not known in nature, and its discovery can be regarded as completing our ideas of radioactive disintegration of heavy elements.

It will be evident from what I have reported above that enormous experimental materials have accumulated in the field of nuclear physics within recent years. Sufficient materials are now available for the theoretical physicists to speculate about the process of atom-building from elementary particles, and the preliminary work in this direction has already begun. The neutron and the proton seem to be the two exclusive constituents of all atomic nuclei. The α-particles can themselves be looked upon us composite bodies, built up again from neutrons and protons.

Though the idea of the elementary atom has undergone revolutionary changes in recent years, in a certain way the progress achieved has been satisfactory, as it has simplified the number of ultimate and fundamental particles to only two, instead of the 92 elements of the older atomic theory. This idea of the evolution of the material world from comparatively few primordial stuffs is not however new. Here, as in other fields of physics, old ideas have returned, renovated in a new garb and clothed with more significance. The quantum theory of photons has to a certain extent resuscitated the ancient corpuscular theory of Newton. The recent developments of the nuclear physics have brought back the old Proutian hypothesis, of the evolution of all elements from one or rather two primordial stuffs—the proton and the neutron.*

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Anomalous Dielectric Constant of Artificial Ionosphere

In a recent communication in *Nature*, Mitra and Roy[1] have pointed out an interesting feature in the ionospheric dispersion formula. They have shown that the formula can yield values of the dielectric constant of an ionized medium greater, equal to, or less than unity depending on the degree of ionization, collisional frequency and the exciting wave frequency. They therefore hold that the value of dielectric constant of an ionized gas greater than unity which has been recorded by many investigators for comparatively large ionizations is only an outcome of the complete dispersion formula. The object of this note is twofold. Taking the ionospheric dispersion formula we shall *first* deduce the conditions under which the dielectric constant may assume values greater or less than unity and *secondly* we shall show that in the experiments where anomalous values of the dielectric constant of an ionized gas have hitherto been obtained, the experimental conditions are such that the dispersion formula alone cannot explain the anomaly.

The dispersion formula is given by

$$\left(\mu - \frac{ick}{p}\right)^2 = 1 + \frac{1}{\alpha + i\beta},$$

where $\alpha = -\dfrac{mp^2}{4\pi Ne^2}$ and $\beta = \dfrac{mpv}{4\pi Ne^2}$

Separating the real and imaginary parts, we have

$$m^2 - \frac{c^2k^2}{p^2} = 1 + \frac{\alpha}{\alpha^2+\beta^2} \qquad \ldots\ldots (1)$$

and $$\frac{2ck}{p}\cdot\mu = \frac{\beta}{\alpha^2+\beta^2} \qquad \ldots\ldots (2)$$

Thus $\dfrac{c^2k^2}{p^2} - \mu^2 = \dfrac{|\alpha|}{\alpha^2+\beta^2} - 1$ so that

$$\mu^2 \begin{matrix} > \\ = \\ < \end{matrix} 1 \text{ according as } \frac{c^2k^2}{p^2} \begin{matrix} > \\ = \\ < \end{matrix} \frac{|\alpha|}{\alpha^2+\beta^2}$$

which is equivalent to the condition :

$$\mu^2 \begin{matrix} > \\ = \\ < \end{matrix} 1 \; if \; \frac{\mu^2c^2k^2}{p^2} \begin{matrix} > \\ = \\ < \end{matrix} \frac{|\alpha|}{\alpha^2+\beta^2},$$

but as $\dfrac{\mu^2c^2k^2}{p^2} = \dfrac{\beta^2}{4(\alpha^2+\beta^2)^2}$ from (2)

the condition reduces to —

$$\mu^2 \begin{matrix} > \\ = \\ < \end{matrix} 1 \; if \; \frac{\beta^2}{4(\alpha^2+\beta^2)^2} \begin{matrix} > \\ = \\ < \end{matrix} \frac{|\alpha|}{\alpha^2+\beta^2}$$

or $$\frac{\beta^2}{\alpha^2+\beta^2} \begin{matrix} > \\ = \\ < \end{matrix} 4|\alpha|.$$

Now the left-hand side is always a proper fraction; therefore, if $4|\alpha| > 1$, *i.e.* $|\alpha| > \cdot 25$ we have then always

$$\frac{\beta^2}{\alpha^2+\beta^2} < 4|\alpha| > 1; \; i.e. \; \mu^2 < 1.$$

If however $|\alpha| < \cdot 25$, $\mu^2 > 1$ will require the further condition

$$\frac{\beta^2}{\alpha^2+\beta^2} > 4|\alpha| \text{ or } \beta^2 > \frac{4|\alpha|\,\alpha^2}{1-4|\alpha|}.$$

Calling $\dfrac{\pi Ne^2}{m} = p_0^2$ or

$$p_0 = \sqrt{\frac{\pi e^2}{m}}\sqrt{N} = 2\cdot 8\times 10^4\sqrt{N},$$

we have $4|\alpha| = \dfrac{p^2}{p_0^2}$ and we can say that if

$p > p_0 = 2{\cdot}8 \times 10^4 \sqrt{N}$, μ is always less than unity, whereas if $p < 2{\cdot}8 \times 10^4 \sqrt{N}$, or $|\alpha| < {\cdot}25$, for $\mu^2 > 1$, we must have

$$\nu^2 > \frac{p^4}{p_0^2 - p^2}$$

i..e., the collisional frequency must exceed the value

$$\frac{p^2}{\sqrt{p_0^2 - p^2}}$$

Thus the equations show that even when $p < 2{\cdot}8 \times 10^4 \sqrt{N}$, there exists a critical collisional frequency which has to be executed if μ is to be greater than unity.

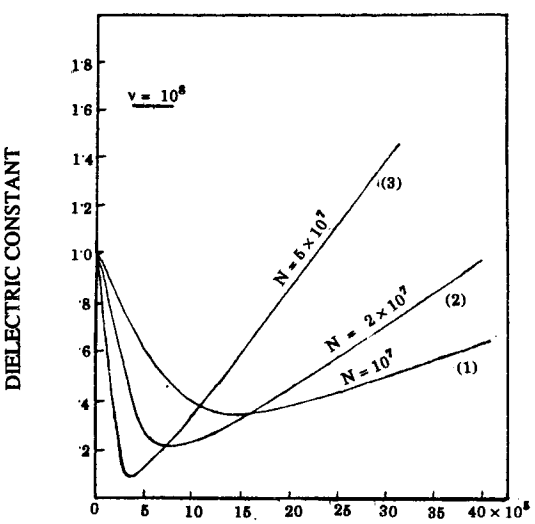


Fig. I

In the experiments recently performed in this laboratory, we have obtained results similar to those of Appleton and Childs[2]. Using a wavelength of about 3·8 metres the dielectric constant of ionized air in a discharge tube has been found to be less than unity when the tube current is small. As the ionization is increased by increasing the tube current, the dielectric constant at first decreases and then increases gradually and even becomes greater than unity. The maximum electron density N in our discharge tube has been within 10^7. Taking $N=10^7$, $p=5 \times 10^8$, we get $|\alpha| = 7{\cdot}8$. This is much greater than ·25 so that the dielectric constant must be less than unity. Even when $N=10^8$, $|\alpha| = {\cdot}78$ and the dielectric constant should be theoretically less than unity.

In Fig. I are plotted the calculated values of the dielectric constants against λ^2 for some definite values of N and ν. Curve (2) which corresponds to Mitra and Banerjee's[3] experimental conditions shows a turning point at λ of the order 9m (λ^2 is of the order 8×10^5 sq.cm.) which is very different from the experimental value λ of the order 6m (λ^2 is of the order $3{\cdot}6 \times 10^5$ sq.cm.). In the experiments of Ali Imam and one[4] of us, which have been recently repeated by a different method, the turning point appears unmistakably at λ of the order 4m (λ^2 is of the order $1{\cdot}6 \times 10^5$ sq.cm.). Curve (1) which approximately corresponds to our experimental conditions shows however the turning point at λ of the order 12m (λ^2 is of the order 14×10^5 sq.cm.). It appears extremely doubtful if the disposition of the experimental apparatus can account for such wide discrepancy. It is likely that there must be some other explanation of these experimental results.

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ON THE TOTAL REFLECTION OF ELECTROMAGNETIC WAVES IN THE IONOSPHERE

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The conditions under which the electromagnetic waves get totally reflected in the upper atmosphere appear still worth a closer investigation. The usual procedure has been to attribute to the medium a refractive index which is calculated according to the classical method of Lorentz from the mechanical equations of motion of electrons. In an absorbing medium, this refractive index is a complex quantity, and depends not only on the properties of the medium but also on the frequency of the waves : as progressive waves through the ionosphere are damped, the co-efficient of damping will also enter into the expression of the refractive index. The usual Appleton-Hartree condition for total reflexion has been deduced by neglecting the damping, as a first approximation, and by putting the refractive index equal to zero. Discussion of the conditions when the damping is not negligible, gets very complicated indeed. Certain workers in Allahabad have suggested the further condition that the group-velocity of the wave-train is zero when total reflection takes place. This is a reasonable hypothesis, but the method by which the group-velocity is calculated neglects damping, an essential factor in the physical process. It appears also that the calculation of the group-velocity of waves in an absorbing medium is not amenable to the ordinary methods unless the damping is negligible, an approximation which, as has been already pointed out, does not seem to agree closely with experience. It seems therefore necessary to change the method of analysis and to take, instead of the equations of Maxwell with a complex refractive-index, the microscopic equations of Lorentz as basis for calculation. The method sketched here is a general one suitable for problems of refractive-index of material media as well as for discussion of propagation of waves in the ionosphere. It seems further amenable to modifications to suit the requirements of modern wave mechanics.

We take the familiar equations of Lorentz

$$\frac{I}{c}\frac{\partial E}{\partial t} - \text{curl } H = -\frac{\rho V}{c}\,; \quad \frac{I}{c}\frac{\partial H}{\partial t} + \text{curl } E = 0.$$

$$\text{div } E = \rho, \quad \text{and} \quad \text{div } H = 0.$$

The density and the convection current, ρ and ρV, can be regarded as decomposable into two parts ρ_+, and ρ_- and $(\rho V)_+$ and $(\rho V)_-$, respectively, which thus takes

account of the corpuscular distribution of positive and negative charges. A wave disturbance in the medium can be considered by putting

$$E = E_0 + Ee^S,\ H = H_0 + He^S;\quad \rho_+ = \rho_+^0 + q_+e^S,\ \rho_- = \rho_-^0 + q_-e^S,$$

$V_+ = V_+^0 + W_+e^S$ and $V_- = V_-^0 + W_-e^S$, where the elements with 0 indices correspond to the undisturbed state of the medium. The convection current, and the electric-density become on this assumption equal to

$$\overline{\rho V} = \rho_+V_+ + \rho_-V_- = (\overline{\rho V})_0 + \bar{\theta}e^S$$

where $$\bar{\theta} = [\rho_+^0\ W_+ + q_+V_+^0 + \rho_-^0W_- + q_-V_-^0) \quad \text{and } \theta_0 = q_+ + q_-\ .$$

Thus the components $(\theta_x, \theta_y, \theta_z)$ of the vector $\bar{\theta}$ correspond to the amplitudes of the fluctuating part of the convection current, and θ_0, similarly to the fluctuating part of the electric density due to the disturbance.

The process is a familiar one : When S is a linear function of x, y, z, t, with a complex factor $2\pi\iota$, as is generally chosen for discussing monochromatic wave-propagation, $2\pi\iota\ S$ represents the phase of the disturbance and the surfaces for various values of S, represent the totality of all wave fronts as function of (x, y, z, t). In order to tackle the more general case of a damped wave-train we shall not at first put any limitation on the form of S but shall assume it to be any function whatever, whose differential co-efficients may also have complex values. Removing the contributions on the both side of the equation by the various quantities corresponding to the undisturbed state, we get

$$\frac{1}{c}\frac{\partial}{\partial t}(Ee^S) - \text{curl}\ (He^S) = -\frac{\theta}{c}\ e^S$$

$$\frac{1}{c}\frac{\partial}{\partial t}(He^S) + \text{curl}\ (Ee^S) = 0$$

$$\text{div}\ (Ee^S) = \theta_0e^S, \text{ and div } (He^S) = 0.$$

We can also verify easily that

$$\text{div}\ [Ee^S] = [\text{div}\ E + (\text{grad}\ S.E)]e^S$$

$$\text{curl}\ [Ee^S] = [\text{curl}\ E + \text{grad}\ S \times E]e^S$$

$$\frac{\partial}{\partial t}\left[\ Ee^S\ \right] = \left[\frac{dE}{dt} + E.\frac{\partial S}{\partial t}\ \right]e^S$$

etc.

Under the usual conditions of propagation E, H, representing the amplitudes of the electric and magnetic waves, are slowly varying quantities whose differential co-efficients with regard to time and space co-ordinates can be neglected in comparison with the rapid variations of the function S which represents the phase of the disturbance. We shall therefore put the differential co-efficients of E, H equal to zero,

and thus get finally the equations reduced to the following form :—

$$\frac{1}{c}\frac{\partial S}{\partial t}E-(\text{grad } S\times H)=-\theta/c$$

$$\frac{1}{c}\frac{\partial S}{\partial t}H+(\text{grad } S\times E)=0$$

$$(\text{grad } S\cdot E)=\theta_0$$
$$(\text{grad } S\cdot H)=0$$

These can be regarded as the characteristic set of the wave-equations of Lorentz, and since there are eight equations, if $\frac{\partial S}{\partial x}$, $\frac{\partial S}{\partial y}$, $\frac{\partial S}{\partial z}$, $\frac{\partial S}{\partial t}$ are taken as unknown, certain compatibility conditions have further to be satisfied which will give us the conditions under which wave-propagation can take place. We proceed to calculate these conditions by the usual method of vector-analysis. We have only to remember that, as the components of vectors may have complex-values, the geometrical interpretation of the various quantities may not be so immediate as in the usual cases : also a relation $A^2=0$, does not mean that the vector vanishes, as the components of the vector A_x, A_y, A_z, need not be zero, when $A_x^2+A_y^2+A_z^2=0$.

We shall call such vectors as singular, and in the problem that we have got before us, considerations of singular-vectors become important.

A few remarks regarding the geometrical interpretation of a singular vector may not be out-of-place; regarding the components of a singular vector A, as decomposable into real and imaginary parts, we can always choose a real direction (λ, μ, ν) which is perpendicular to a complex vector, *viz.*, $(A_x+A_x^*\iota, \ldots)$

i.e., such that $\lambda A_x+\mu A_y+\nu A_z=0$

$$\lambda A_x^*+\mu A_y^*+\nu A_z^*=0$$

i.e., an imaginary vector A, may be regarded as defining a real plane.

If co-ordinates are chosen so as to represent this plane as the plane of (xy), a singular vector A, i.e., whose $A^2=0$, can be represented as

$(A, \iota A, 0)$ where A, may be a complex quantity, i.e., A is of the form $[\iota\lambda+\iota\mu, -\mu+\iota\lambda, 0]$. Any vector perpendicular to a singular vector, will have in this co-ordinate system, the general form $(P, \iota P, Q)$ where P and Q may be complex quantities. A non-singular vector, C perpendicular to A, will have $C^2\neq 0$, *i.e.*, $Q\neq 0$; a singular vector perpendicular to A, will also necessarily have $Q=0$, *i.e.*, if A and B are two singular vectors, both conditions $(A\cdot B)=0$, and $(A\times B)=0$, may be satisfied at the

same time and all singular vectors perpendicular to one another may be regarded as lying in one real plane.

If A and B are two perpendicular vectors of which one is singular while the other is not, any vector which is perpendicular to both A and B, must necessarily be singular, and will lie in the plane of A. This proposition follows immediately by writing out the general form of A and B in the manner explained above, and trying to write down the general form of C which satisfies the orthogonality condition. These preliminary remarks will be useful in discussing the singular case of total reflection, as we shall see later on. The other two relations in vector analysis which we shall have to use pretty frequently are

$$A.(B\times C) = B.(C\times A) = -B.(A\times C) = \text{etc.} \qquad \ldots \quad (\text{I})$$

and

$$A\times(B\times C) = B(AC) - C(AB). \qquad \ldots \quad (\text{II})$$

These have unrestricted validity even when the components of vectors have complex values, as they are formal identities, involving only re-arrangement of terms.

We shall at first deduce certain general conclusions from our equations

$$\frac{1}{c}\frac{\partial S}{\partial t} E - (\text{grad } S\times H) = -\theta/c \qquad \ldots \quad (1)$$

$$\frac{1}{c}\frac{\partial S}{\partial t} H + (\text{grad } S\times E) = 0 \qquad \ldots \quad (2)$$

$$(\text{grad } S\cdot E) = \theta_0 \qquad \ldots \quad (3)$$

$$(\text{grad } S\cdot H) = 0. \qquad \ldots \quad (4)$$

Scalar multiplication with grad S of (1) gives, taking account of (3),

$$\frac{1}{c}\left[\frac{\partial S}{\partial t}\theta_0 + (\theta \text{ grad } S)\right] = 0, \qquad \ldots \quad (\text{A})$$

a relation which expresses the conservation of charge in the disturbed state of the medium.

Vector multiplication by grad S of (2) gives

$$\frac{1}{c}\frac{\partial S}{\partial t}(\text{grad } S\times H) + \text{grad } S\times(\text{grad } S\times E) = 0;$$

taking account of (1) we get

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-(\text{grad } S)^2\right]E+\frac{\theta}{c^2}\frac{\partial S}{\partial t}+\theta_0 \text{ grad } S=0. \quad \ldots \text{ (B)}$$

Scalar multiplication of (2) with E gives $(EH)=0$ ... (C)

Scalar multiplication (1) with H gives $(\theta H)=0$ ... (D)

Relations (C), (D) and (4) show that the vectors E, θ, and grad S, are all in one plane perpendicular to H ; also (1) shows that grad S is perpendicular to $\frac{\partial S}{\partial t}E+\theta$. This vector can be regarded as representing the rate of change of induction, or $\frac{dD}{dt}$ of Maxwell's equation; thus H and D can be regarded as lying in the plane perpendicular to grad S while E and θ separately do not lie in it.

The arrangement of the various vectors thus agrees with our usual ideas about electromagnetic waves.

Energy and momentum relations in the wave field

Scalar multiplication of (1) and (2) by E and H respectively give

$$\left[\frac{1}{c}\frac{\partial S}{\partial t}E^2+\frac{(E\theta)}{c}\right]=E\cdot(\text{grad } S\times H)=-\text{grad } S.\ [E\times H] \quad \ldots \text{ (E)}$$

$$\frac{1}{c}\frac{\partial S}{\partial t}H^2=-\text{ grad } S.\ [E\times H]\ ; \quad \ldots \text{ (F)}$$

addition and subtraction of (E) and (F) give two further important relations

$$\frac{1}{c}\frac{\partial S}{\partial t}\frac{[E^2+H^2]}{2}+\frac{(E\theta)}{2c}=-\text{ grad } S.\ [E\times H] \quad \ldots \text{ (G)}$$

$$\frac{1}{c}\frac{\partial S}{\partial t}(E^2-H^2)=-\frac{(E\theta)}{c}\ . \quad \ldots \text{ (H)}$$

Also vector multiplication of (1), and (2) by H and E similarly gives

$$\frac{1}{c}\frac{\partial S}{\partial t}(E\times H)+H\times(\text{grad } S\times H)=-\frac{\theta\times H}{c}$$

or

$$\frac{1}{c}\frac{\partial S}{\partial t}(E\times H)+H^2\,\text{grad } S=-\frac{\theta\times H}{c}\ ; \quad \ldots \text{ (K)}$$

also $$\frac{1}{c}\frac{\partial S}{\partial t}(E\times H)+E^2\,\text{grad}\,S-E\,(\text{grad}\,S.E)=0$$

or $$\frac{1}{c}\frac{\partial S}{\partial t}\,(E\times H)+E^2\,\text{grad}\,S=\theta_0 E. \qquad \ldots \text{(L)}$$

Subtracting (L) from (K) we have

$$\text{grad}\,S\,(H^2-E^2)=-\left[\,\theta_0 E+\frac{\theta\times H}{c}\right]. \qquad \ldots \text{(M)}$$

Multiplying (K) and (L) by grad S we have the further relation

$$\frac{1}{c}\frac{\partial S}{\partial t}\cdot\text{grad}\,S.(E\times H)+\text{grad}\,S.\,\frac{(\theta\times H)}{c}=-H^2\,(\text{grad}\,S)^2$$

$$\frac{1}{c}\frac{\partial S}{\partial t}\,\text{grad}\,S.\,(E\times H)=\theta_0^2-(\text{grad}\,S)^2E^2;$$

but $$\text{grad}\,S.\,(\theta\times H)=-\,\theta.\,(\text{grad}\,S\times H)=-\frac{1}{c}\left(\theta^2+\frac{\partial S}{\partial t}\,(E\theta)\right)$$

or $$\frac{1}{c}\,\text{grad}\,S.\,(\theta\times H)=-\left[\frac{\theta^2}{c^2}+\frac{1}{c^2}\frac{\partial S}{\partial t}\,(E\theta)\right].$$

These two relations thus become

$$\frac{1}{c}\frac{\partial S}{\partial t}\,\text{grad}\,S.\,(E\times H)=\frac{\theta^2}{c^2}+\frac{1}{c^2}\frac{\partial S}{\partial t}\,(E\theta)-(\text{grad}\,S)^2H^2$$

$$\frac{1}{c}\frac{\partial S}{\partial t}\,\text{grad}\,S.\,(E\times H)=-(\text{grad}\,S)^2E^2+\theta_0^2.$$

From these we get

$$\frac{\theta^2}{c^2}-\theta_0^2+\frac{1}{c^2}\frac{\partial S}{\partial t}\,(E\theta)-(\text{grad}\,S)^2(H^2-E^2)=0,$$

or remembering (H) we see that

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-(\text{grad}\,S)^2\right](H^2-E^2)=S\,\theta_0^2-\frac{\theta^2}{c^2}. \qquad \ldots \text{(P)}$$

Also $(\text{grad}\,S)^2H^2=(\text{grad}\,S.H)^2+(\text{grad}\,S\times H)^2$

$$=\frac{1}{c^2}\left[\left(\frac{\partial S}{\partial t}\right)^2E^2+2\,\frac{\partial S}{\partial t}\,(E\theta)+\theta^2\right];$$

we get

$$\frac{1}{c}\frac{\partial S}{\partial t}\,\text{grad}\,S\,.\,(E\times H)=-\frac{1}{c^2}\frac{\partial S}{\partial t}\left[E^2\frac{\partial S}{\partial t}+(E\theta)\right]$$

$$=\theta_0^2-(\text{grad}\,S)^2E^2, \qquad \ldots\ (Q)$$

which can be written in the form

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-(\text{grad}\,S)^2\right]E^2+\frac{(E\theta)}{c^2}\frac{\partial S}{\partial t}+\theta_0^2=0. \qquad \ldots\ (Q)$$

This equation could have been immediately deduced from (B) by taking scalar multiplication with E.

Equation (B) :

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-(\text{grad}\,S)^2\right]E+\frac{\theta}{c^2}\frac{\partial S}{\partial t}+\text{grad}\,S(\text{grad}\,S\cdot E)=0$$

gives us generally the mutual disposition of E, θ and grad S when wave-propagation is possible, whereas the equation (Q) written in the form

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-(\text{grad}\,S)^2\right]+\frac{1}{c^2}\frac{\partial S}{\partial t}\frac{(E\theta)}{E^2}+\frac{(\text{grad}\,S\cdot E)}{E^2}=0 \qquad \ldots\ (Q)$$

may be regarded as the equation for the refractive index, determining the velocity of propagation in any direction prescribed by grad S.

The Notion of the Refractive-index

A surface $S(x, y, z, t) = C$ may be regarded as moving perpendicular to itself with a velocity given by

$$W=\frac{\dfrac{\partial S}{\partial t}}{|\text{grad}\,S|}\ ;$$

the refractive index can be defined as $\frac{c}{W}=\mu=\dfrac{|\text{grad}\,S|}{\frac{1}{c}\frac{\partial S}{\partial t}}$, agreeing with our usual definition, which however can be complex in the general case.

Introducing μ, equation (Q) can be written in the form

$$(1-\mu^2)+\frac{(E\theta)}{E^2}\left|\frac{\partial S}{\partial t}+\frac{c^2(\text{grad } S\cdot E)^2}{E^2}\right|\left(\frac{\partial S}{\partial t}\right)^2=0.$$

On Relation Between E and θ

Early in the paper we have expressed θ, in terms of motion of the positive and negative charges, namely,

$$\theta=[\rho_0^+ W_0^+ + q^+ V_0^+ + \rho_0^- W^- + q^- V_0^-].$$

These involve in general both the original distribution of the charges, their velocities as well as the amplitudes of velocities, and densities of positive and negative electricity due to the disturbance in the medium.

From general considerations we can express the relation of θ, and E in the following way.

We have seen that during propagation of disturbance both θ and E lie in the plane perpendicular to H. Therefore, θ can be decomposed into two components, one along E and another perpendicular to E, in this plane, *i.e.*, in the direction of $(E\times H)$.

θ thus can be put as equal to $\alpha E+\beta\dfrac{[E\times H]}{|E|}$

$$\because\quad (\theta E)=\alpha E^2,\quad \theta\times E=\frac{\beta E^2}{|E|}H;$$

also from equations of wave--propagation

$$\theta\times E=cH\,(\text{grad } S\cdot E)\text{ or }\beta^2/c^2=\theta_0^2/E^2.$$

Writing therefore

$$\theta=\alpha E+\beta\frac{[E\times H]}{|E|}$$

we see that

$$\alpha=\frac{(E\theta)}{E^2},\ \beta=\frac{C\theta_0}{|E|},$$

so that the refractive-index equation can be expressed in terms of α and β in the form

$$(1-\mu^2)+\frac{\alpha}{\partial S/\partial t}+\frac{\beta^2}{(\partial S/\partial t)^2}=0.$$

Further,

$$\theta^2 = \alpha^2 E^2 + \beta^2 H^2.$$

As the vectors E and H are perpendicular to one another it follows that α and β are two co-efficients which stand in very simple relation to the total polarisation current; α and β however are not constants but may involve grad$S$ as well as $\frac{\partial S}{\partial t}$, *i.e.*, may be function of direction and frequency of the waves.

Condition for total Reflection

It is well-known that for electro-magnetic waves in vacuum we have $E^2 - H^2 = 0$ as well as $(E \cdot H) = 0$. For waves in material media however $E^2 - H^2 \neq 0$, but $(EH) = 0$ remains still valid. The ratio of the electric and magnetic vectors depends generally upon the properties of the medium as well as on the frequency of the wave and we can express this fact by the relations (H) and (M)

which are $$\frac{1}{c}\frac{\partial S}{\partial t}(E^2 - H^2) = -\frac{(E\theta)}{c}$$

$$\text{grad } S(H^2 - E^2) = -F = -\left(E\theta_0 + \frac{\theta \times H}{c}\right)$$

The ratio of the amplitudes of the vectors, H^2/E^2, can therefore be expressed by the following relations :

$$\frac{\partial S}{\partial t} + \frac{(E\theta)}{E^2} = \frac{H^2}{E^2}\frac{\partial S}{\partial t}$$

$$\text{grad } S - \frac{F}{E^2} = \frac{H^2}{E^2}\text{ grad } S,$$

Depending upon the property of the medium and the direction of disposition of the vector E, $\frac{(E\theta)}{E^2}$ will generally vary with $\frac{\partial S}{\partial t}$ as well as with grad S. If, however, $\frac{\partial S}{\partial t}$ tends to such a value that $\frac{\partial S}{\partial t} + \frac{E\theta}{c^2}. = 0$, it follows that $H^2 = 0$ $(E \neq 0)$, independent of the amplitude of the wave traversing the medium; the disturbance in such a case loses its wave-character, so that we can say the wave cannot penetrate beyond that region and gets totally reflected. We shall therefore take this condition $\frac{H^2}{E^2} = 0$ or an equivalent condition as characterising the condition of total reflection

and we can also correlate it with another physical idea.

We have also proved the following relation (F)

$$\frac{1}{c}\frac{\partial S}{\partial t}H^2 = -\text{grad } S.\,[E\times H]$$

and also

$$\frac{1}{c}\frac{\partial S}{\partial t}\left[\frac{E^2}{2}+\frac{H^2}{2}\right]+\frac{E\theta}{2c} = -\text{grad } S.\,(E\times H).$$

grad $S.(E\times H)$ may thus be regarded as proportional to the flow of energy across the surface S = constant, so that grad $S.(E\times H) = 0$ may be interpreted to mean that the flow of energy across the wave-front vanishes. This agrees with our usual idea of total-reflection at a boundary of two media, where the energy flows along the boundary interface in the limiting case.

We have also seen that

$$\frac{|\text{grad } S|}{\frac{1}{c}\frac{\partial S}{\partial t}} = \mu = \text{the refractive index of the medium.}$$

From (2) we have $\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 H^2 = (\text{grad } S\times E)^2 = E^2(\text{grad } S)^2-(\text{grad } S.E)^2$

$$= -\frac{1}{c}\frac{\partial S}{\partial t}\text{ grad } S.[E\times H),$$

we have

$$\frac{H^2}{E^2} = \frac{(\text{grad } S)^2}{\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2} - \frac{(\text{grad } S.E)^2}{E^2\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2} = -\frac{\text{grad } S.[E\times H]}{\frac{1}{c}\frac{\partial S}{\partial t}E^2} = \frac{H.(\text{grad } S\times E]}{\frac{1}{c}\frac{\partial S}{\partial t}E^2}.$$

In case (grad S·E) = 0, we see that the condition $\frac{H^2}{E^2} = 0$ is equivalent to the condition $\mu^2 = 0$, as has been assumed by Appleton to characterise the case of total reflection. If we now examine the condition (B) which gives the relation between the vectors E, θ and grad S, the condition $\theta = 0$ will mean that the wave is propagated under such conditions that the vector E is parallel to θ.

Multiplying (1) with E, we have

$$E\times\theta = cE\times(\text{grad } S\times H)$$
$$= -cH(\text{grad S}\cdot E),$$

so that (grad $S\cdot E$) = 0 will mean $E\times\theta = 0$, *i.e.*, E is paralled to θ.

We thus see that if the wave is such that E is $\parallel$ to θ the condition of total reflection $H = 0$, or grad $S.(E \times H) = 0$ is equivalent to the assumption $\mu = 0$; in the general case of wave propagation when E is not parallel to θ we would expect on the other hand the condition $H^2 = 0$ as giving a new result.

$H^2 = 0$, or grad $S.(E \times H) = 0$, may be regarded as equivalent to the assumption that the group-velocity vanishes. The condition of total-reflection postulated above is obviously suitable for a train of damped waves, because our analysis is general and the condition of non-penetrability has been deduced independent of all ideas of frequency.

We shall now apply the results of our analysis in the particular case of the ionosphere.

The contribution of the positive charges, as well as of the bound electrons in 0, may be easily seen to be negligible in comparison with the contribution of the free electrons.

We can assume for the ionosphere

$$\theta = [\rho_0^- W^- + q^- V_0^-]$$

where these quantities refer only to the free-electrons present. If we average over a volume containing a large number of electrons, but small compared with the wavelength of the disturbance,

$$\rho_0^- = Ne, \quad \theta = \overline{NeW} ;$$

the average value of the second term $q\bar{V}_0$ may be regarded as zero as the initial velocities of the charges may be regarded as distributed chaotically in all directions.

Writing the equation of motion of a free electron after Lorentz as

$$m\ddot{x} + gx = e\left[E_x + \frac{v_y h_z - v_z h_y}{c}\right], \text{ etc.}$$

where (h_x, h_y, h_z) may be regarded as the components of a steady magnetic force, we have, dividing by m and multiplying by $\rho = Ne$, and averaging, the following equation

$$\dot{\theta} + \frac{g}{m}\theta = \frac{Ne^2}{m} E + \left(\theta \times \frac{eh}{cm}\right)$$

where $(\theta_x = \overline{Nev_x}$, etc...).

Remembering that for an oscillatory disturbance $\theta = \theta \frac{\partial S}{\partial t} e^S$ as before also that the

other terms in the equation varies at e^S, the equation can be written as

$$\left(\frac{\partial S}{\partial t}+\frac{g}{m}\right)\theta=\frac{Ne^2}{m}E+\left(\theta\times\frac{eh}{cm}\right) \qquad \ldots \quad (Z)$$

cancelling out e^S. $\frac{g}{m}=\nu$ is usually called the collision frequency of the electrons. We shall use this equation (Z) to calculate $\frac{(E\theta)}{E^2}$ or $\frac{\theta}{E}$.

Case I : This can be regarded as decomposable into two subcases.

(*a*) When $\frac{\theta\times eh}{cm}=0$, *i.e.*, θ is parallel to the external magnetic field, equation (Z) shows that θ is then parallel to E

and $$\frac{E\theta}{E^2}=\frac{\theta}{E}=\frac{Ne^2}{m}\left(\frac{\partial S}{\partial t}+\nu\right).$$

The equation for determining the critical frequency for total reflection becomes

$$\frac{\partial S}{\partial t}+\frac{Ne^2}{m}\Big/\left(\frac{\partial S}{\partial t}+\nu\right)=0$$

or $$\left(\frac{\partial S}{\partial t}\right)^2+\nu\left(\frac{\partial S}{\partial t}\right)+\frac{Ne^2}{m}=0$$

or $$\frac{\partial S}{\partial t}=-\frac{\nu}{2}\pm\iota\sqrt{\frac{Ne^2}{m}-\nu^2/4}$$

This determines the dependence of the critical frequency on the ionic density; the train totally reflected is therefore of the form

$$e^{-\nu/2t}e^{\pm\iota t\sqrt{Ne^2/m-\nu^2/4}}$$

or $$w_c^2=Ne^2/m-\nu^2/4$$

and the damping co-efficient is $\nu/2$.

Case II : When $\theta\parallel E$ and $\theta^2=0$

that is the vector θ is singular, $\theta^2=0$ but $\theta\neq 0$

a vector $\left(\theta\times\frac{eh}{cm}\right)$ will represent a vector parallel to θ, when h is non-singular and perpendicular to θ according to our former remarks about singular vectors. Choosing the direction of h, as the direction of Z, we can write the relation

$$\left(\frac{\partial S}{\partial t}+\nu\right)\theta=\frac{Ne^2}{m}E+\left(\theta\times\frac{e\dot{h}}{cm}\right)$$

in the form

$$\left(\frac{\partial S}{\partial t}+\nu\right)\theta_x=\frac{Ne^2}{m}E_x+\frac{eh}{cm}\theta_y$$

$$\left(\frac{\partial S}{\partial t}+\nu\right)\theta_y=\frac{Ne^2}{m}E_y-\frac{eh}{cm}\theta_x$$

or

$$\left(\frac{\partial S}{\partial t}+\nu\pm\iota\frac{eh}{cm}\right)(\theta_x+\iota\theta_y)=\frac{Ne^2}{m}(E_x\pm\iota E_y)$$

$\frac{\theta}{E}$ becomes equal to $\frac{Ne^2/m}{\left[\frac{\partial S}{\partial t}+\nu\pm\iota\frac{eh}{cm}\right]}$,

according as θ and E are of the form $(A, \mp\iota A, 0)$.

The condition of total reflection now becomes, from (B),

$$\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t}+\nu\pm\iota\frac{eh}{cm}\right)+\frac{Ne^2}{m}=0;$$

calling

$$\frac{\partial S}{\partial t}+\nu/2\pm\iota\frac{eh}{2cm}=x$$

we have

$$x^2+\frac{Ne}{m}-\nu^2/4+\frac{e^2h^2}{4c^2m^2}\pm\frac{eh\iota\nu}{2cm}=0$$

Solving the above equation, we find that the two reflected waves are circularly polarised and the critical frequencies are

$$\omega_{c\pm}=\sqrt{\frac{Ne^2}{m}-\nu^2/4+\frac{e^2h^2}{4c^2m^2}}\mp\frac{eh}{2cm}$$

$$T_{c\pm}=\nu/2\left[1\pm\frac{eh}{2cm\omega'}\right]\text{ where }\omega'=\sqrt{\frac{Ne^2}{m}-\nu^2/4+\frac{e^2h^2}{4c^2m^2}}$$

So that when we have the critical frequency $=\frac{eh}{cm}$, *i.e.*, $\omega'=\frac{eh}{2cm}$, one of the circularly polarised wave will be suppressed, and the other will be reflected with undiminished intensity : as this will necessarily mean $\frac{Ne^2}{m}=\nu^2/4$ the reflection of plane-

polarised wave, discussed above in case I, will not occur as the condition of total reflection gives $\omega_c = 0$, if $\frac{Ne^2}{m} = \nu^2/4$.

These two cases correspond to the cases discussed by Appleton by putting $\mu = 0$ and the result agrees with the usual formula, when the collision-frequency is neglected.
Case III. When $\theta \neq E$:

We shall now calculate the value of $\frac{(E\theta)}{E^2}$ in the general case.

We have

$$\left(\frac{\partial S}{\partial t} + \nu\right)\theta = \frac{Ne^2}{m} E + \left(\theta \times \frac{eh}{cm}\right).$$

Scalar multiplication with E gives

$$\left(\frac{\partial S}{\partial t} + \nu\right)(E\,\theta) = \frac{Ne^2}{m} E^2 - \theta\left[E \times \frac{eh}{cm}\right];$$

also vector multiplication with E gives

$$\left(\frac{\partial S}{\partial t} + \nu\right)(E \times \theta) = \theta.\left(E \cdot \frac{eh}{cm}\right) - \frac{eh}{cm}(E\theta).$$

Scalar multiplication by $\frac{eh}{cm}$ of the above relation gives

$$\left(\frac{\partial S}{\partial t} + \nu\right)\frac{eh}{cm} \cdot (E \times \theta) = \left(\frac{eh}{cm} \cdot \theta\right)\left(E \cdot \frac{eh}{cm}\right) - \left(\frac{eh}{cm}\right)^2 (E\theta).$$

Again, scalar multiplication of the original equation with $\frac{eh}{cm}$ gives

$$\left(\frac{\partial S}{\partial t} + \nu\right)\left(\frac{eh}{cm}\theta\right) = \frac{Ne^2}{m}\left(E \frac{eh}{cm}\right).$$

Combining (a), (b) and (c) we can easily deduce

$$\frac{(E\theta)\left[\left(\frac{\partial S}{\partial t}\right) + \nu\right]^2 + \left(\frac{eh}{cm}\right)^2}{\left(\frac{\partial S}{\partial t} + \nu\right)} = \frac{\frac{Ne^2}{m}(E^2)\left[\left(\frac{\partial S}{\partial t} + \nu\right)^2 + \frac{1}{E^2}\left(E \cdot \frac{eh}{cm}\right)^2\right]}{\left(\frac{\partial S}{\partial t} + \nu\right)^2}$$

or $$\frac{(E\theta)}{E^2} = \frac{Ne^2}{m} \frac{\left[\left(\frac{\partial S}{\partial t}+\nu\right)^2 + \frac{1}{E^2}\left(E_1 \frac{eh}{cm}\right)^2\right]}{\left(\frac{\partial S}{\partial t}+\nu\right)\left[\left(\frac{\partial S}{\partial t}+\nu\right)^2 + \left(\frac{eh}{cm}\right)^2\right]}$$

when E and θ are non-singular vectors.

Now if we consider the case of total reflection as defined by grad $S \cdot [E \times H]$ $= 0$, [Q] gives $\frac{\partial S}{\partial t} = -\frac{(E\theta)}{E^2}$ as well as $\theta_0^2 - (\text{grad } S)^2 E = 0$.

Now also grad $S \cdot [E \times H]$ can be written in the form $H(\text{grad } S \times E)$, we see that grad $S \| E$

and $$\frac{1}{E^2}\left(E.\frac{eh}{cm}\right)^2 = \frac{1}{(\text{grad } S)^2}\left(\text{grad } S \frac{eh}{cm}\right)^2$$

$\frac{1}{E^2}\left(E_1 \frac{eh}{cm}\right)^2 = \left(\frac{eh}{cm}\right)^2 \cos^2\alpha$ if grad S is a vector whose components have real ratios.

The relation $\frac{\partial S}{\partial t} + \frac{(E\theta)}{E^2} = 0$ becomes

$$\frac{\partial S}{\partial t} + \frac{\frac{Ne^2}{m}\left[\left(\frac{\partial S}{\partial t}+\nu\right)^2 + \frac{1}{E^2}\left(E \frac{eh}{cm}\right)^2\right]}{\left(\frac{\partial S}{\partial t}+\nu\right)\left[\left(\frac{\partial S}{\partial t}+\nu\right)^2 + \left(\frac{eh}{cm}\right)^2\right]} = 0$$

If the disturbance is plane-polarised and the collision frequency is small we can put

$$\nu = 0, \quad \frac{1}{E^2}\left(E_1 \frac{eh}{cm}\right)^2 = \cos^2\alpha \left(\frac{eh}{cm}\right)^2$$

where α is the angle between the direction of propagation and the magnetic lines. The formula becomes, putting $\frac{\partial S}{\partial t} = \iota\, \omega_c$

$$\omega_c^2 = \frac{\frac{Ne^2}{m}\left[\omega_c^2 - \left(\frac{eh}{cm}\right)^2 \cos^2\alpha\right]}{\omega_c^2 - \left(\frac{eh}{cm}\right)^2}$$

agreeing with the formula supposed to be derived by putting group-velocity = zero by Bajpai and Mathur.[1]

We shall have to remember however that E may be a complex-vector in the general case, any complex vector $E = [E_x + \iota E_x$, etc.], may be regarded as perpendicular to a real direction, and choosing this direction as the direction of Z, we can choose as components of the complex vector, $(E_x, \iota E_y, 0)$ where x, and y are certain special directions and the external field is regarded as having components (h_x, h_y, h_z) in these special directions.

$\left(E_1 \frac{eh}{cm}\right)$ then becomes equal to $\frac{e}{cm}(E_x h_x + \iota E_y h_y)$

$$E^2 = E_x^2 - E_y^2 \neq 0$$

$$\frac{1}{E^2}\left(E_1 \frac{eh}{cm}\right)^2 = \frac{e^2}{c^2m^2}\left[\frac{E_x^2h_x^2 - E_y^2h_y^2}{E_x^2 - E_y^2} + 2\iota h_x h_y \frac{E_x E_y}{E_x^2 - E_y^2}\right]$$

The frequency equation for the elliptically polarised beams would be

$$0 = \frac{\partial S}{\partial t} + \frac{Ne^2}{m} \frac{\left[\left(\frac{\partial S}{\partial t} + \nu\right)^2 + \frac{e^2}{c^2m^2}\left[\frac{E_x^2h_x^2 - E_y h_y^2}{E_x^2 - E_y^2} + \frac{2\iota E_x E_y}{E_x^2 - E_y^2} h_x h_y\right]\right]}{\left(\frac{\partial S}{\partial t} + \nu\right)\left[\left(\frac{\partial S}{\partial t} + \nu\right)^2 + \frac{e^2h^2}{c^2m^2}\right]};$$

when the collision frequency is neglected this similarly becomes

$$\omega^2 = \frac{Ne^2}{m} \frac{\left[\omega^2 - \frac{e^2}{c^2m^2}\left[\frac{E_x^2h_x^2 - E_y^2h_y^2}{E_x^2 - E_y^2}\right]\right]}{\omega^2 - e^2h^2/c^2m^2} \text{ when } \frac{E_x E_y}{E_x^2 - E_y^2} h_x h_y = 0.$$

This shows that the two cases will be either $h_x = 0$, or $h_y = 0$

$$\omega^2 = \frac{\frac{Ne^2}{m}\left[\omega^2 - \frac{e^2h_x^2E_x^2}{E_x^2 - E_y^2}\right]}{\omega^2 - \frac{e^2h^2}{c^2m^2}}, \quad \text{or} \quad \frac{\frac{Ne^2}{m}\left[\omega^2 + \left[\frac{e^2}{c^2m^2}\frac{h_y^2E_y^2}{(E_x^2 - E_y^2)}\right]\right]}{\omega^2 - \frac{e^2h^2}{c^2m^2}},$$

We have therefore

or $$\omega^2 = \frac{\frac{Ne^2}{m}\left[\omega^2 + \frac{e^2h^2}{c^2m^2}\frac{E_y^2}{E_x^2 - E_y^2}\cos^2\alpha\right]}{\omega^2 - \frac{e^2h^2}{c^2m^2}}$$

when h is $\perp$ to the major axis,

$$\omega^2 = \frac{\frac{Ne^2}{m}\left[\omega^2 - \frac{e^2h^2}{c^2m^2}\frac{E_x^2\cos^2\alpha}{E_x^2-E_y^2}\right]}{\omega^2 - \frac{e^2h^2}{c^2m^2}}$$

when h is $\perp$ to the minor axis.

The Propagation of the Disturbance

We shall analyse now the relation (B), *i.e.*,

$$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - (\text{grad } S)^2\right]E + \frac{\theta}{c^2}\frac{\partial S}{\partial t}\theta_0 \text{ grad } S = 0$$

which may be regarded as prescribing the relation between the three vectors E, θ and grad S, when propagation takes place.

For this purpose, we shall have to consider the relation (A) which expresses the conservation of charge, *i.e.*,

$$\frac{1}{c}\left[\frac{\partial S}{\partial t}\theta_0^2 + (\theta \text{ grad } S)\right] = 0,$$

and the relation (Z), *i.e.*,

$$\left(\frac{\partial S}{\partial t} + \nu\right)\theta = \frac{Ne^2}{m}E + \left(\theta \times \frac{eh}{cm}\right).$$

We can utilise (*A*) and (Z) to eliminate E and θ_0 from (B) and thus get

$$\frac{\theta}{c^2}\frac{\partial S}{\partial t} + \frac{\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \text{grad}^2 S\right]\left[\left(\frac{\partial S}{\partial \nu} + \nu\right)\theta - \left(\theta \times \frac{eh}{cm}\right)\right]}{\frac{Ne^2}{m}} = \frac{\text{grad } S\,(\theta.\ \text{grad } S)}{\frac{\partial S}{\partial t}}$$

or by re-arranging terms,

$$\left\{\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \text{grad}^2 S\right]\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + \nu\right) + \frac{Ne^2}{mc^2}\left(\frac{\partial S}{\partial t}\right)^2\right\}\theta$$

$$- \frac{\partial S}{\partial t}\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - (\text{grad}^2 S)\right]\left(\theta \times \frac{eh}{cm}\right) = \frac{Ne^2}{m}\text{grad } S\,(\theta.\ \text{grad } S). \quad \text{(R)}$$

This vector equation is equivalent to three homogeneous linear equations in $(\theta_x, \theta_y, \theta_z)$,

whose co-efficients involve the differential co-efficients of S and the collision-frequency; when the relation is satisfied for non-zero values of $(\theta_x, \theta_y, \theta_z)$ the discriminant of the simultaneous equation must vanish. This will give a relation between the differential co-efficients of S alone, where the characteristic numbers of the ionospheric layer, *i.e.*, N, h and ν, will also be involved, and may be regarded as giving the equation of the characteristic surface, corresponding to the Ei-konal equation of Hamilton for a material medium

or equation $$\frac{1}{c^2}\left[\left(\frac{\partial S}{\partial t}\right)^2-(\text{grad } S)^2\right]=0 \qquad \text{for vacuum.}$$

It is better for perspicacity to introduce certain abbreviations at this stage; we shall put

$$g(S)=\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-\text{grad}^2 S$$

also $$\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-\text{grad}^2 S\right]\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t}+\nu\right)+\frac{Ne^2}{mc^2}\left(\frac{\partial S}{\partial t}\right)^2 \text{ is put}$$

$$= g(S)\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t}+\nu\right)+\frac{Ne^2}{m}\left[g(S)+\text{grad}^2 S\right]= L(S)$$

$$= g(S)\,p(S)+\frac{Ne^2}{m}(\text{grad } S)^2 =\frac{p(S)}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t}+\nu\right)\text{grad}^2 S$$

where $$p(S)=\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t}+\nu\right)+\frac{Ne^2}{m}.$$

The vector equation (R) is thus written

$$L(S)\,\theta-g(S)\frac{\partial S}{\partial t}\left(\theta\times\frac{eh}{cm}\right)=\frac{Ne^2}{m}\text{grad } S(\theta\cdot\text{grad } S).$$

Scalar multiplication with θ gives

$$L(S)\theta^2=\frac{Ne^2}{m}(\theta\cdot\text{grad } S)^2,$$

so that (grad $S\cdot\theta$) $= 0$, either when $\theta^2 = 0$ or when $L(S) = 0$, so that the vector equation becomes simplified, in both of these cases.

$E \parallel \theta$

Case I. When $L(S) = 0$, $\theta^2 \neq 0$.

The vector equation (R) is satisfied only when

$$\left(\theta \times \frac{eh}{cm}\right) = 0, \text{ or } g(S) = 0, \text{ or } \frac{\partial S}{\partial t} = 0.$$

We reject $\frac{\partial S}{\partial t} = 0$, as we are considering wave propagation. We can easily see that $g(S) = 0$ is a singular case, which is not relevant to the problem before us, as $g(S) = 0$ will lead to the vanishing of the vector θ.

θ is parallel to h, the external magnetic field.

The characteristic equation thus becomes

$$\frac{p(S)}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\frac{\partial S}{\partial t} + v\right)\frac{\partial S}{\partial t}\,\text{grad}^2 S = 0 \qquad \ldots (1)$$

or
$$\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2 = \frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 \frac{\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + v\right) + \frac{Ne^2}{m}}{\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + v\right)}$$

or putting
$$\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2 = \frac{\mu}{c^2}\left(\frac{\partial S}{\partial t}\right)^2$$

$$\mu^2 = 1 + \frac{Ne^2}{m} \Big/ \frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + v\right)$$

which agrees with the ordinary expression for the refractive index for plane polarised ordinary ray. The singular solution of the equation (1) when $p(S)$ and $|\text{grad } S| = 0$ gives the case of total reflection.

Case II. When $\theta^2 = 0$

$$(\theta \text{ grad } S) = 0.$$

The vector equation (R) can be satisfied only when

$$L(S)\theta - g(S)\frac{\partial S}{\partial t}\left(\theta \times \frac{eh}{cm}\right) = 0$$

i.e., when $\left(\theta \times \frac{eh}{cm}\right)$ becomes parallel to θ.

This singular case, already discussed, means that θ is perpendicular to h, and

choosing a new set of perpendicular axis, with h in the direction of Z, we see $L(S)$, $p(S)$, $g(S)$ are invariants for orthogonal transformations. The vector equation can be split up into two equations

$$L(S)\theta_x - g(S)\frac{\partial S}{\partial t}\,\theta_y\,\frac{eh}{cm} = 0$$

$$L(S)\theta_y + g(S)\,\frac{\partial S}{\partial t}\,\theta_x\frac{eh}{cm} = 0.$$

So that remembering $\theta_x^2+\theta_y^2 = 0$

we have $$L^2(S)+g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\left(\frac{eh}{cm}\right)^2 = 0$$

which breaks up into two equations

$$g(S)\,p(S)+\frac{Ne^2}{m}(\text{grad } S)^2 \pm \iota g(S)\frac{\partial S}{\partial t}\,\frac{eh}{cm} = 0$$

or $$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \text{grad}^2 S + \frac{\frac{Ne^2}{mc^2}\left(\frac{\partial S}{\partial t}\right)^2}{\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t}+\nu \pm \iota\frac{eh}{cm}\right)} = 0$$

which can again be put in the form

$$\frac{\mu^2}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - (\text{grad } S)^2 = 0.$$

If $$\mu^2 = 1+\frac{Ne^2/m}{\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t}+\nu\pm\iota\frac{eh}{cm}\right)};$$

total reflection is obtained as before by putting $\mu = 0$

$\text{grad}^2 S$ can also become infinite, if $\frac{\partial S}{\partial t} = \pm\frac{\iota eh}{cm} - \nu$

i.e., for a frequency equal to the gyromagnetic frequency.

Case III.

Neither $L(S)$ nor $\theta^2 = 0$

i.e., when $(\text{grad } S\theta) \neq 0$

and θ is not parallel to E.

We have to proceed to further elimination to get the refractive index surface.

The elimination work becomes simplified if we choose a new set of axis, with Z axis, in the direction of h.

The vector equation is now equivalent to the three scalar equations

$$L(S)\theta_x - g(S)\frac{\partial S}{\partial t}\theta_y\frac{eh}{cm} = \frac{Ne^2}{m}S_x(S_x\theta_x+S_y\theta_y+S_z\theta_z) \quad \ldots (i)$$

$$L(S)\theta_y + g(S)\frac{\partial S}{\partial t}\theta_x\frac{eh}{cm} = \frac{Ne^2}{m}S_y(S_x\theta_x+S_y\theta_y+S_z\theta_z) \quad \ldots (ii)$$

$$L(S)\theta_z = \frac{Ne^2}{m}S_z(S_x\theta_x+S_y\theta_y+S_z\theta_z) \quad \ldots (iii)$$

where $\left(S_x = \frac{\partial S}{\partial x}, S_y = \frac{\partial S}{\partial y}, S_z = \frac{\partial S}{\partial z} \right)$.

From (*i*) and (*ii*) we easily get the relations

$$\theta_x\left[L^2(S)+g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\frac{e^2h^2}{c^2m^2}\right]$$

$$= \frac{Ne^2}{m}\left[L(S)S_x + S_y g(S)\frac{\partial S}{\partial t}\frac{eh}{cm}\right](S_x\theta_x+S_y\theta_y+S_z\theta_z)$$

$$\theta_y\left[L^2(S)+g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\frac{e^2h^2}{c^2m^2}\right]$$

$$= \frac{Ne^2}{m}\left[L(S)S_y - S_x g(S)\frac{\partial S}{\partial t}\frac{eh}{cm}\right][S_x\theta_x+S_y\theta_y+S_z\theta_z]$$

and . $$L_z\theta_z = \frac{Ne^2}{m}S_z[S_x\theta_x+S_y\theta_y+S_z\theta_z].$$

$$\frac{\theta_x}{S_x\theta_x+S_y\theta_y+S_z\theta_z} = \frac{L(S)S_x+S_y g(S)\frac{\partial S}{\partial t}\frac{eh}{cm}}{L^2(S)+g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\frac{e^2h^2}{c^2m^2}}\cdot\frac{Ne^2}{m} \quad \ldots (iv)$$

$$\frac{\theta_y}{S_x\theta_x+S_y\theta_y+S_z\theta_z} = \frac{L(S)S_y-S_x g(S)\frac{\partial S}{\partial t}\frac{eh}{cm}}{L^2(S)+g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\frac{e^2h^2}{c^2m^2}}\cdot\frac{Ne^2}{m} \quad \ldots (v)$$

$$\frac{\theta_z}{S_x\theta_x+S_y\theta_y+S_z\theta_z} = \frac{\frac{Ne^2}{m}S_z}{L(S)}; \quad \ldots (vi)$$

multiplying (*iv*), (*v*) and (*vi*) by S_x, S_y, S_z, we have

$$1 = \frac{Ne^2}{m}\left[\frac{L(S)(S_x^2+S_y^2)}{L^2(S)+g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\left(\frac{eh}{cm}\right)^2}+\frac{S_z^2}{L(S)}\right]$$

or $$L(S)\left[L^2(S)+g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\left(\frac{eh}{cm}\right)^2\right]$$

$$= \frac{Ne^2}{m}\left[L^2(S)(S_x^2+S_y^2+S_z^2+S_z^2g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\left(\frac{eh}{cm}\right)^2\right].$$

Now writing $$S_z^2 = \text{grad}^2 S\cos^2\alpha$$

where α = the angle between the wavenormal and the magnetic field, we have

$$L^2(S)\left[L(S)-\frac{Ne^2}{m}\text{grad}^2 S\right]+g^2(S)\,L(S)\left(\frac{\partial S}{\partial t}\right)^2\left(\frac{eh}{cm}\right)^2$$

$$= \frac{Ne^2}{m}\left(\frac{eh}{cm}\right)^2 g^2(S)\left(\frac{\partial S}{\partial t}\right)^2\text{grad}^2 S\cos^2\alpha.$$

Remembering $$L(S)-\frac{Ne^2}{m}\text{grad}^2 S = p(S)g(S)$$

and rejecting the factor $g(S)$ the equation finally assumes the form

$$p(S)L^2(S)+g(S)\left(\frac{\partial S}{\partial t}\right)^2\left(\frac{eh}{cm}\right)^2 L(S)$$

$$-\frac{Ne^2}{m}\left(\frac{\partial S}{\partial t}\right)^2 g(S)\left(\frac{eh}{cm}\right)^2\text{grad}^2 S.\cos^2\alpha = 0.$$

When this equation is regarded as a quadratic in $(\text{grad}^2 S)$ *i.e.*, when the value of $\frac{\partial S}{\partial t}$ is supposed to be prescribed, the two roots of grad^{2}S, become, let us say, $f_1\left(\frac{\partial S}{\partial t}, \alpha\right)$ and $f_2\left\{\frac{\partial S}{\partial t}, \alpha\right\}$. So that we get two rays corresponding to any prescribed α.

Corresponding to the two refractive indices

$$\mu_1^2 = \frac{f_1\left\{\frac{\partial S}{\partial t}, \alpha\right\}}{\frac{1}{c^2}\left\{\frac{\partial S}{\partial t}\right\}^2}, \text{ and } \mu_2^2 = \frac{f_2\left\{\frac{\partial S}{\partial t}, \alpha\right\}}{\frac{1}{c^2}\left\{\frac{\partial S}{\partial t}\right\}^2}$$

When damping is neglected, $\frac{\partial S}{\partial t} = \iota\,\omega$.

We see from (*iv*), (*v*) and (*vi*), $\theta_x : \theta_y : \theta_z$

defines a complex-ratio. As $L(S)$ will become real, and $\frac{\partial S}{\partial t}$ pure imaginary, the two split

rays will there be elliptically polarised, and the ratio of the axis, projected on a plane perpendicular of the magnetic field, is easily seen to be

$$\frac{g(S)\frac{\partial S}{\partial t}\frac{eh}{cm}}{L(S)} = \frac{g(S)\frac{\partial S}{\partial t}\left(\frac{eh}{cm}\right)}{\left[\frac{p(S)}{c^2} - \text{grad}^2 S\right]} = \frac{\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \text{grad}^2 S\right]\frac{eh}{cm}}{\frac{\partial S}{\partial t}\left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \text{grad}^2 S + \frac{Ne^2}{mc^2}\left(\frac{\partial S}{\partial t}\right)^2\right]};$$

for large values of $\text{grad}^2 S$ this tends to $\rightarrow \frac{eh}{cm\omega_c \iota}$.

Writing out the equation in extenso and putting $\text{grad}^2 S = y$, we see the equation

$$0 = p(S)\left[\frac{p(S)}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + \nu\right)y\right]^2 + \left[\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - y\right]\left(\frac{eh}{cm}\right)^2\left(\frac{\partial S}{\partial t}\right)^2$$

$$\left\{\frac{p(S)}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - y\left(\frac{\partial S}{\partial t} + \nu\right)\frac{\partial S}{\partial t} + \frac{Ne^2}{m}\cos^2\alpha\right]\right\}.$$

The co-efficient of $y^2 = p(S)\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + \nu\right)^2 + \left(\frac{eh}{cm}\right)^2\left(\frac{\partial S}{\partial t}\right)^2\left[\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + \nu\right)\right.$

$$\left. + \frac{Ne^2}{m}\cos^2\alpha\right].$$

The absolute term on the other hand is equal to

$$\frac{p^3(S)}{c^4}\left(\frac{\partial S}{\partial t}\right)^4 + \frac{1}{c^4}\left(\frac{\partial S}{\partial t}\right)^6 p(S)\left(\frac{eh}{cm}\right)^2 = \frac{p(S)}{c^4}\left(\frac{\partial S}{\partial t}\right)^4\left[p^2(S) + \left(\frac{\partial S}{\partial t}\right)^2\left(\frac{eh}{cm}\right)^2\right].$$

We get therefore

$$\frac{\partial S}{\partial t}\left(\frac{\partial S}{\partial t} + \nu\right) + \frac{\frac{Ne^2}{m}\left[\left(\frac{\partial S}{\partial t} + \nu\right)^2 + \left(\frac{eh}{cm}\right)^2\cos^2\alpha\right]}{\left[\left(\frac{\partial S}{\partial t} + \nu\right)^2 + \left(\frac{eh}{cm}\right)^2\right]} = 0$$

as defining the critical frequencies for total reflection. This is in the general case when collision frequency is not neglected an equation of the fourth degree having two different conjugate complex roots corresponding to two elliptically polarised rays: or when collision frequency is neglected,

and $$\frac{\partial S}{\partial t} = \iota\omega,$$

the above relation reduces to

$$\omega_c^2 = \frac{\frac{Ne^2}{m}\left[\omega^2 - \left(\frac{eh}{cm}\right)^2 \cos^2\alpha\right]}{\omega^2 - \left(\frac{eh}{cm}\right)^2}$$

which agrees with the expression arrived at by Rai, Bajpai, and Mathur, and also with the result arrived at by another method.

From energy consideration, we have seen that when $E \parallel \theta$ the condition of total reflection grad $S.[E \times H) = 0$, is satisfied (grad $S)^2 = 0$: Here the propagation stops from penetrating further, as H vanishes, and the field beyond becomes electrostatic in character. When (grad S $E) \neq 0$, which will happen when the direction of propagation makes an angle with the magnetic field, the electric vector is not the wave front, and the total-reflection condition results when electric vector coincides with the wave normal. This happens as (grad $S)^2 \to \alpha$. The Poynting vector then lies in the wave front. The flow of energy across the surface stops, and the waves get totally reflected.

Conclusion

We have discussed the question of ionosphere at some length, but it will be easily seen, that the method can be applied with the same ease for discussion of propagation of light in a material medium. We have only to utilise the corresponding Lorentz-equation for motion of bound-electrons, which can be made to yield a relation similar to the equation (Z) deduced for the ionosphere. Readers interested in mathematical analysis will easily recognise the method as an application of the method of characteristics, used for discussions of wave-propagation by Hadamard, Debye, and others. So far as the writer is aware the method has not been applied to the microscopic equations of Lorentz, where however the extension does not seem to create any difficulty when the waves are looked upon as possible modes of oscillation of the stationary electro-magnetic field, generated by positive and negative charges, present in the medium.

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Studies in Lorentz Group

By
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1. The decomposition of any Lorentz transformation into two commutable factors is a well-known result, and forms a convenient starting point for the theory of spinors. Einstein arrived at the decomposition from consideration of the infinitesimal elements of the Lorentz group. The result is here obtained from purely algebraic consideration, which gives at the same time a slightly more general result inasmuch as the factorisation can be shown to hold for elements of the general orthogonal group C_4, with $\det C_4 = +1$. The Lorentz group L is really a sub-group of C_4 inasmuch as the co-efficients are restricted by certain reality conditions, for every matrix C in L has C_{rs} real ($r, s = 1, 2, 3$), while C_{4r}, C_{r4}, $r = 1, 2, 3$ are pure imaginaries, and C_{44} real.

If C is a matrix, the transposed matrix will be denoted by $\bar{C}$; a symmetric matrix for which $\bar{C} = C$ will be denoted by S. If $\bar{C}+C = 0$, the matrix is anti-symmetric and will be denoted by A.

A non-symmetric matrix $C = S+A$ while $\bar{C} = S-A$. We shall assume here the elements of a matrix to be complex numbers.

2. Prop. I. An anti-symmetric matrix A having six non-zero elements can be further decomposed into two component matrices P and Q, which have only three non-zero elements such that

$$\left.\begin{aligned} P_{12} = P_{34}, \quad & P_{23} = P_{14}, \quad & P_{31} = P_{24} \\ Q_{12} = -Q_{34}, \quad & Q_{23} = -Q_{14}, \quad & Q_{31} = -Q_{24} \end{aligned}\right\}. \qquad \ldots (1)$$

If

$$A = P+Q,$$

$$A_{rs} = P_{rs}+Q_{rs}$$

where

$$\left.\begin{aligned} P_{12} = \tfrac{1}{2}(A_{12}+A_{34}), \quad & P_{23} = \tfrac{1}{2}(A_{23}+A_{14}), \quad & P_{31} = \tfrac{1}{2}(A_{31}+A_{24}) \\ Q_{12} = \tfrac{1}{2}(A_{12}-A_{34}), \quad & Q_{23} = \tfrac{1}{2}(A_{23}-A_{14}), \quad & Q_{31} = \tfrac{1}{2}(A_{31}-A_{24}) \end{aligned}\right\}. \qquad \ldots (2)$$

If further we introduce certain basic matrices, defined by

$$P_1=\begin{vmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}, \quad P_2=\begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}, \quad P_3=\begin{vmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{vmatrix},$$

and

$$Q_1=\begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{vmatrix}, \quad Q_2=\begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix}, \quad Q_3=\begin{vmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{vmatrix},$$

in which the first set P have the properties of the above P matrices in (1), and the second set Q those of the Q matrices. We note that

$$P_1^2 = P_2^2 = P_3^2 = Q_1^2 = Q_2^2 = Q_3^2 = -E$$

$$P_1 = P_2P_3 = -P_3P_2, \quad P_2 = P_3P_1 = -P_1P_3, \quad P_3 = P_1P_2 = -P_2P_1$$

$$Q_1 = Q_2Q_3 = -Q_3Q_2, \quad Q_2 = Q_3Q_1 = -Q_1Q_3, \quad Q_3 = Q_1Q_2 = -Q_2Q_1,$$

$$P_r Q_s = Q_s P_r. \qquad \ldots \quad (3)$$

Any P matrix can then be represented as

$$P = a_1P_1+a_2P_2+a_3P_3, \qquad \ldots \quad (4)$$

and any Q matrix as

$$Q = b_1Q_1+b_2Q_2+b_3Q_3. \qquad \ldots \quad (5)$$

Hence

$$A = a_1P_1+a_2P_2+a_3P_3+b_1Q_1+b_2Q_2+b_3Q_3 \qquad \ldots \quad (6)$$

Carrying out the multiplication of all P_i's and Q_i's ($i = 1, 2, 3$), we get in all sixteen matrices which we can utilise as basic matrices for the representation of any arbitrary matrix C thus (the indices of Q will be raised in what follows when Q's appear with summations)

$$C = C_0^0 E+\Sigma C_0^r P_r+\Sigma C_r^0 Q^r+\Sigma C_\mu^\lambda P_\lambda Q^\mu \qquad \ldots \quad (7)$$

$$\text{spur of } C = 4C_0^0.$$

Any symmetric matrix

$$S = S_0E+\sum_{r,s} S_s^r P_r Q^s. \qquad \ldots \quad (7')$$

Since $P_i^2 = -E$, we have generally

$$P^2 = -(a_1^2+a_2^2+a_3^2)E. \qquad \ldots \quad (8)$$

Prop. II. If S is any orthogonal matrix, *i.e.*, $S^2 = E$, det $S \neq 0$, then, either (*i*) det $S = -1$, spur $S \neq 0$, or (*ii*) det $S = +1$, spur $S = 0$.

We take a symmetric matrix

$$S = \begin{vmatrix} a & h & g & x \\ h & b & f & y \\ g & f & c & z \\ x & y & z & d \end{vmatrix}$$

Since $S^2 = E$, $S = S^{-1}$, and as det $S \neq 0$, the inverse can be written down by utilising the minors of the det S from which the following results follow easily.

If $s = \det S$, and A, B, C, etc. are minors of the determinant

$$D = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

then

$$\begin{aligned} s(ad-x^2) &= A \quad s(fd-yz) = F \quad BC-F^2 = aD \\ s(bd-y^2) &= B \quad s(gd-zx) = G \quad \text{etc.} \quad \text{etc.} \\ s(cd-z^2) &= C \quad s(hd-xy) = H \quad ,, \quad ,, \end{aligned}$$

We also remember the usual orthogonality relations of the type

$$\begin{aligned} ax+hy+gz+xd &= 0, \quad \text{etc.} \\ a^2+h^2+g^2+x^2 &= 1, \quad \text{etc.} \end{aligned}$$

So we easily deduce

$$\begin{aligned} Ax+Hy+Gz+sx &= 0 \\ Hx+By+Fz+sy &= 0 \\ Gx+Fy+Cz+sz &= 0, \end{aligned}$$

so that, if x, y, z be not all zero, we have by elimination

$$s^3+s^2(A+B+C)+sD(a+b+c)+D^2 = 0,$$

or, as $s^2 \neq 0$, and $D = sd$

$$(s+A+B+C) = -d(a+b+c+d).$$

Again

$$\begin{aligned} A+B+C &= s[d(a+b+c)-x^2-y^2-z^2] \\ &= s[d(a+b+c+d)-1]; \end{aligned}$$

so that

$$(A+B+C+s) = sd(a+b+c+d).$$

Hence either $s = -1$, or $a+b+c+d = 0$, or $d = 0$.

The special cases

$$d = 0 \quad \text{and} \quad x = y = z = 0$$

can be treated in the same way, and we will have in every case $s = -1$ or spur $S = 0$, which proves our general result.

Prop. III. If $S^2 = E$ and spur $S = 0$ (so that $s = +1$), then S is always decomposable into two commutative anti-symmetric factors.

As spur $S = 0$, we can write by (7)

$$S = \Sigma S^{\lambda}_{\mu} P_{\lambda} Q^{\mu}.$$

Now putting

$$S^{\lambda}_{\mu} P_{\lambda} = \Delta_{\mu} \qquad \ldots \quad (9)$$

where Δ_{μ} are certain matrices of the P class, we have

$$S = \Delta_1 Q_1 + \Delta_2 Q_2 + \Delta_3 Q_3. \qquad \ldots \quad (10)$$

Again as

$$S^2 = E = \sum_r \Delta_r^2 + Q_1(\Delta_2\Delta_3 - \Delta_3\Delta_2) + Q_2(\Delta_3\Delta_1 - \Delta_1\Delta_3) + Q_3(\Delta_1\Delta_2 - \Delta_2\Delta_1),$$

we must have

$$\begin{aligned} \Delta_2\Delta_3 &= \Delta_3\Delta_2 \\ \Delta_3\Delta_1 &= \Delta_1\Delta_3 \\ \Delta_1\Delta_2 &= \Delta_2\Delta_1. \end{aligned} \qquad \ldots \quad (11)$$

Hence the Δ matrices are *commutable*.

It follows easily, as the Δ's are all of the same P class,

$$\Delta_1 : \Delta_2 : \Delta_3 = k_1 : k_2 : k_3$$

where k_1, k_2, k_3 are certain numbers:

hence

$$\frac{\Delta_1}{k_1} = \frac{\Delta_2}{k_2} = \frac{\Delta_3}{k_3} = (l_1 P_1 + l_2 P_2 + l_3 P_3) \qquad \ldots \quad (12)$$

so that

$$S = (l_1 P_1 + l_2 P_2 + l_2 P_3)(k_1 Q_1 + k_2 Q_2 + k_3 Q_3) \qquad \ldots \quad (13)$$

which puts the symmetric orthogonal matrix with zero spur into two commutative anti-symmetric factors.

Also

$$\det S = |\det (l_1 P_1 + l_2 P_2 + l_3 P_3)| \; |\det (k_1 Q_1 + k_2 Q_2 + k_3 Q_3)|$$

whence

$$1 = (l_1^2+l_2^2+l_3^2)^2(k_1^2+k_2^2+k_3^2)^2. \quad \ldots \quad (14)$$

When the elements of S are given, we can determine the elements of the factor matrices in the following way.

As

$$S = \begin{vmatrix} a & h & g & x \\ h & b & f & y \\ g & f & c & z \\ x & y & z & d \end{vmatrix}, \text{ and } \quad a+b+c+d = 0,$$

we can put

$$S = \Delta_1 Q_1+\Delta_2 Q_2+\Delta_3 Q_3.$$

Then

$$\left.\begin{aligned} SQ_1 &= -\Delta_1+(\Delta_3 Q_2-\Delta_2 Q_3) \\ SQ_2 &= -\Delta_2+(\Delta_1 Q_3-\Delta_3 Q_1), \text{ etc.} \end{aligned}\right\} \quad \ldots \quad (15)$$

Hence $\Delta_1, \Delta_2, \Delta_3$ are obtained by separating the anti-symmetric parts of SQ_1, SQ_2 and SQ_3. These are expressible as follows :

$$\left.\begin{aligned} \Delta_1 &= \tfrac{1}{2}[(a+d)P_1+(h-z)P_2+(g+y)P_3] \\ \Delta_2 &= \tfrac{1}{2}[(h+z)P_1+(b+d)P_2+(f-x)P_3] \\ \Delta_3 &= \tfrac{1}{2}[(g-y)P_1+(f+x)P_2+(c+d)P_3]. \end{aligned}\right\} \quad \ldots \quad (16)$$

as

$$\frac{a+d}{2k_1} = \frac{h+z}{2k_2} = \frac{g-y}{2k_3} = l_1$$

$$\frac{h-z}{2k_1} = \frac{b+d}{2k_2} = \frac{f+x}{2k_3} = l_2$$

$$\frac{g+y}{2k_1} = \frac{f-x}{2k_2} = \frac{c+d}{2k_3} = l_3,$$

hence

$$a+d = 2k_1 l_1, \quad b+d = 2k_2 l_2, \quad c+d = 2k_3 l_3,$$

Also

$$a+b+c+d = 0,$$

so

$$\begin{aligned} d &= k_1 l_1+k_2 l_2+k_3 l_3 \\ a &= k_1 l_1-k_2 l_2-k_3 l_3, \text{ etc.} \\ h &= k_1 l_2+k_2 l_1, \\ z &= k_2 l_1-k_1 l_2. \end{aligned}$$

Hence,

$$S = \begin{vmatrix} k_1l_1-k_2l_2-k_3l_3, & k_1l_2+k_2l_1, & k_1l_3+k_3l_1, & k_3l_2-k_2l_3 \\ — , & k_2l_2-k_1l_1-k_3l_3, & k_2l_3+k_3l_2, & k_1l_3-k_3l_1 \\ — & — , & k_3l_3-k_1l_1-k_2l_2, & k_2l_1-k_1l_2 \\ — & — & — , & k_1l_1+k_2l_2+k_3l_3 \end{vmatrix}$$

$$= (k_1P_1+k_2P_2+k_3P_3)(l_1Q_1+l_2Q_2+l_3Q_3)\,; \qquad \ldots \quad (17)$$

where also

$$k_1^2+k_2^2+k_3^2 = +1,$$
$$l_1^2+l_2^2+l_3^2 = +1.$$

3. *Decomposition of Non-symmetric Matrices*

A general orthogonal matrix can be written as

$$C = S+A \qquad \ldots \quad (18)$$

$$C\overline{C} = \overline{C}C = E.$$

where

From this follows

$$SA-AS = 0, \quad S^2-A^2 = E. \qquad \ldots \quad (19)$$

Decomposing

$$A = P+Q,$$

where P and Q are linear combinations of P and Q type matrices, we have

$$S^2-P^2-Q^2-2PQ = E$$

i.e.,

$$S^2 = (1+P^2+Q^2)E+2PQ. \qquad \ldots \quad (20)$$

Now

$$\text{spur } C = \text{spur } S = \mu \text{ (say)},$$

hence by (7),

$$S = \mu E+\lambda PQ.$$

Again as

$$S^2 = \mu^2+\lambda^2P^2Q^2+2\mu\lambda PQ. \qquad \ldots \quad (21)$$

we have comparing (20) and (21), firstly

$$\lambda = \frac{1}{\mu}. \qquad \ldots \quad (22)$$

Thus

$$C = \mu E+P+Q+\frac{PQ}{\mu},$$

or $$\mu C = [\mu+Q][\mu+P]. \quad \ldots (23)$$

Hence if

$$\mu = k_1 k_2$$

$$\mu C = [k_1k_2+Q][k_1k_2+P]$$

$$= k_1k_2\left[k_2+\frac{Q}{k_1}\right]\left[k_1+\frac{P}{k_2}\right].$$

Identifying (20) with (21) we put

$$k_2^2-\frac{Q^2}{k_1^2} = 1, \quad k_1^2-\frac{P^2}{k_2^2} = 1,$$

$$\mu^2-Q^2 = k_1^2, \quad \mu^2-P^2 = k_2^2.$$

Hence

$$(\mu^2-Q^2)(\mu^2-P^2) = \mu^2$$

which relation is equivalent to $$\mu^2+\frac{Q^2P^2}{\mu^2} = 1+P^2+Q^2.$$

The numbers k_1 and k_2 are thus determined. Thus (23) which is the factorisation aimed at is completely established.

The general form of C can also be calculated.

If

$$C = AB$$

where

$$\left.\begin{aligned} A &= A_0+A_1P_1+A_2P_2+A_3P_3 \\ B &= B_0+B_1Q_1+B_2Q_2+B_3Q_3 \end{aligned}\right\} \quad \ldots (24)$$

and

$$A_0^2+A_1^2+A_2^2+A_3^2 = 1$$
$$B_0^2+B_1^2+B_2^2+B_3^2 = 1.$$

Explicitly,

$$A = \begin{vmatrix} A_0 & -A_3 & A_2 & -A_1 \\ A_3 & A_0 & -A_1 & -A_2 \\ -A_2 & A_1 & A_0 & -A_3 \\ A_1 & A_2 & A_3 & A_0 \end{vmatrix} \quad B = \begin{vmatrix} B_0 & -B_3 & B_2 & +B_1 \\ B_3 & B_0 & -B_1 & +B_2 \\ -B_2 & B_1 & B_0 & +B_3 \\ -B_1 & -B_2 & -B_3 & B_0 \end{vmatrix} \quad \ldots (25)$$

Multiplying we have

$$C = \begin{vmatrix} (A_0B_0+A_1B_1) & -(A_3B_0+A_0B_3) & +(A_2B_0+A_0B_2) & -(A_1B_0-A_0B_1) \\ -(A_2B_2+A_3B_3) & +(A_1B_2+A_2B_1) & +(A_1B_3+A_3B_1) & +(A_2B_3-A_3B_2) \\ (A_3B_0+A_0B_3) & (A_0B_0+A_2B_2) & -(A_0B_2+A_1B_0) & -(A_2B_0-A_0B_2) \\ +(A_1B_2+A_2B_1) & -(A_3B_3+A_1B_1) & +(A_2B_3+A_3B_2) & +(A_3B_1-A_1B_3) \\ -(A_2B_0+A_0B_2) & +(A_1B_0+A_0B_1) & (A_0B_0+A_3B_3) & -(A_3B_0-A_0B_3) \\ +(A_1B_3+A_3B_1) & (A_2B_3+A_3B_2) & -(A_1B_1+A_2B_2) & +(A_1B_2-A_2B_1) \\ (A_1B_0-A_0B_1) & +(A_2B_0-A_0B_2) & (A_3B_0-A_0B_3) & +(A_1B_1+A_2B_2) \\ +(A_2B_3-A_3B_2) & +(A_3B_1-A_1B_3) & +(A_1B_2-A_2B_1) & +(A_3B_3+A_0B_0) \end{vmatrix} \quad \ldots \quad (26)$$

Now

$$\det C = (A_0^2+A_1^2+A_2^2+A_3^2)^2(B_0^2+B_1^2+B_2^2+B_3^2)^2 = 1$$

and

$$\text{spur } C = 4A_0B_0. \quad \ldots \quad (27)$$

Two special forms of C can be at once noticed :

If C is a matric of the proper Lorentz group, then

$$C_{14},\ C_{24},\ C_{34},\ C_{41},\ C_{42},\ C_{43} \textit{ are pure imaginaries,}$$

so that

$$(A_2B_3-A_3B_2)+(A_1B_0-B_1A_0) = i\lambda,$$

$$(A_2B_3-A_3B_2)-(A_1B_0-B_1A_0) = i\lambda',$$

from which follows

$$(A_2B_3-A_3B_2) \text{ as well as } (A_1B_0-B_1A_0),$$

etc, are all pure imaginaries.

A_0B_0, A_1B_1, A_2B_2, A_3B_3 are real quantities as well as $(A_2B_3+A_3B_2)$, etc. and $(A_3B_0+A_0B_3)$, etc. are real.

We have then

$$\frac{A_0}{B_0^*} = \frac{A_1}{B_1^*} = \frac{A_2}{B_2^*} = \frac{A_3}{B_3^*} = k, \text{ real} \quad \ldots \quad (28)$$

as $A_0^2+A_1^2+A_2^2+A_3^2 = B_0^2+B_1^2+B_2^2+B_3^2 = +1$ (B_0^* representing the conjugate of B_0, etc.).

We thus see that the sets (A_0, A_1, A_2, A_3) and (B_0, B_1, B_2, B_3) *are conjugate to each other.*

There are now two cases possible.

Case I. When (A_0, A_1, A_2, A_3) are real, then

$$A_0 = B_0, \quad A_1 = B_1, \quad A_2 = B_2, \quad A_3 = B_3$$

and from (26) further

$$C_{14} = C_{24} = C_{34} = C_{41} = C_{42} = C_{43} = 0$$

$$C_{44} = 1.$$

This represents a three dimensional rotation.

The axis of rotation is easily seen to be

$$\frac{l}{A_1} = \frac{m}{A_2} = \frac{n}{A_3} \qquad \ldots (29)$$

The angle of rotation is given by

$$\tan \psi = \frac{2A_0 G}{A_0^2 - G^2}, \text{ where } G^2 = A_1^2 + A_2^2 + A_3^2.$$

Case II. A_1, A_2, A_3 are pure imaginaries, A_0 real.

The resultant transformation is here seen to be an Einstein transformation, with translation along the line

$$\frac{l}{A_1} = \frac{m}{A_2} = \frac{n}{A_3}$$

and velocity given by

$$\frac{V}{c} = \frac{2A_0 G}{A_0^2 + G^2}$$

where

$$A_0^2 - G^2 = 1, \quad \text{and} \quad G^2 = A_1^2 + A_2^2 + A_3^2.$$

4.

We shall now consider certain general matrices of the form

$$\left.\begin{aligned} A &= \mu E + aP_1 + bP_2 + cP_3 \\ \text{and} \qquad B &= \mu E + aQ_1 + bQ_2 + cQ_3. \end{aligned}\right\} \qquad \ldots (30)$$

We can easily see that

$$|\det A| = [\mu^2 + a^2 + b^2 + c^2]^2$$

$$|\det B| = [\mu^2 + a^2 + b^2 + c^2]^2.$$

So when a, b, c are complex quantities and $|\det A|$ and $|\det B|$ real and positive we must have

$$\mu^2+a^2+b^2+c^2 = \mu^{*2}+a^{*2}+b^{*2}+c^{*2}$$

where μ^*, etc. are conjugates of μ, etc.

We now consider A and B to be factors of decomposition of C, and take the determinant of each of the factors to be $+1$, so that

$$\mu^2+a^2+b^2+c^2 = \pm 1.$$

The matrices of A and B types are orthogonal matrices; but since their elements are generally complex, they do not belong to the Lorentz sub-group of Group C_4.

We will now discuss certain general characteristics of the A and B matrices, which will bring into light also the intimate relation that exists between the decompositions as carried out here and the spinor theory.

Prop. All matrices of A and B types are reducible :

$$A = \left|\begin{array}{cc:cc} p_0 & -p_3 & p_2 & -p_1 \\ p_3 & p_0 & -p_1 & -p_2 \\ \hdashline -p_2 & p_1 & p_0 & -p_3 \\ p_1 & p_2 & p_3 & p_0 \end{array}\right| = \left|\begin{array}{cc} S & -Q \\ Q & S \end{array}\right| \qquad \ldots \quad (31)$$

where S and Q are certain matrices of the second order. That such matrices are reducible is a well-known result.

Let

$$\psi = A\phi$$

define a transformation of a four-component ϕ to a four component ψ.

We have

$$\begin{aligned} \psi_1 &= p_0\phi_1 - p_3\phi_2 + p_2\phi_3 - p_1\phi_4 \\ \psi_2 &= p_3\phi_1 + p_0\phi_2 - p_1\phi_3 - p_2\phi_4 \\ \psi_3 &= -p_2\phi_1 + p_1\phi_2 + p_0\phi_3 - p_3\phi_4 \\ \psi_4 &= p_1\phi_1 + p_2\phi_2 + p_3\phi_3 + p_0\phi_4. \end{aligned} \qquad \ldots \quad (32)$$

From this it can be easily deduced that

$$\left|\begin{array}{c} \psi_1+i\psi_2 \\ \psi_3-i\psi_4 \end{array}\right| = \left|\begin{array}{cc} p_0+ip_3 & p_2-ip_1 \\ -p_2-ip_1 & p_0-ip_3 \end{array}\right| \left|\begin{array}{c} \phi_1+i\phi_2 \\ \phi_3-i\phi_4 \end{array}\right| \qquad \ldots \quad (33)$$

and there is also the corresponding conjugate relation obtained by putting $-i$ for i, (33) can also be written as

$$\begin{vmatrix} \psi_3+i\psi_4 \\ -\psi_1+i\psi_2 \end{vmatrix} = \begin{vmatrix} p_0+ip_3 & p_2-ip_1 \\ -p_2-ip_1 & p_0-ip_3 \end{vmatrix} \begin{vmatrix} \phi_3+i\phi_4 \\ i\phi_2-\phi_1 \end{vmatrix} \quad \ldots \quad (34)$$

Combining (33) and (34) we obtain

$$\begin{vmatrix} \psi_3+i\psi_4 & \psi_1+i\psi_2 \\ i\psi_2-\psi_1 & \psi_3-i\psi_4 \end{vmatrix} = \begin{vmatrix} p_0+ip_3 & p_2-ip_1 \\ -p_2-ip_1 & p_0-ip_3 \end{vmatrix} \begin{vmatrix} \phi_3+i\phi_4 & \phi_1+i\phi_2 \\ i\phi_2-\phi_1 & \phi_3-i\phi_4 \end{vmatrix} \quad \ldots \quad (35)$$

which is a well-known spinor transformation.

Again, choosing two sets of variables as (x_1, x_2, x_3, x_4) and (y_1, y_2, y_3, y_4) the corresponding transformation formula for B matrices can be written as

$$\begin{vmatrix} y_3+iy_4 & -y_1+iy_2 \\ y_1+iy_2 & y_3-iy_4 \end{vmatrix} = \begin{vmatrix} q_0-iq_3 & -q_2-iq_1 \\ q_2-iq_1 & q_0+iq_3 \end{vmatrix} \begin{vmatrix} x_3+ix_4 & -x_1+ix_2 \\ x_1+ix_2 & x_3-ix_4 \end{vmatrix} \quad \ldots \quad (36)$$

which can be also written in the form

$$\begin{vmatrix} y_3+iy_4 & y_1+iy_2 \\ -y_1+iy_2 & y_3-iy_4 \end{vmatrix} = \begin{vmatrix} x_3+ix_4 & x_1+ix_2 \\ -x_1+ix_2 & x_3-ix_4 \end{vmatrix} \begin{vmatrix} q_0-iq_3 & q_2-iq_1 \\ -q_2-iq_1 & q_0+iq_3 \end{vmatrix}. \quad \ldots \quad (37)$$

(35) can also be written in this notation with the transformation matrix to the left on the right-hand side.

Thus A can be regarded as inducing a front transformation while B induces a back transformation, or *vice-versa*, which shows the commutative nature of the transformation at once.

The general reducibility of the A and B types of matrices shows that the Lorentz Group can be studied advantageously in the two-dimensional representation of the A and B class of transformation.

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THE COMPLETE SOLUTION OF THE EQUATION:

$$\nabla^2\phi - \frac{\partial^2\phi}{c^2\,\partial t^2} - k^2\phi = -4\pi\rho(xyzt).$$

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1. In what follows we desire to present what appears to us to be a complete Kirchhoff-like solution of the equation above and indeed by two distinct methods. The one is a method of complex integration and the other an extension of a method adopted by Love * for ordinary retarded potentials. The equation itself is one which seems to have acquired some importance in view of a recent work of J. H. Bhabha † on the mesotron, who has stated a solution of the equation, in which the part involving a surface integral does not appear. To the best of our knowledge and belief the solution at which we arrive and which exhibits the surface integral is original. As the whole work, however, was started by a successful derivation, by an adaptation of the well-known method of Herglotz and Sommerfeld ‡, of the potentials of a moving mesotron, such as Bhabha uses, we let this derivation precede the main investigation, since this way of arriving at the potentials appears to us to be also new and of sufficient interest.

2. To start with, we seek a solution of the equation

$$\nabla^2\psi - \frac{\partial^2\psi}{c^2\,\partial\theta^2} - k^2\psi = 0 \qquad .. \qquad .. \qquad .. \qquad (1)$$

of the form: $\psi = \psi(s)$,

where $$s^2 = c^2(\theta-t)^2 - \Sigma(\xi-x)^2 = c^2(\theta-t)^2 - r^2 \qquad .. \qquad .. \qquad (2)$$

and x, y, z, t are parameters. The transformation of eq. (1) to the single variable s leads to the equation

$$\frac{d^2\psi}{ds^2} + \frac{3}{s}\frac{d\psi}{ds} + k^2\psi = 0$$

or $$\frac{d^2}{ds^2}(s\psi) + \frac{1}{s}\frac{d}{ds}(s\psi) + \left(k^2 - \frac{1}{s^2}\right)(s\psi) = 0\,. \qquad .. \qquad .. \qquad (3)$$

Hence we infer: $\psi = \dfrac{J_1(ks)}{s}$ or $\dfrac{Y_1(ks)}{s}$, where $J_1(ks)$ and $Y_1(ks)$ are Bessel functions of the first and second kinds respectively.

3. *Retarded potentials of a moving mesotron.*

We put $\chi = \dfrac{kY_1(ks)}{s}$

$$= \frac{kJ_1(ks)}{s} ln(ks) - \frac{k}{s} \sum_{m=0}^{\infty} (-1)^m \frac{2m+1}{m(m+1)} J_{2m+1}(ks) - \frac{J_0(ks)}{s^2} \S \quad .. \quad (4)$$

and choose for the four potentials (ϕ, A_1, A_2, A_3) of the mesotron solutions of eq. (1), stated as an equation in x, y, z, t, with ξ, η, ζ, θ as parameters, in the forms

$$\phi = \int_{\theta_1}^{\theta_2} d\theta f_4(\theta)\chi, \quad A_1 = \int_{\theta_1}^{\theta_2} d\theta f_1(\theta)\chi, \text{ etc.} \quad .. \quad (5)$$

The equation of restraint on the four potentials becomes

$$0 = \frac{\partial \phi}{c\,\partial t} + \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

$$= -\int_{\theta_1}^{\theta_2} d\theta \left[f_4 \frac{\partial}{c\,\partial \theta} + f_1 \frac{\partial}{\partial \xi} + f_2 \frac{\partial}{\partial \eta} + f_3 \frac{\partial}{\partial \zeta} \right] \chi$$

$$= -B \int_{\theta_1}^{\theta_2} d\theta \left[\frac{\partial}{\partial \theta} + \frac{d\xi}{d\theta} \frac{\partial}{\partial \xi} + \frac{d\eta}{d\theta} \frac{\partial}{\partial \eta} + \frac{d\zeta}{d\theta} \frac{\partial}{\partial \zeta} \right] \chi$$

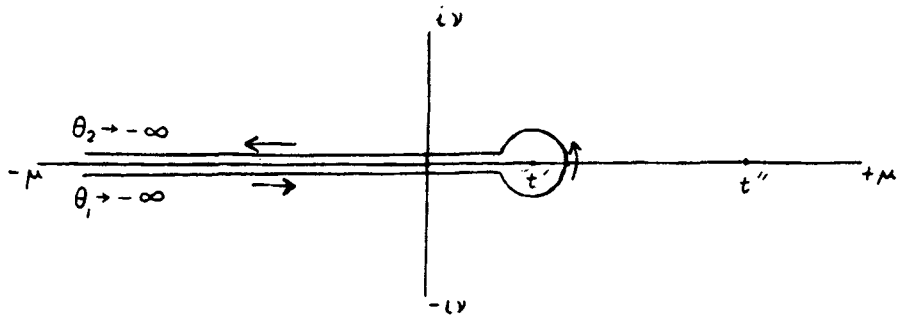
$$= -B \int_{\theta_1}^{\theta_2} d\theta \frac{d\chi}{d\theta} \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (6)$$

if, after Herglotz and Sommerfeld, we treat ξ, η, ζ as functions of θ and put

$$(f_4, f_1, f_2, f_3) = B\left(c, \frac{d\xi}{d\theta}, \frac{d\eta}{d\theta}, \frac{d\zeta}{d\theta}\right), \quad .. \quad .. \quad (7)$$

where B is an absolute constant.

We assume now θ to have been complex of the form $\mu + i\nu$ and proceed to perform the integrations in eqs. (5) along a path on the complex plane shown in the diagram below.



$$\nabla^2\phi - \frac{\partial^2\phi}{c^2\,\partial t^2} - k^2\phi = -4\pi\rho(xyzt). \qquad 95$$

The following considerations govern the choice of this path. Since, from eq. (2),

$$s^2 = c^2(\theta - t')(\theta - t''), \qquad .. \quad .. \quad .. \quad (8)$$

where $t' = t - \frac{r}{c}$ and $t'' = t + \frac{r}{c}$, the only singularities of χ are those at $\theta = t'$ and $\theta = t''$ and since we are seeking, for the moment, the retarded potentials we let the path embrace just the one singularity at $\theta = t'$. Since, further, lns in χ is multiple-valued we cannot satisfy the equation of constraint (6) by just having a closed contour about t' but have to let the path run along the real axis from $-\infty$ on one Riemann surface round t' to $-\infty$ on a second, as, with $\theta \to -\infty$, $\chi \to 0$.

Retaining only such terms in χ as involve a singularity we may write eqs. (5) as

$$\phi = Bc \int_{\theta_1}^{\theta_2} d\theta \left[\frac{kJ_1(ks)}{s} lns - \frac{1}{s^2}\right],$$

$$A_1 = B \int_{\theta_1}^{\theta_2} d\theta \left[\frac{kJ_1(ks)}{s} lns - \frac{1}{s^2}\right] \frac{d\xi}{d\theta},$$

etc.,

and evaluate the integrals as follows:

$$\int_{\theta_1}^{\theta_2} d\theta \frac{1}{s^2}\frac{d\xi}{d\theta} = \oint d\theta \frac{1}{s^2}\frac{d\xi}{d\theta} = \frac{2\pi i}{\left[\frac{d(s^2)}{d\theta}\right]_{t'}} \cdot \frac{dx'}{dt'} = -\frac{\pi i}{cr\left(1 - \frac{v_r}{c}\right)} v_x, \text{ etc.},$$

$$\int_{\theta_1}^{\theta_2} d\theta \frac{J_1(ks)}{s}\frac{d\xi}{d\theta} lns = \frac{1}{2}\int_{\theta_1}^{\theta_2} d\theta \frac{J_1(ks)}{s}\frac{d\xi}{d\theta} ln(\theta - t')$$

$$= -\pi i \int_{-\infty}^{t'} d\theta \frac{J_1(ks)}{s}\frac{d\xi}{d\theta}, \text{ etc.}$$

We identify now $B\pi i$ with the charge e of the mesotron and obtain as its potentials

$$\left.\begin{aligned} \phi &= \frac{e}{r\left(1 - \frac{v_r}{c}\right)} - eck \int_{-\infty}^{t'} d\theta \frac{J_1(ks)}{s}, \\ (A_1, A_2, A_3) &= \frac{e\vec{v}}{cr\left(1 - \frac{v_r}{c}\right)} - ek \int_{-\infty}^{t'} d\theta \frac{J_1(ks)}{s}\left(\frac{d\xi}{d\theta}, \frac{d\eta}{d\theta}, \frac{d\zeta}{d\theta}\right), \end{aligned}\right\} \quad .. \quad (9)$$

which are the forms used by Bhabha.

In concluding this section we remark that the advanced potentials of the mesotron, also used by Bhabha, may be obtained in like manner by the choice of a path of integration made to run along the real axis from $+\infty$ on one Riemann surface round the other singularity at $\theta = t''$ back to $+\infty$ on a second.

4. *Solution of the proposed equation for retarded values of t : First method.* We write

$$\nabla^2\phi - \frac{\partial^2\phi}{c^2\,\partial\theta^2} - k^2\phi = -4\pi\rho(\xi\eta\zeta\theta) \qquad .. \quad .. \quad (10)$$

and

$$\nabla^2\chi - \frac{\partial^2\chi}{c^2\,\partial\theta^2} - k^2\chi = 0,$$

where χ is given by (4). Hence we obtain firstly,

$$\sum \frac{\partial}{\partial\xi}\left(\chi\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial\chi}{\partial\xi}\right) - \frac{\partial}{c\,\partial\theta}\left(\chi\frac{\partial\phi}{c\,\partial\theta} - \phi\frac{\partial\chi}{c\,\partial\theta}\right) = -4\pi\chi\rho(\xi\eta\zeta\theta)$$

and then, upon integration with respect to θ,

$$\int_{\theta_1}^{\theta_2} d\theta \sum \frac{\partial}{\partial\xi}\left(\chi\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial\chi}{\partial\xi}\right) - \frac{1}{c}\left[\chi\frac{\partial\phi}{c\,\partial\theta} - \phi\frac{\partial\chi}{c\,\partial\theta}\right]_{\theta_1}^{\theta_2} = -4\pi\int_{\theta_1}^{\theta_2} d\theta\,\chi\rho(\xi\eta\zeta\theta).$$

We assume now θ to have been complex of the form $\mu + i\nu$ and proceed to perform the integrations along the same path on the complex plane as we have used in the previous section. Since, with $\theta \to -\infty$, both χ and $\frac{\partial\chi}{\partial\theta} \to 0$, we have

$$\int_{\theta_1}^{\theta_2} d\theta \sum \frac{\partial}{\partial\xi}\left(\chi\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial\chi}{\partial\xi}\right) = -4\pi\int_{\theta_1}^{\theta_2} d\theta\,\chi\rho(\xi\eta\zeta\theta).$$

This we may write again in the form

$$\sum \frac{\partial}{\partial\xi}\int_{\theta_1}^{\theta_2} d\theta\left(\chi\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial\chi}{\partial\xi}\right) = -4\pi\int_{\theta_1}^{\theta_2} d\theta\,\chi\rho(\xi\eta\zeta\theta), \qquad .. \quad (11)$$

if we observe that we may treat the path of integration as independent of ξ, η, ζ, since the path about the singularity at t' may always be slightly varied to accommodate the shift of t' involved in the operations, $\frac{\partial}{\partial\xi}$, etc.

We set down now in brief the necessary calculations.

(i) $$\int_{\theta_1}^{\theta_2} d\theta\,\chi\rho(\xi\eta\zeta\theta) = \int_{\theta_1}^{\theta_2} d\theta\left[\frac{kJ_1(ks)}{s}\,ins - \frac{1}{s^2}\right]\rho(\xi\eta\zeta\theta)$$

$$= -\pi ik\int_{-\infty}^{t'} d\theta\,\frac{J_1(ks)}{s}\,\rho(\xi\eta\zeta\theta) + \frac{\pi i}{cr}\,\rho(\xi\eta\zeta t').$$

$$\nabla^2\phi - \frac{\partial^2\phi}{c^2\,\partial t^2} - k^2\phi = -4\pi\rho(xyzt).$$

(ii) $$\int_{\theta_1}^{\theta_2} d\theta\,\chi\frac{\partial\phi}{\partial\xi} = \int_{\theta_1}^{\theta_2} d\theta\left[\frac{kJ_1(ks)}{s}\,lns - \frac{1}{s^2}\right]\frac{\partial\phi}{\partial\xi}$$

$$= -\pi ik\int_{-\infty}^{t'} d\theta\,\frac{J_1(ks)}{s}\,\frac{\partial\phi}{\partial\xi} + \frac{\pi i}{cr}\left(\frac{\partial\phi}{\partial\xi}\right)_{t'}.$$

(iii) $$\int_{\theta_1}^{\theta_2} d\theta\,\phi\frac{\partial\chi}{\partial\xi} = -(\xi - x)\int_{\theta_1}^{\theta_2} d\theta\,\phi\,\frac{1}{s}\,\frac{d\chi}{ds}$$

$$= -(\xi - x)\int_{\theta_1}^{\theta_2} d\theta\,\phi\,\frac{1}{s}\,\frac{d}{ds}\left[\frac{kJ_1(ks)}{s}\,lns - \frac{J_0(ks)}{s^2}\right]$$

$$= -(\xi - x)\int_{\theta_1}^{\theta_2} d\theta\,\phi\left[k\frac{d}{ds}\left(\frac{J_1(ks)}{s}\right)\frac{lns}{s} + \frac{2kJ_1(ks)}{s^3} + \frac{2J_0(ks)}{s^4}\right]$$

$$= -(\xi - x)\left[k\int_{\theta_1}^{\theta_2} d\theta\,\phi\,\frac{d}{ds}\left(\frac{J_1(ks)}{s}\right)\frac{lns}{s} + \frac{k^2}{2}\int_{\theta_1}^{\theta_2} d\theta\,\frac{\phi}{s^2} + 2\int_{\theta_1}^{\theta_2} d\theta\,\frac{\phi}{s^4}\right]$$

Replacing $\frac{1}{s^4}$ by $\frac{1}{c^4(t''-t')^2}\left[\frac{1}{(\theta-t')^2} - \frac{2}{(\theta-t')(\theta-t'')} + \frac{1}{(\theta-t'')^2}\right]$

we have

$$\int_{\theta_1}^{\theta_2} d\theta\,\phi\frac{\partial\chi}{\partial\xi}$$

$$= -(\xi - x)\left[-\pi ik\int_{-\infty}^{t'} d\theta\,\phi\,\frac{d}{ds}\left(\frac{J_1(ks)}{s}\right)\frac{1}{s} - \frac{k^2\pi i}{2cr}(\phi)_{t'} + \frac{\pi i}{cr^2}\left(\frac{\partial\phi}{c\,\partial\theta}\right)_{t'} + \frac{\pi i}{cr^3}(\phi)_{t'}\right]$$

$$= -\pi ik\int_{-\infty}^{t'} d\theta\,\phi\,\frac{\partial}{\partial\xi}\left(\frac{J_1(ks)}{s}\right) + \frac{k^2\pi i}{2c}(\phi)_{t'}\frac{\partial r}{\partial\xi} - \frac{\pi i}{cr}\frac{\partial r}{\partial\xi}\left(\frac{\partial\phi}{c\,\partial\theta}\right)_{t'} + \frac{\pi i}{c}(\phi)_{t'}\frac{\partial}{\partial\xi}\left(\frac{1}{r}\right).$$

Substituting these results in eq. (11) and dropping the factor $\frac{\pi i}{c}$ we get

$$-4\pi\left[\frac{\rho(\xi\eta\zeta t')}{r}-ck\int_{-\infty}^{t'}d\theta\,\frac{J_1(ks)}{s}\rho(\xi\eta\zeta\theta)\right]$$

$$=\sum\frac{\partial}{\partial\xi}\left[\frac{1}{r}\left(\frac{\partial\phi}{\partial\xi}\right)_{t'}+\frac{1}{r}\frac{\partial r}{\partial\xi}\left(\frac{\partial\phi}{c\,\partial\theta}\right)_{t'}-(\phi)_{t'}\frac{\partial}{\partial\xi}\left(\frac{1}{r}\right)-ck\int_{-\infty}^{t'}d\theta\,\frac{J_1(ks)}{s}\frac{\partial\phi}{\partial\xi}\right.$$

$$\left.+ck\int_{-\infty}^{t'}d\theta\,\phi\frac{\partial}{\partial\xi}\left(\frac{J_1(ks)}{s}\right)-\frac{k^2}{2}(\phi)_{t'}\frac{\partial r}{\partial\xi}\right]. \quad \ldots \quad (12)$$

We integrate this equation now further through a closed volume about the point P (x, y, z) but exclude the volume of a small sphere about that point. We obtain thus

$$-4\pi\iiint d\xi\,d\eta\,d\zeta\left[\frac{\rho(\xi\eta\zeta t')}{r}-ck\int_{-\infty}^{t'}d\theta\,\frac{J_1(ks)}{s}\rho(\xi\eta\zeta\theta)\right]$$

$$=\iint dS\left[\frac{1}{r}\left(\frac{\partial\phi}{\partial n}\right)_{t'}+\frac{1}{r}\frac{\partial r}{\partial n}\left(\frac{\partial\phi}{c\,\partial\theta}\right)_{t'}-(\phi)_{t'}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right.$$

$$\left.-ck\int_{-\infty}^{t'}d\theta\,\frac{J_1(ks)}{s}\frac{\partial\phi}{\partial n}+ck\int_{-\infty}^{t'}d\theta\,\phi\frac{\partial}{\partial n}\left(\frac{J_1(ks)}{s}\right)-\frac{k^2}{2}(\phi)_{t'}\frac{\partial r}{\partial n}\right]$$

$$+\iint dS'\left[\frac{1}{r}\left(\frac{\partial\phi}{\partial n}\right)_{t'}+\frac{1}{r}\frac{\partial r}{\partial n}\left(\frac{\partial\phi}{c\,\partial\theta}\right)_{t'}-(\phi)_{t'}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right.$$

$$\left.-ck\int_{-\infty}^{t'}d\theta\,\frac{J_1(ks)}{s}\frac{\partial\phi}{\partial n}+ck\int_{-\infty}^{t'}d\theta\,\phi\frac{\partial}{\partial n}\left(\frac{J_1(ks)}{s}\right)-\frac{k^2}{2}(\phi)_{t'}\frac{\partial r}{\partial n}\right],$$

where the second surface integral is over the surface of the small sphere about P. Its limiting value, with $r\to 0$, is seen to be

$$-\phi_{P,t}\iint dS'\frac{\partial}{\partial n}\left(\frac{1}{r}\right)=-4\pi\phi_{P,t}.$$

$$\nabla^2\phi - \frac{\partial^2\phi}{c^2\,\partial t^2} - k^2\phi = -4\pi\rho(xyzt).$$

We arrive hence at the complete Kirchhoff-like solution for retarded values of t in the form

$$\phi(xyzt) = \iiint d\xi\,d\eta\,d\zeta\left[\frac{\rho(\xi\eta\zeta t')}{r} - ck\int_{-\infty}^{t'} d\theta\,\frac{J_1(ks)}{s}\rho(\xi\eta\zeta\theta)\right]$$

$$+\frac{1}{4\pi}\iint dS\left[\frac{1}{r}\left(\frac{\partial\phi}{\partial n}\right)_{t'} + \frac{1}{r}\frac{\partial r}{\partial n}\left(\frac{\partial\phi}{c\,\partial\theta}\right)_{t'} - (\phi)_{t'}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right.$$

$$\left. -ck\int_{-\infty}^{t'} d\theta\left\{\frac{J_1(ks)}{s}\frac{\partial\phi}{\partial n} - \phi\frac{\partial}{\partial n}\left(\frac{J_1(ks)}{s}\right)\right\} - \frac{k^2}{2}(\phi)_{t'}\frac{\partial r}{\partial n}\right]. \quad .. \quad (13)$$

In closing this section we remark that a like procedure with the second path of integration pointed out in the previous section yields the complete Kirchhoff-like solution for advanced values of t in the form

$$\phi(xyzt) = \iiint d\xi\,d\eta\,d\zeta\left[\frac{\rho(\xi\eta\zeta t'')}{r} - ck\int_{t''}^{\infty} d\theta\,\frac{J_1(ks)}{s}\rho(\xi\eta\zeta\theta)\right]$$

$$+\frac{1}{4\pi}\iint dS\left[\frac{1}{r}\left(\frac{\partial\phi}{\partial n}\right)_{t''} - \frac{1}{r}\frac{\partial r}{\partial n}\left(\frac{\partial\phi}{c\,\partial\theta}\right)_{t''} - (\phi)_{t''}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right.$$

$$\left. -ck\int_{t''}^{\infty} d\theta\left\{\frac{J_1(ks)}{s}\frac{\partial\phi}{\partial n} - \phi\frac{\partial}{\partial n}\left(\frac{J_1(ks)}{s}\right)\right\} - \frac{k^2}{2}(\phi)_{t''}\frac{\partial r}{\partial n}\right]. \quad .. \quad (14)$$

5. *Solution of the proposed equation for retarded values of t : Second method.*

To begin with, we replace θ by $t - \frac{r}{c}\,(= t')$ in eq. (10). With the notation

$$\left.\begin{aligned}\rho\left(\xi, \eta, \zeta, t - \frac{r}{c}\right) &= [\rho],\\ \phi\left(\xi, \eta, \zeta, t - \frac{r}{c}\right) &= [\phi],\end{aligned}\right\} \quad .. \quad .. \quad .. \quad (15)$$

etc.,

equation (10) takes then the form

$$[\nabla^2\phi] - \left[\frac{\partial^2\phi}{c^2\,\partial\theta^2}\right] - k^2[\phi] = -4\pi[\rho]. \quad .. \quad .. \quad (16)$$

Now

$$\left.\begin{aligned}
\frac{\partial}{\partial\xi}[\phi] &= \left[\frac{\partial\phi}{\partial\xi}\right] - \left[\frac{\partial\phi}{\partial\theta}\right]\frac{1}{c}\frac{\partial r}{\partial\xi}, \\
\frac{\partial}{\partial t}[\phi] &= \left[\frac{\partial\phi}{\partial\theta}\right], \\
\frac{\partial^2}{\partial\xi^2}[\phi] &= \left[\frac{\partial^2\phi}{\partial\xi^2}\right] - 2\left[\frac{\partial^2\phi}{\partial\xi\,\partial\theta}\right]\frac{1}{c}\frac{\partial r}{\partial\xi} \\
&\qquad + \left[\frac{\partial^2\phi}{\partial\theta^2}\right]\left(\frac{1}{c}\frac{\partial r}{\partial\xi}\right)^2 - \left[\frac{\partial\phi}{\partial\theta}\right]\frac{1}{c}\frac{\partial^2 r}{\partial\xi^2}, \\
\frac{\partial^2}{\partial\xi\,\partial t}[\phi] &= \left[\frac{\partial^2\phi}{\partial\xi\,\partial\theta}\right] - \left[\frac{\partial^2\phi}{\partial\theta^2}\right]\frac{1}{c}\frac{\partial r}{\partial\xi}.
\end{aligned}\right\} \quad .. \quad (17)$$

Hence we deduce firstly

$$\frac{\partial^2}{\partial\xi^2}[\phi] + \frac{2}{c}\frac{\partial r}{\partial\xi}\frac{\partial^2}{\partial\xi\,\partial t}[\phi] + \frac{1}{c}\frac{\partial^2 r}{\partial\xi^2}\frac{\partial}{\partial t}[\phi] = \left[\frac{\partial^2\phi}{\partial\xi^2}\right] - \left(\frac{1}{c}\frac{\partial r}{\partial\xi}\right)^2\left[\frac{\partial^2\phi}{\partial\theta^2}\right]$$

and then

$$\nabla^2[\phi] + \frac{2}{c}\sum\frac{\partial r}{\partial\xi}\frac{\partial^2}{\partial\xi\,\partial t}[\phi] + \frac{2}{cr}\frac{\partial}{\partial t}[\phi] - k^2[\phi] = [\nabla^2\phi] - \left[\frac{\partial^2\phi}{c^2\,\partial\theta^2}\right] - k^2[\phi]$$

$$= -4\pi[\rho] \quad .. \quad .. \quad .. \quad (18)$$

in view of eq. (16). Dividing both sides of eq. (18) by r, we have, since $\nabla^2\frac{1}{r} = 0$,

$$\sum\frac{\partial}{\partial\xi}\left\{\frac{1}{r}\frac{\partial}{\partial\xi}[\phi] - [\phi]\frac{\partial}{\partial\xi}\left(\frac{1}{r}\right) + \frac{2}{c}\frac{1}{r}\frac{\partial r}{\partial\xi}\frac{\partial}{\partial t}[\phi]\right\} - \frac{k^2[\phi]}{r} = -4\pi\frac{[\rho]}{r}. \quad .. \quad (A)$$

We next proceed to combine eq. (10) with

$$\nabla^2 Q - \frac{\partial^2 Q}{c^2\,\partial\theta^2} - k^2 Q = 0,$$

where

$$Q = \frac{J_1(ks)}{s},$$

and obtain firstly

$$\sum\frac{\partial}{\partial\xi}\left(Q\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial Q}{\partial\xi}\right) - \frac{\partial}{c\,\partial\theta}\left(Q\frac{\partial\phi}{c\,\partial\theta} - \phi\frac{\partial Q}{c\,\partial\theta}\right) = -4\pi Q\rho(\xi\eta\zeta\theta)$$

and then, since both Q and $\frac{\partial Q}{\partial\theta} \to 0$ with $\theta \to -\infty$,

$$\sum\int_{-\infty}^{t'} d\theta\,\frac{\partial}{\partial\xi}\left(Q\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial Q}{\partial\xi}\right) - \frac{1}{c}\left[Q\frac{\partial\phi}{c\,\partial\theta} - \phi\frac{\partial Q}{c\,\partial\theta}\right] = -4\pi\int_{-\infty}^{t'} d\theta\,Q\rho(\xi\eta\zeta\theta)$$

$$\nabla^2\phi - \frac{\partial^2\phi}{c^2\,\partial t^2} - k^2\phi = -4\pi\rho(xyzt).$$

$$\text{or } \sum \frac{\partial}{\partial\xi}\int_{-\infty}^{t'} d\theta\left(Q\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial Q}{\partial\xi}\right) + \sum\left[Q\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial Q}{\partial\xi}\right]\frac{1}{c}\frac{\partial r}{\partial\xi} - \frac{1}{c}\left[Q\frac{\partial\phi}{c\,\partial\theta} - \phi\frac{\partial Q}{c\,\partial\theta}\right]$$
$$= -4\pi\int_{-\infty}^{t'} d\theta\, Q\rho(\xi\eta\zeta\theta).$$

Rearranging the terms in this equation we write

$$\sum \frac{\partial}{\partial\xi}\int_{-\infty}^{t'} d\theta\left(Q\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial Q}{\partial\xi}\right) - \frac{[\phi]}{c}\left[\sum\frac{\partial Q}{\partial\xi}\frac{\partial r}{\partial\xi} - \frac{\partial Q}{c\,\partial\theta}\right] + \frac{[Q]}{c}\left[\sum\frac{\partial\phi}{\partial\xi}\frac{\partial r}{\partial\xi} - \frac{\partial\phi}{c\,\partial\theta}\right]$$
$$= -4\pi\int_{-\infty}^{t'} d\theta\, Q\rho(\xi\eta\zeta\theta) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

and observe that

$$\left[\frac{\partial Q}{\partial\theta}\right] = 0 = \left[\frac{\partial Q}{\partial\xi}\right] = \left[\frac{\partial Q}{\partial\eta}\right] = \left[\frac{\partial Q}{\partial\zeta}\right],$$
$$[Q] = \frac{k}{2},$$

and, in view of eqs. (17),

$$\left[\sum\frac{\partial\phi}{\partial\xi}\frac{\partial r}{\partial\xi} - \frac{\partial\phi}{c\,\partial\theta}\right] = \sum\frac{\partial r}{\partial\xi}\frac{\partial}{\partial\xi}[\phi]$$
$$= \sum\frac{\partial}{\partial\xi}\left(\frac{\partial r}{\partial\xi}[\phi]\right) - \frac{2}{r}[\phi].$$

Equation (19) now takes the form

$$\sum\frac{\partial}{\partial\xi}\left\{\int_{-\infty}^{t'} d\theta\left(Q\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial Q}{\partial\xi}\right) + \frac{k}{2c}\frac{\partial r}{\partial\xi}[\phi]\right\} - \frac{k}{c}\frac{[\phi]}{r}$$
$$= -4\pi\int_{-\infty}^{t'} d\theta\, Q\rho(\xi\eta\zeta\theta). \quad \ldots \quad \ldots \quad \ldots \quad (B)$$

Combining now equations (A) and (B) we get

$$\sum\frac{\partial}{\partial\xi}\left\{\frac{1}{r}\frac{\partial}{\partial\xi}[\phi] - [\phi]\frac{\partial}{\partial\xi}\left(\frac{1}{r}\right) + \frac{2}{c}\frac{1}{r}\frac{\partial r}{\partial\xi}\frac{\partial}{\partial t}[\phi] - ck\int_{-\infty}^{t'} d\theta\left(Q\frac{\partial\phi}{\partial\xi} - \phi\frac{\partial Q}{\partial\xi}\right) - \frac{k^2}{2}\frac{\partial r}{\partial\xi}[\phi]\right\}$$
$$= -4\pi\left\{\frac{[\rho]}{r} - ck\int_{-\infty}^{t'} d\theta\, Q\rho(\xi\eta\zeta\theta)\right\}$$

or, in view of eqs. (17),

$$\sum \frac{\partial}{\partial \xi}\left\{\frac{1}{r}\left[\frac{\partial \phi}{\partial \xi}\right]+\frac{1}{r}\frac{\partial r}{\partial \xi}\left[\frac{\partial \phi}{c\,\partial \theta}\right]-[\phi]\frac{\partial}{\partial \xi}\left(\frac{1}{r}\right)-ck\int_{-\infty}^{t'} d\theta\left(Q\frac{\partial \phi}{\partial \xi}-\phi\frac{\partial Q}{\partial \xi}\right)\right.$$

$$\left.-\frac{k^2}{2}\frac{\partial r}{\partial \xi}[\phi]\right\}$$

$$=-4\pi\left\{\frac{[\rho]}{r}-ck\int_{-\infty}^{t'} d\theta\, Q\rho(\xi\eta\zeta\theta)\right\}. \qquad \ldots \quad \ldots \quad (20)$$

This equation (20) is easily seen to be identical with eq. (12) and subsequent integration through a closed volume, as in the last section, will now obviously lead to the complete solution of the proposed equation in the form (13).

It is evident that we may in like manner obtain the complete solution for advanced values of t if we replace θ in eq. (10) at the outset by $t+\frac{r}{c}$ $(=t'')$ and later perform integrations with respect to θ from t'' to $+\infty$.

6. In closing this communication we like to make a few observations on the methods here employed. Firstly, the method of complex integration is obviously also available for the Kirchhoff solution of the equation

$$\nabla^2\phi-\frac{\partial^2\phi}{c^2\,\partial\theta^2}=-4\pi\rho(\xi\eta\zeta\theta).$$

We may have this solution, of course, immediately out of the solutions (13) and (14) with $k\to 0$; we may have it, too, by a like procedure repeated with a solving function $\chi(s)=\frac{1}{s^2}$. Secondly, both methods may be adapted without difficulty for solution of the equation

$$\nabla^2\phi-\frac{\partial^2\phi}{c^2\,\partial\theta^2}+k^2\phi=-4\pi\rho(\xi\eta\zeta\theta)$$

and the only difference should be that real Bessel functions of imaginary arguments would take the place of those we have used above. Lastly, underlying the method of complex integration there is obviously the assumption that ρ and ϕ are analytic functions of θ over a region of the complex plane wide enough to embrace the path of integration—an assumption from which the second method is happily free.

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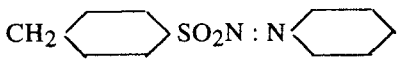
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REACTION OF SULPHONAZIDES WITH PYRIDINE : SALTS AND DERIVATIVES OF PYRIDINE-IMINE

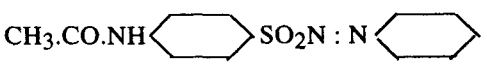
Curtius and Kraemers[1] heated p-toluene-sulphonazide with pyridine and isolated a crystalline compound, M.P. 210°C. They supposed it to be p-toluenesulphon—amido-pyridine. When, however, the sulphonic acid residue was removed by hydrolysis, the new base yielded a picrate of M.P. 138-9°C. which did not agree with M.P. of the picrate of any of the known amino-pyridines. The structure of this compound was thus not established by this work of Curtius.

Reactions of p-acetyl-amino-benzene-sulphonazide and p-toluene-sulphonazide with pyridine have been studied here for the last year and a half. This has led to the discovery of a new derivative of pyridine, N-imino-pyridine, and the compound of Curtius has now been shown to be a sulphonic acid derivative of this N-amino-pyridine or pyridine-imine.



(I)

When p-acetyl-amino-benzene-sulphonazide is boiled in dry pyridine in an inert atmosphere, nitrogen is slowly evolved and from among other products a crystalline compound insoluble in pyridine can be isolated. After repeated crystallisation the substance melts with decomposition at 283°-4°C. The analysis shows that the substance has the same empirical formula as p-acetyl-amino-benzene-sulphonamido-pyridine but closer investigation reveals it to be also a derivative of pyridine-imine having the formula



(II)

The acetyl-group can be removed easily and this process leads to an extremely hygroscopic hydrochloride of a base. The free base has M.P. 228-229°C. Estimation of Pt. in the platini-chloride compound, M.P. 224°C, indicats the formula

$$(C_{11}H_{11}O_2N_3S)H_2PtCl_6$$

showing that it has two basic groups.

When further hydrolysed with hydrochloric acid, sulphanilic acid separates out and the solution of an extremely hygroscopic hydrochloride of a base is obtained.

The perchlorate of this base has M.P. 204°C., the picrate, 149°C. and the platini-chloride, 237°C. and the analyses completely agree with the expected formulae.

Attempts to liberate the free-base with alkali lead to polymerisation.

Alkaline-ferricyanide liberates nitrogen from the salts of this base. Nitrous acid decomposes the salts and from the solution pyridine can be isolated, as perchlorate.

The same base is obtained from the hydrolysis of paratoluene-sulphonazide-compound. This has been confirmed by allowing p-acetyl-amino-benzene-sulpho-chloride to react on the base from (I) which leads to the compound (II).

A confirmation of the imino-structure has been obtained by a synthesis of the compound (I), from glutaconic-dialdehyde. Mono-benzoyl-derivative of glutaconic-dialdehyde[2] is allowed to react with para-toluene sulpho-hydrazide leading to the compound of the form

$$C_6H_5COO\,CH = CH - CH = CH - CH = N.NHSO_2C_7H_7$$

With the action of alcoholic hydrochloric acid the benzoyl group is eliminated, and there is a ring closure leading to the formation of the pyridine ring, and the hydrochloride of compound (I). This synthesis is similar to the synthesis of pyridine-oxide by Baumgarten.[3]

Further investigations about this interesting class of compound are in progress. Full details will be published elsewhere.

In April number of *Current Science*, Ganapathi and Miss Alamela[4] reported about their attempts to prepare sulphonamide derivatives of heterocyclic compounds by Curtius process. They mentioned the compound (II) and assumed this to be a 3-amino-pyridine derivative without any justification. It is expected that this note may be of some interest to them.

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A NOTE ON DIRAC EQUATIONS AND THE ZEEMAN EFFECT

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ABSTRACT. A new treatement has been given for solving Dirac's equations for hydrogenic atoms, and the radial functions are expressed in terms of a combination of Sonine's polynomials $T_n^{(\mu)}(\rho)$, $T_{n-1}^{(\mu)}(\rho)$ of only two consecutive degrees n, $n-1$; and the elementary properties of such polynomials have enabled us to tackle the Zeeman effect problem in general (homogeneous field) leading to the standard quadratic equation in energy for the effect.

1. With the help of the two-dimensional matrices s_x, s_y, s_z, of Pauli the wave-equations of Dirac can be put in the well-known matrix form :

$$\frac{\iota}{\hbar c}\left[E+E_0+\frac{Ze^2}{r}\right]X+DY=0,\quad \frac{\iota}{\hbar c}\left[E-E_0+\frac{Ze^2}{r}\right]Y+DX=0 \tag{1.1}$$

where D is the operator $\quad s_x\frac{\partial}{\partial x}+s_y\frac{\partial}{\partial y}+s_z\frac{\partial}{\partial z}.$

If, similarly $S = xs_x+ys_y+zs_z$, and $s = \frac{I}{r}(xs_x+ys_y+zs_y)$, then

$S.D. = r\frac{d}{dr}+L$, where $L = \iota\,(M_x s_x+M_y s_y+M_z s_z)$.

And the following commutation rules can be easily deduced :

$$s(L-1)+(L-I)s = 0,\ D(L-1)+(L-1)D = 0;$$

also $\qquad s^2 \equiv 1.$

Hence multiplying the equations on the left by S we have

$$\frac{\iota}{\hbar c}\left[E+E_0+\frac{Ze^2}{r}\right]SX+\left[r\frac{d}{dr}+1\right]Y+(L-1)Y=0; \tag{1.2}$$

$$\frac{\iota}{\hbar c}\left[E-E_0+\frac{Ze^2}{r}\right]SY+\left[r\frac{d}{dr}+1\right]X+(L-1)X=0.$$

Assuming $X \equiv \begin{vmatrix} u_1 \\ u_2 \end{vmatrix} = \frac{f(r)}{r}\Phi$; $Y \equiv \begin{vmatrix} u_3 \\ u_4 \end{vmatrix} = \frac{g(r)}{r}\Psi$ where Φ, ψ are similar one-column matrices and functions of (θ, ϕ), the equations can be rewritten as

$$\frac{\iota}{\hbar c} f(r)\Big[E+E_0+\frac{Ze^2}{r}\Big]s\Phi+\frac{dg(r)}{dr}\cdot\Psi+\frac{g(r)}{r}(L-1)\Psi=0, \tag{1.3}$$

$$\frac{\iota}{\hbar c} g(r)\Big[E-E_0+\frac{Ze^2}{r}\Big]s\Psi+\frac{df(r)}{dr}\cdot\Phi+\frac{f(r)}{r}(L-1)\Phi=0.$$

The equations become easily separable if the matrices Φ and Ψ are so chosen that at first $(L-1)\Psi = k\Psi$ and $s\Phi = \Psi$; and as $s^2 \equiv 1$, it follows therefore from commutation rules that

$$\Phi = s\Psi \text{ and } (L-1)s\Psi = -ks\Psi.$$

We observe at once that $L(L-1)\ \Psi = -\nabla_2\Psi = k(k+1)\Psi$, where

$$\nabla_2 \equiv \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\Big(\sin\theta\frac{\partial}{\partial\theta}\Big)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2};$$

and therefore k can be either a positive or a negative integer.

Secondly, if (Φ, Ψ) are the matrices for positive k, then (Ψ,Φ) are the matrices for negative k. Also remembering that the operator

$-\Big[\iota\Big(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\Big)+\frac{1}{2}s_z\Big]$ commutes with the equation-system, the angular matrices can be expressed in terms of spherical harmonics of order k and $k-1$ in the following form :

$$\Phi \equiv \begin{vmatrix} \sqrt{\frac{k+m}{2k-1}}\,Y^{\mu}_{k-1} \\ \sqrt{\frac{k-m-1}{2k-1}}\,Y^{\mu+1}_{k-1} \end{vmatrix}, \Psi \equiv \begin{vmatrix} \sqrt{\frac{k-m}{2k+1}}\,Y^{\mu}_{k} \\ \sqrt{\frac{k+m+1}{2k+1}}\,Y^{\mu+1}_{k} \end{vmatrix} \quad (k, \text{ a positive integer}) \tag{1.4}$$

where $Y^{\mu}_{k} \equiv \sqrt{\frac{2k+1}{2}\cdot\frac{(k-\mu)!}{(k+\mu)!}}\,\frac{1}{2^k k!}\,\frac{d^{k+\mu}}{dx^{k+\mu}}(x^2-1)^k\,\frac{e^{i\mu\Phi}}{\sqrt{2\pi}}$. $(x = \cos\theta)$

The functions are interchanged for negative values of k. Writing $f(r) = \iota F(r)$ so as to remove the imaginary from the radial equations, we get

$$\left[\frac{dG}{dr} + k\frac{G}{r} \right] - \frac{1}{hc}\left[E + E_0 + \frac{Ze^2}{r} \right] F = 0,$$

$$\left[\frac{dF}{dr} - k\frac{F}{r} \right] + \frac{1}{hc}\left[E - E_0 + \frac{Ze^2}{r} \right] G = 0. \qquad (1.5)$$

2. To solve the equation we assume

$$F = f_0 e^{-\lambda r} r^{\mu} F_1(r),\ G = g_0 e^{-\lambda r} r^{\mu} G_1(r), \text{ and } N = aE/(E_0^2 - E^2)^{\frac{1}{2}} \qquad \ldots \quad (2.1)$$

where $Ze^2/hc = a$.

Substituting in (1.5), F_1 and G_1 are easily seen to satisfy the following equations :

$$\frac{dG_1}{dr} - \lambda G_1 + \lambda F_1 + \frac{k+\mu}{r} G_1 + \frac{N}{E}(E_0 - E)\frac{F_1}{r} = 0, \qquad (2.2)$$

$$\frac{dF_1}{dr} - \lambda F_1 + \lambda G_1 - \frac{k-\mu}{r} F_1 - \frac{N}{E}(E_0 + E)\frac{G_1}{r} = 0,$$

where $f_0/g_0 = -(E_0 - E)/hc\lambda = \dfrac{\lambda hc}{E_0 + E} = -\left(\dfrac{E_0 - E}{E_0 + E}\right)^{\frac{1}{2}}$ and $\lambda = \left(\dfrac{E_0^2 - E^2}{hc}\right)^{\frac{1}{2}}$ (2.3)

If $F_1 + G_1 = 2\chi^+$, $G_1 - F_1 = 2\chi^-$, elimination leads easily to the following equations :

$$r\frac{d^2\chi^+}{dr^2} + (2\mu + 1 - 2\lambda r)\frac{d\chi^+}{dr} + 2\lambda(N - \mu)\chi^+ = 0, \qquad (2.4)$$

$$r\frac{d\chi^+}{dr} - (N - \mu)\chi^+ = \left(N\frac{E_0}{E} - k\right)\chi^-, \qquad (2.5)$$

provided $k^2 - \mu^2 = N^2(E_0^2 - E^2)/E^2 = \alpha^2$.

Taking the first equation it can be easily seen that it admits of polynomial solutions if $N - \mu = n$ (an integer). Writing ρ for $2\lambda r$, and writing a Sonine polynomial in the form

$$T_n^{(\mu)}(\rho) = B_n^{(\mu)}\left[\rho^n - \frac{n(n+2\mu)}{1!}\rho^{n-1} + \frac{n(n-1)(n+2\mu)(n+2\mu-1)}{2!}\rho^{n-2} - \ldots \right]$$

$$= B_n^{(\mu)} \cdot (-1)^n e^{\rho} \rho^{-2\mu} \frac{d^n}{d\rho^n}\left[e^{-\rho}\rho^{n+2\mu} \right] \qquad (2.6)$$

with constant $B_n^{(\mu)}$ so chosen that it is normalized according to

$$\int_0^\infty e^{-\rho}\rho^{2\mu}\, T_n^{(\mu)}(\rho)T_n^{(\mu)}(\rho)d\rho = 1$$

we get $B_n^{(\mu)} = 1/\{n\,!\,\Gamma(n+2\mu+1)\}^{\frac{1}{2}}$

Hence it easily follows that χ^+ and χ^- will have solutions as given below :

$$\chi^+ = \delta\left(N\frac{E_0}{E} - k\right)^{\frac{1}{2}} T_n^{(\mu)}(\rho) \qquad (2.7)$$

$$\left[\, N = n+\mu = \frac{\alpha\epsilon}{\sqrt{1-\epsilon^2}}\ ;\ \epsilon = \frac{E}{E_0}\,\right]$$

$$\chi^- = \delta\left(N\frac{E_0}{E} + k\right)^{\frac{1}{2}} T_{n-1}^{(\mu)}(\rho) \qquad (2.8)$$

where the normalising factor δ is to be determined presently.

Our radial functions of eqs. (2.1) stand thus

$$F = -\delta\sqrt{\left(1-\frac{E}{E_0}\right)}\ e^{-\frac{\rho}{2}}\rho^\mu\left[\left(N\frac{E_0}{E}-k\right)^{\frac{1}{2}}T_n^{(\mu)}(\rho) - \left(N\frac{E_0}{E}+k\right)^{\frac{1}{2}}T_{n-1}^{(\mu)}(\rho)\right] \qquad (2.9)$$

$$G = \delta\sqrt{\left(1+\frac{E_0}{E}\right)}\ e^{-\frac{\rho}{2}}\rho^\mu\left[\left(N\frac{E_0}{E}-k\right)^{\frac{1}{2}}T_n^{(\mu)}(\rho) + \left(N\frac{E_0}{E}+k\right)^{\frac{1}{2}}T_{n-1}^{(\mu)}(\rho)\right] \qquad (2.10)$$

Normalisation requires

$$\int_0^\infty (F^2+G^2)dr = 1,$$

which gives

$$\delta = \sqrt{\frac{\lambda}{2N}\,\frac{E}{E_0}}\,.$$

We can write the two solutions (A) and (B), corresponding to positive and negative values of k respectively.

$$\left[\begin{array}{ll} u_1 = \sqrt{\dfrac{k+\mu}{2k-1}}\ .\iota\dfrac{f_+(r)}{r}\ Y_{k-1}^{\mu}\,, & u_3 = \sqrt{\dfrac{k-\mu}{2k+1}}\ \dfrac{g_+(r)}{r}\ Y_k^{\mu} \\ \\ u_2 = -\sqrt{\dfrac{k-\mu-1}{2k-1}}\ \iota\dfrac{f_+(r)}{r}\ Y_{k-1}^{\mu+1}, & u_4 = \sqrt{\dfrac{k+\mu+1}{2k+1}}\ \dfrac{g_+(r)}{r}\ Y_k^{\mu+1} \end{array}\right] \qquad (A)$$

$$\left[\begin{array}{ll} v_1 = \sqrt{\frac{k-\mu}{2k+1}}\ \iota\frac{f_-(r)}{r}\ Y^{\mu}_{k} & v_3 = \sqrt{\frac{k+\mu}{2k-1}}\ \frac{g_-(r)}{r}\ Y^{\mu}_{k-1} \\ v_2 = \sqrt{\frac{k+\mu+1}{2k+1}}\ \iota\frac{f_-(r)}{r}\ Y^{\mu+1}_{k},\ v_4 = -\sqrt{\frac{k-\mu-1}{2k-1}}\ \frac{g_-(r)}{r}\ Y^{\mu+1}_{k-1} & \end{array}\right] \tag{B}$$

wherein it is understood that f_- and g_- are obtained by changing the sign of k, in the expressions for F and G in (1.5) above.

3. When the atom is in an electromagnetic field defined by the vector potential $(A_xA_yA_z)$, the Dirac-equations become

$$\frac{\iota}{hc}\left[E+E_0+\frac{Ze^2}{r}\right]X+DY+\frac{e}{hc}AY=0 \tag{3.1}$$

$$\frac{\iota}{hc}\left[E-E_0+\frac{Ze^2}{r}\right]Y+DX+\frac{e}{hc}AX=0 \tag{3.2}$$

where $$A \equiv \iota[A_x\beta_x+A_y\beta_y+A_z\beta_z]$$

In the case of a constant magnetic field H, in the direction of Z-axis

$$A \equiv \tfrac{1}{2}Hr \begin{vmatrix} 0 & e^{-i\mu\phi}\sin\theta \\ -e^{i\mu\phi}\sin\theta & 0 \end{vmatrix} \equiv \tfrac{1}{2}Hra. \tag{3.3}$$

We observe in passing that in the absence of A, the wave function X is generally small compared with Y—the radial component of $F(r)$ has the factor $\sqrt{1-\frac{E}{E_0}}$, while $G(r)$ has $\sqrt{1+\frac{E}{E_0}}$; so that the perturbation effect of $\frac{e}{hc}\ AX$ is small as compared with effect due to the term $\frac{e}{hc}\ AY$ in eq. (3.1)

Remembering

$$e^{-i\phi}\,Y^{\mu+1}_{k}\sin\theta = -\sqrt{\frac{(k-\mu)(k-\mu+1)}{(2k+1)(2k+3)}}\ Y^{\mu}_{k+1}+\sqrt{\frac{(k+\mu)(k+\mu+1)}{(2k+1)(2k-1)}}\ Y^{\mu}_{k-1};$$

$$e^{i\phi}\,Y^{\mu}_{k}\sin\theta = \sqrt{\frac{(k+\mu+1)(k+\mu+2)}{(2k+1)(2k+3)}}\ Y^{\mu+1}_{k+1}-\sqrt{\frac{(k-\mu)(k-\mu-1)}{(2k-1)(2k+1)}}\ Y^{\mu+1}_{k-1};$$

we seek an approximate solution of the equation by choosing one set of angular func-

tion $(Y_k^\mu,\ Y_k^{\mu+1})$, and assuming the existence of both the sets $(Y_{k-1}^\mu,\ Y_{k-1}^{\mu+1})$ and $(Y_{k+1}^\mu,\ Y_{k+1}^{\mu+1})$ in X. This can be done by suitably combining the (A) and (B) types, thus

Take $u_j = C_l^+ u_j^+ + C_{-(l+1)}^- v_j^-$, $(j = 1, 2, 3, 4)$, where $(u_1^+, u_2^+, u_3^+, u_4^+)$ corresponds to $k = +l$ and $(v_1^-, v_2^-, v_3^-, v_4^-)$ corresponds to $k = -(l+1)$. More explicitly, their values are

$$u_1^+ = \sqrt{\frac{l+\mu}{2l-1}}\ \iota\frac{f_+(r)}{r}\,Y_{l-1}^\mu; \qquad v_1^- = \sqrt{\frac{l-\mu+1}{2l+3}}\ Y_{l+}^\mu\ \iota\frac{f_-(r)}{r};$$

$$u_1^+ = -\sqrt{\frac{l-\mu-1}{2l-1}}\ \iota\frac{f_+(r)}{r}\,Y_{l-1}^{\mu+1}; \quad v^- = \sqrt{\frac{l+\mu+2}{2l+3}}\ Y_{l+1}^{\mu+1}\,\iota\frac{f_-(r)}{r};$$

$$u_3^+ = \sqrt{\frac{l-\mu}{2l+1}}\ \frac{g_+(r)}{r}\ Y_l^\mu; \qquad v_3^- = \sqrt{\frac{l+\mu+1}{2l+1}}\ \frac{g_-(r)}{r}\ Y_l^\mu;$$

$$u_1^+ = \sqrt{\frac{l+\mu+1}{2l+1}}\ \frac{g_+(r)}{r}\,Y_l^{\mu+1}; \quad v_4^- = -\sqrt{\frac{l-\mu}{2l+1}}\ \frac{g_-(r)}{r}\ Y_l^{\mu+1}.$$

The constants λ, N, μ of the two types of solutions are different and are expressed by the following relations

$$\lambda_+ = (E_0^2 - E_+^2)^{\frac{1}{2}}/\hbar c, \quad l^2 - \alpha^2 = \mu_+^2, \quad N_+ = \alpha E_+/[E_0^2 - E_+^2]^{\frac{1}{2}}, \tag{3.4}$$

and $\quad N_+ = n_+ + \mu_+$[for (A)−type with $k = l$].

$$\lambda_- = (E_0^2 - E_-^2)^{\frac{1}{2}}/\hbar c), \quad (l+1)^2 - a^2 = \mu_-^2, \quad N_- = aE_-/[E_0^2 - E_0^2]^{\frac{1}{2}}, \tag{3.5}$$

and $\quad N_- = n_- + \mu_-$[for (B)−type with $k = -(l+1)$].

If E_+ and E_- differ slightly from one another, we have the following approximate relations

$$\lambda_+ = \lambda_-,\ \mu_- = \mu_+ + 1,\ N_+ = N_-,\ n_- = n_+ - 1,\ \text{and}\ \frac{E_+}{E_0} \sim 1. \tag{3.6}$$

By following the usual method of perturbation we see easily that the characteristic equation for determining the Eigen value E would be

$$\begin{vmatrix} \frac{\iota}{\hbar_c}\left[E - E_+\right] + pa_{11} & pa_{12} \\ pa_{21} & \frac{\iota}{\hbar_c}\left[E - E_-\right] + pa_{22} \end{vmatrix} = 0 \tag{3.7}$$

where $p = eH/2\hbar c$, and

$$a_{11} = \int [\bar{X}_+^* r\boldsymbol{a} Y_+ + Y_+^* r\boldsymbol{a} X_+] dV$$

$$a_{22} = \int [\bar{X}_-^* r\boldsymbol{a} Y_- + \bar{Y}_-^* r\boldsymbol{a} X_-] dV$$

$$a_{12} = \int [\bar{X}_+^* r\boldsymbol{a} Y_- + \bar{Y}_-^* r\boldsymbol{a} X_-] dV \qquad (3.8)$$

$$a_{21} = \int [\bar{X}_-^* r\boldsymbol{a} Y_+ + \bar{Y}_-^* r\boldsymbol{a} X_+] dV$$

and $\bar{X}_+^*$ etc. are transposed conjugates of X_+ (the $\boldsymbol{a}$ is given in (3.3))

Carrying out integrations over the angle-variable-space we get

$$a_{11} = -\frac{2\iota(4\mu+2)l}{(2l-1)(2l+1)} \int_0^\infty r f_+ g_+ dr \ ;$$

$$a_{22} = \frac{2\iota(4\mu+2)(l+1)}{(2l+1)(2l+3)} \int_0^\infty r f_- g_- dr \ ;$$

$$a_{12} = a_{21} = \frac{\iota\sqrt{l(-\mu)(l+\mu+1)}}{2l+1} \int_0^\infty r(f_- g_+ + f_+ g_-)\, dr.$$

4. To evaluate the integral {I} or {II}, we substitute the values of the corresponding f, g, and remembering $r = \rho/2\lambda$, we obtain

$$\{\mathrm{I}\} = -\frac{\lambda_+}{2N_+} \frac{E_+}{E_0^2} \frac{(E_0^2 - E_+^2)^{\frac{1}{2}}}{4\lambda^2} \left\{ \left(N_+ \frac{E_0}{E_+} - k \right) \int_0^\infty e^{-\rho} \rho^{2\mu+1} \left[T_n^{(\mu)}(\rho) \right]^2 d\rho \right.$$

$$\left. - \left(N_+ \frac{E_0}{E_+} + k \right) \int_0^\infty e^{-\rho} \rho^{2\mu+1} \left[T_{n-1}^{(\mu)}(\rho) \right]^2 d\rho \right\}.$$

The integrals are evaluated quickly by repeated partial integration; thus

$$\int_0^\infty e^{-\rho} \rho^{2\mu+1} \left[T_n^{(\mu)}(\rho) \right]^2 d\rho = \frac{(-)^n}{\sqrt{n!\,\Gamma(n+2\mu+1)}} \int_0^\infty \rho T_n^{(\mu)}(\rho) \frac{d^n}{d\rho} \left[e^{-\rho} \rho^{2\mu+n} \right] d\rho$$

$$= \int_0^\infty \frac{e^{-\rho}}{n!} \frac{\rho^{2\mu+n}}{\Gamma(n+2\mu+1)} d\rho \cdot \frac{d^n}{d\rho^n} \left[\rho \left(\rho^n \frac{n(n+2\mu)}{1!} \rho^{n-1} + \dots \right) \right]$$

$$= (n+1)(n+2\mu+1) - n(n+2\mu) = 2N_+ + 1.$$

Similarly

$$\int_0^\infty e^{-\rho}\rho^{2\mu+1}\Big[\ T^{(\mu)}_{n-1}\ (\rho)\Big]^2 d\rho = 2N_+ - 1.$$

An easy substitution of the value of λ_+ in the foregoing leads to the following results :

$$\int_0^\infty rf_+g_+dr = -\frac{h_c}{4E_0}\left(1-2l\,\frac{E_0}{E_+}\ \right); \tag{4.1}$$

$$\int_0^\infty rf_-g_-dr = -\frac{h_c}{4E_0}\left(1+2(l+1)\,\frac{E_0}{E_-}\ \right). \tag{4.2}$$

The evaluation of the third integral cannot, however, be exactly expressed in a neat form; for the simple reason that λ_+, N_+, E_+ as well as λ_-, N_-, E_- are different in the two solutions, as also $\rho_+ = 2\lambda_+ r$ and $\rho_- = 2\lambda_- r$. If however, we introduce approximations in the beginning, and take

$$\lambda_+ = \lambda_- = |\lambda|,\ E_+ = E_-,\ \ \mu_- = \mu_+ + 1,\ \ n_- = n_+ - 1,\ \ \rho_+ = \rho_-, \tag{4.3}$$

we see the third integral

$$\{\mathrm{III}\} = -\frac{2|\lambda|}{2N}\ \frac{|E|}{E_0^2}\,\frac{(E_0-|E|^2)^{\frac{1}{2}}}{4|\lambda|^2}\Big\{\Big(N\frac{E_0}{E}-l\Big)^{\frac{1}{2}}\Big(N\frac{E_0}{E}+l+1\Big)^{\frac{1}{2}}\times$$

$$\int_0^\infty e^{-\rho}\rho^{2\mu+2}T^{(\mu)}_n(\rho)T^{(\mu+1)}_{n-1}(\rho)d\rho - \Big(N\frac{E_0}{E}+l\Big)^{\frac{1}{2}}\Big(N\frac{E_0}{E}-l-1\Big)^{\frac{1}{2}}$$

$$\times\int_0^\infty e^{-\rho}\rho^{2\mu+2}T^{(\mu)}_n(\rho)\,T^{(\mu+2)}_{n-2}(\rho)d\rho\Big\}.$$

The two definite integrals can be easily evaluated in the same way as before. We give the results below ;

$$\int_0^\infty e^{-\rho}\rho^{2\mu+2}\ T^{(\mu)}_n(\rho)\,T^{(\mu+1)}_{n-1}(\rho)d\rho = \int_0^\infty \frac{e^{-\rho}\rho^{n+2\mu}}{\sqrt{n-1!\,n\,!\ \Gamma(n+2\mu+1)\ \Gamma(n+2\mu+2)}} \tag{4.4}$$

$$\times\frac{d^n}{d\rho^n}\Big[\ \rho^{n+1}-(n-1)(n+2\mu+1)\rho^n+\dots\ \Big] = 2\sqrt{(N-l)(N+l+1)}.$$

$$\int_0^\infty e^{-\rho}\rho^{2\mu+2}T^{(\mu)}_{n-2}(\rho)T^{(\mu+1)}_{n-}\,d\rho = 2\sqrt{(N+l)(N-l-1)}. \tag{4.5}$$

Hence

$$\{\text{III}\} = -\frac{1}{2N}\frac{h_c}{E_0}\left[\sqrt{N\frac{E_0}{E}-l}\sqrt{N\frac{E_0}{E}+l+1}\sqrt{N-l}\sqrt{N+l+1}\right]$$

$$-\sqrt{N\frac{E_0}{E}+l}\sqrt{N\frac{E_0}{E}-l-1}\sqrt{N+l}\sqrt{N-l-1}\Big].$$

Observing $E_0/E \sim 1$, we see

$$\{\text{III}\} = -\frac{1}{2N}\cdot\frac{h_c}{E_0}2N = -\frac{h_c}{E_0}.$$

Finally, putting $p = eH/2h_c$ and $eh/2m_0c = \mu_0$ (Bohr-magneton) and substituting the values of the three integrals (and making $E_0/E \sim 1$) we see that the determinantal equation (3.7) takes the form :

$$\begin{vmatrix} E-E_+-\frac{2l}{2l+1}m\mu_0H & -\frac{\mu_0H}{2l+1}\sqrt{(l-m+\frac{1}{2})(l+m+\frac{1}{2})} \\ -\frac{\mu_0H}{2l+1}\sqrt{(l-m+\frac{1}{2})(l+m+\frac{1}{2})} & E-E_--\frac{2l+2}{2l+1}m\mu_0H \end{vmatrix} = 0 \qquad (4.6)$$

wherein we have put $\mu+\frac{1}{2} = m$ (magnetic quantum number).

The result (4.6) agrees completely with that quoted by Bethe (1933), supposed to have been worked out by him from Pauli's equations. Condon and Shortley (1935) obtained similar determinant from principles of quantum mechanics by applying two-fold perturbations (spin-orbit and magnetic) simultaneously. It may be noted that Darwin (1928) has many similar features with our mode of attack, and we can claim some elegance by our introduction of Sonines properties, which exhibit our solutions in a good perspective.

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Thirtyfirst Indian Science Congress

DELHI, 1944

PRESIDENTIAL ADDRESS

CONGRESS PRESIDENT : PROFESSOR S. N. BOSE

THE CLASSICAL DETERMINISM AND THE QUANTUM THEORY

(Delivered on Jan. 3, 1944)

I wish to express sincere thanks for the great honour you have done me. The Presidentship of the Science Congress is a great distinction, and I confess, I have my own misgivings about the wisdom of your choice. Your first decision had raised high hopes. Many of us expected that a deliberate programme of the future scientific activities of the country would probably be a feature of the opening speech of this Congress. Pandit Jawaharlal had studied the needs of the country. Many of our front-rank scientists and industrialists had met under his leadership, not long ago, and given to questions of future reconstruction much time and anxious thought. The results of this deliberation would have been invaluable at the present moment. My regret is keen that chance has deprived us of the benefits of a sustained and careful study of the problems of the day. I would have liked to present here the results, if they were available. Unfortunately they are not, as most of the reports are inaccessible to me.

One of your former Presidents had remarked that "a scientist is apt to become a man that knows more and more about less and less, so that his opinion upon subjects outside his field of special study is not necessarily of special value". I realise the wisdom of this warning and hope to have your indulgence, if I seem to be more at home with doubts and criticisms than with useful knowledge.

I would like to present before you certain aspects of modern physics and draw your attention to the profound changes in the principle of scientific explanation of natural phenomena brought about by the quantum theory. The last fifty years record remarkable discoveries. I need only mention the electron and the neutron, X-rays and Radio-activity to remind you of the increase of our knowledge. Our equipment has gained in power range and accuracy. We possess powerful telescopes to scan the furthest corners of the universe, also precise and delicate instruments to probe into the interior of the atoms and molecules. The alchemists' dream of transmutation has become a reality. Atoms are now disintegrated and synthesised. X-ray reveals invisible worlds and wireless links up the furthest ends of the earth with possibility of immediate intercommunication. These discoveries have their repercussions in the realm of ideas. Fifty years ago the belief in causality and determination was absolute. To-day physicists have gained knowledge but lost their faith. To understand properly the significance of such a profound change it will be necessary to discuss briefly how it all came about. Classical physics had begun with the study of astronomy. With his laws of gravitation and his dynamics Newton had explained planetary motion. Subsequent study has shown astronomical prediction to be possible and sure. Physicists had taken the equations of celestial mechanics as their model of a universal law. The atomic theory had in the mean time gained

universal acceptance ; since matter had resolved into a conglomeration of particles, the ideal scheme was to explain all phenomena in terms of their motions and interactions. It was only necessary to set up a proper set of equations, and to take account of all possible mutual interactions. If the mass, position, and velocity of all the particles were known at any instant, these equations would theoretically enable the physicist to predict the position and motion of every particle at any other subsequent moment.

The phenomena of light did not at first fit into this simple scheme. To regard it as a stream of particles was impossible due to the discovery of interference. Accordingly the wave theory of light was originated by Huyghens and perfected by Maxwell. With the discovery of the electron as a universal constituent of matter, the electromagnetic theory of Maxwell was converted into an electronic theory by Lorentz. To the dynamical laws were added the electromagnetic equations and the two together apparently gave an exact and ideal formulation of the laws of causality. In the forces of interaction henceforth, were to be included not only the gravitational forces but also those interactions which depended on the charge and the motion of the particles. These interactions were brought about by influences which spread out as waves with the velocity of light. They superimposed, interfered and constituted the field of force in the neighbourhood of the particles, modified their motion and were in turn modified by them. The motions of all particles throughout the universe were thus interlocked. These out-going influences also constituted light, invisible radiation, X-rays and wireless waves. Thus a set of universal laws was supposed to have been discovered and we had only to apply them suitably to find explanations of all conceivable natural phenomena. In physical science we do not however always proceed in the above way and turn to the "microscopic" equations whenever we have to explain events. We often study materials *en masse*, consisting of an enormous number of corpuscles, and we use either the principle of the conservation of energy or the laws of thermodynamics to explain their behaviour. These laws were however regarded either as simple consequences of the fundamental equations or as statistical laws derivable from them by a suitable averaging. Though in the latter cases we talk about probabilities and fluctuations, it was more or less a matter of faith to maintain that if it were possible for us to obtain all the necessary data by delicate observations, universal laws would enable us to follow each individual molecule in this intricate labyrinth and we should find in each case an exact fulfilment of the laws and agreement with observation. The above in brief forms an expression of faith of a classical physicist. We see that it involves as necessary consequences, belief in continuity, in the possibility of space-time description of all changes and in the existence of universal laws independent of observers which inexorably determine the course of future events and the fate of the material world for all times.

A few remarks about the general equations will perhaps enable us to follow better the criticisms that have been levelled against the system. The structure of the mechanical equations of particles is different from the field-equations of Maxwell and Lorentz. The principles of conservation of energy and momentum were first discovered as consequences of the mechanical equations. Mass and velocity of the corpuscle furnish means to measure its momentum, and its energy, if we leave aside the potential energy which resides in the field. To maintain the integrity of the principle of conservation, the field must also be considered capable of possessing energy and momentum, which however, being associated with wave-motion, must spread out in all directions with the waves. The transfer

of energy from the field to the particles must thus be a continuous process, whereby, a finite change should come about only in a finite interval and the process should theoretically be capable of an exact description in space and time.

Physics being essentially concerned with relations between quantities, these should all be capable of exact measurement. We measure always intervals of time or inter-distance between points, hence the specification of the reference frame is just as important as the units of measure. Newton had not analysed closely the conception of mass and time. This vagueness persisted in the dynamical equations for the particles. The field-equations which form the basis of the wave-theory of light have a different origin. With the discovery of the principle of the least action, a common derivation of both has been attempted. But a difference in the choice of reference frame in the two apparently subsisted. The wave-equations assumed a fixed ether whereas the material laws contemplated a Galilean inertial-frame. An immediate deduction from this distinction was the possibility of measuring the relative velocity of the observer with reference to ether. The experiment of Michelson and Morley showed it to be unrealisable in practice and formed the starting point of the celebrated Relativity Theory. Einstein had subjected the conception of time-measurement to a searching examination and showed the impossibility of conceiving a time independent of an observer, or an absolute simultaneity of events happening at two different places. The same space-time reference should be chosen for the dynamical equations as well as the equations of the field, this being supplied by the observer. In spite of this apparent limitation Einstein demonstrated the possibility of formulation of natural laws independent of all axes of reference and pointed out that the necessary auxiliaries existed already in the invariant theory and the tensor Calculus of mathematicians. In spite of its apparently revolutionary character, the theory of relativity upheld the ideal of causality and determinism. Einstein himself has continued to seek with great earnestness a unifying field theory which will combine gravitation and electromagnetism and render unnecessary a separate formulation of the dynamical equations. No such theory as yet exists.

II

The development of the quantum theory has raised fundamental issues. Facts have been discovered which demonstrate the breakdown of the fundamental equations which justified our belief in determinism. A critical examination of the way in which physical measurements are made has shown the impossibility of measuring accurately all the quantities necessary for a space-time description of the motion of the corpuscles.

Experiments reveal either the corpuscular or the wave nature for the photon or the electron according to the circumstances of the case, and present us with an apparently impossible task of fusing two contradictory characters into one sensible image. The only solution suggested has been a renunciation of space-time representation of atomic phenomena and with it our belief in causality and determinism.

Let me briefly recapitulate the facts. In 1900 Planck discovered the quantum of action while studying the conditions of equilibrium between matter and the radiation field. Apparently interchange of energy took place in discrete units whose magnitude depended on 'h' and the frequency of the radiation emitted or absorbed by matter. Photo-electric emission had similar disquieting features. Einstein therefore suggested a discrete structure of the radiation field in which energy existed in quanta

instead of being continuously distributed in space as required by the wave-theory. This light-quantum however is not the old light-corpuscle of Newton. The rich experimental materials supporting the wave-theory preclude that possibility altogether. Moreover the fundamental relation, $E=h\nu$, and $p=h\bar{k}$, connecting energy and momentum of the photon with the frequency ν and the vector wave number k, makes a direct reference to idealised plane wave so foreign to the old idea of a corpuscle. Soon afterwards Bohr postulated the existence of radiationless stationary states of atoms and showed how it led to a simple explanation of the atomic spectra. The extreme simplicity of the proposed structure and its striking success in correlating a multitude of experimental facts at once revealed the inadequacy of the ordinary laws of mechanics and electro-dynamics in explaining the remarkable stability of the atoms.

The new ideas found application in different branches of physics. Discontinuous quantum processes furnished solutions to many puzzles. Suitably modified, the theory furnished a reasonable explanation of the periodic classification of elements and thermal behaviour of substances at low temperature. There was however one striking feature. It was apparently impossible to characterise the details of the actual transition processes from one stationary state to another, that is, to visualise it as a continuous sequence of changes determined by any law as yet undiscovered. It became clear that the dynamical laws as well as the laws of electromagnetism failed to account for atomic processes. New laws had to be sought out compatible with the quantum theory capable at the same time of explaining the rich experimental materials of classical physics. Bohr and his pupils utilised for a time a correspondence principle, guessing correct laws for atomic processes from analogy with the results of the classical theory. In every case these appeared as statistical laws concerned with the probabilities of transition between the various atomic states. Einstein tackled the problem of the equilibrium of matter and radiation on the basis of certain hypotheses regarding the probabilities of transition between the various states by absorption and emission. A derivation of the Planck Law was obtained by Bose by a suitable modification of the methods of classical statistics. Heisenberg finally arrived at a satisfactory solution and discovered his matrix-mechanics and a general method for all atomic problems. Dirac and Schrödinger also published simultaneously their independent solutions. Though clothed in apparently dissimilar mathematical symbols, the three theories gave identical results and have now come to be looked upon as different formalisms expressing the same statistical laws.

I have mentioned that the photon gave a simple explanation of many of the properties of radiation and thereby presented its corpuscular aspect while the well-known properties of interference and superposibility brought out its wave character. That the same dual nature may exist in all material corpuscles was first imagined by De Broglie. His phase-waves found quick experimental verification, and raised a similar problem of the real nature of the corpuscle. The formulation of wave-mechanics by Schrödinger, once raised a hope that by a radical modification of our usual ideas about the corpuscle it might be possible to re-establish the law of causality and classical determinism. Subsequent developments have shown such hopes to be illusory. His waves are mathematical fictions utilising the multidimensional representation of a phase-space and are just as incapable of explaining the individuality of the electron, as the photon is incapable of explaining the superposibility of the field. The true meaning of his equations appears in their statistical interpretation.

III

The adherents of the quantum theory interpret the equations in a peculiar way. They maintain that these equations make statements about the behaviour of a simple atom and nothing more than a calculation of the probabilities of transition between its different states is ever possible. There is nothing incomprehensible about such a statistical law even if it relates to the behaviour of a single particle. But a follower of determinism will interpret such statements as betraying imperfect knowledge, either of the attendant circumstances or of the elementary laws. We may record the throws when a certain die is cast a large number of times and arrive at a statistical law which will tell us how many times out of a thousand it will fall on a certain side. But if we can take into account the exact location of its centre of gravity, all the circumstances of the throw, the initial velocity, the resistance of the table and the air and every other peculiarity that may affect it, there can be no question of chance, because each time we can reckon where the die will stop and know in what position it will rest. It is the assertion of the impossibility of even conceiving such elementary determining laws for the atomic system that is disconcerting to the classical physicist.

Von Neumann has analysed the statistical interpretation of the quantum mechanical laws and claims to have demonstrated that the results of the quantum theory cannot be regarded as obtainable from exact causal laws by a process of averaging. He asserts definitely that a causal explanation of quantum mechanics is not possible without an essential modification or sacrifice of some parts of the existing theory.

Bohr has recently analysed the situation and asserted that we cannot hope any future development of the theory will ever allow a return to a description of the atomic phenomena more conformable to the ideal of causality. He points out the importance of the searching analysis of the theory of observation made by Heisenberg, whereby he has arrived at his famous principle of indeterminacy. According to it, it is never possible for us to determine the simultaneous values of momentum, and positional co-ordinates of any system with an accuracy greater than what is compatible with the inequality $\bar{\Delta} p \, \bar{\Delta} q > \frac{h}{4\pi}$.

This natural limitation does not affect the physics of bodies of finite size but makes space-time descriptions of corpuscles and photons impossible. When we proceed to study the behaviour of the elementary particles, our instruments of measurement have an essential influence on the final results. We have also to concede that the contributions of the instrument and the object, are not separately computable from the results as they are interpreted in a classical way with the usual ideas of co-ordinate and momentum accepting thereby a lack of control of all action and reaction of object and instrument due to quantum effects.

It is in this imperative necessity of describing all our knowledge with the usual classical ideas, that Bohr seeks an explanation of the apparently irreconcilable behaviour of corpuscles and radiation in different experiments. For example, if we set our experiments in such a fashion as to determine accurately the space-time co-ordinates, the same arrangement cannot be simultaneously used to calculate the energy momentum relations accurately ; when our arrangements have pushed the accuracy of determining the positional co-ordinates to its utmost limit, the results evidently will be capable only of a corpuscular representation. If, on the other hand, our aim is to determine momentum and energy with the utmost accuracy, the necessary apparatus will not allow us any deter-

mination of positional co-ordinates and the results we obtain can be understood only in terms of the imagery of wave-motion. The apparently contradictory nature of our conclusions is to be explained by the fact, that every measurement has an individual character of its own. The quantum theory does not allow us to separate rigorously the contribution of the object and the instrument and as such the sum total of our knowledge gained in individual cases cannot be synthesised to give a consistent picture of the object of our study which enables us to predict with certainty its behaviour in any particular situation. We are thus doomed to have only statistical laws for these elementary particles and any further development is not likely to affect these general conclusions.

It is clear that a complete acceptance of all the above conclusions would mean a complete break with the ancient accepted principles of scientific explanation. Causality and the universal laws are to be thrown simultaneously overboard. These assertions are so revolutionary that, no wonder, they have forced physicists to opposing camps. There are some who look upon causality as an indispensable postulate for all scientific activities. The inability to apply it consistently because of the limitations of the present state of human knowledge would not justify a total denial of its existence. Granted that physics has outgrown the stage of a mechanistic formulation of the principle, they assert that it is now the task of the scientists to seek for a better formulation. Others of the opposing camp look upon old determinism as an inhuman conception, not only because it sets up an impossible ideal, but also as it forces man to a fatalistic attitude which regards humanity as inanimate automata in the hands of an iron law of causation. For them the new theory has humanised physics. The quantum statistical conception of determinism nestles closer to reality and substitutes a graspable truth for an inaccessible ideal. The theory has brought hope and inspired activity. It constitutes a tremendous step towards the understanding of nature. The features of the present theory may not all be familiar but use will remove the initial prejudice. We are not to impose our reason and philosophy on nature. Our philosophy and our logic evolve and adjust themselves more and more to reality.

In spite of the striking successes of the new theory, its provisional character is often frankly admitted. The field theory is as yet in an unsatisfactory state. In spite of strong optimism, difficulties do not gradually dissolve and disappear. They are relegated to a lumber room, whence the menace of an ultimate divergence of all solutions neutralises much of the convincing force of imposing mathematical symbols. Nor is the problem of matter and radiation solved by the theory of complementary characters. Also we hear already of the limitations of the new theory encountered in its application to nuclear problems.

The quantum theory is frankly utilitarian in its outlook ; but is the ideal of a universal theory completely overthrown by the penetrating criticism of the nature of physical measurements?

Bohr has stressed the unique character of all physical measurements. We try to synthesise their results and we get probabilities to reckon with instead of certainties. But how does the formalism $\frac{h}{2\pi i}\frac{\partial\psi}{\partial t}=H\psi$ emerge as a certain law? The wider the generalisation, the less becomes the content. A universal law would be totally devoid of it. It may nevertheless unfold unsuspected harmonies in the realm of concept. More than ever now, physics does need such a generalisation to bring order in its domain of ideas.

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ON AN INTEGRAL EQUATION ASSOCIATED WITH THE EQUATION FOR HYDROGEN ATOM

By
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I

The Schrödinger functions 'ϕ' characterising the stationary states of hydrogen atom are now very familiar things in analysis as also the differential equation which they satisfy, namely,

$$\nabla^2\phi+\frac{8\pi^2m}{h^2}\left(E+\frac{e^2}{r}\right)\phi=0. \qquad \ldots \quad (1,1)$$

The associated functions M, defined by the relation,

$$\phi=\int M \exp.\, 2\pi\iota(lx+my+nz)dl\, dm\, dn, \qquad \ldots \quad (1,2)$$

can be utilised for defining the probability in momentum space if the momentum variables are introduced by the relation $p_x=hl$, $p_y=hm$, $p_z=hn$.

When inversion is possible, (1,2) implies also

$$M(l, m, n)=\int \phi \exp.-2\pi\iota(lx+my+nz)dx\, dy\, dz \qquad \ldots \quad (1,3)$$

and solutions of (1,1) can be utilised to calculate M's. Elsasser (1933) has followed this method and arrived at fairly complicated formula. Another alternative would be to set up an appropriate equation for them, and investigate its solutions. This is an integral equation, whose complete solution is presented here. The analysis presents several interesting features, and leads to expressions of M's, which can be immediately utilised to study their properties or to apply them to physical problems.

II

If we use the semi-convergent integral

$$\frac{1}{R}=\frac{1}{\pi}\int \frac{dl\, dm\, dn}{l^2+m^2+n^2} \exp.\, 2\pi\iota(lx+my+nz), \quad R^2=x^2+y^2+z^2, \qquad \ldots \quad (2,1)$$

then combination with (1,2) leads to the following sex-tuple integral for ϕ/R, after a change of variable and order of integration :

$$\frac{\phi}{R}=\frac{1}{\pi}\int \frac{M(\lambda-l, \mu-m, \nu-n)}{l^2+m^2+n^2} \exp.\, 2\pi\iota(\lambda x+\mu y+\nu z)dl\, dm\, dn\, d\lambda\, d\mu\, d\nu.$$

M and

$$\frac{1}{\pi}\int \frac{M(\lambda-l, \mu-m, \nu-n)dl\, dm\, dn}{l^2+m^2+n^2}$$

can thus be regarded as Fourier transforms of ϕ and ϕ/R.

Using then in the Schrödinger equation (1,1) we deduce the following integral equation

$$(l^2+m^2+n^2-k^2)M = \lambda \int \frac{M(l-x, m-y, n-z)}{x^2+y^2+z^2}\, dx\, dy\, dz, \qquad \ldots \quad (A)$$

where

$$k^2 = 2mE/h^2. \quad \text{and} \quad \lambda = 2me^2/\pi h^2.$$

M is here assumed to be finite and single-valued throughout the domain of integration, as also

$$\int |M|^2\, dx\, dy\, dz = 1.$$

(A) is the characteristic integral equation for M-functions of hydrogen; for $k^2 < 0$, it leads to the discrete spectrum, while $k^2 > 0$: yields the continuous spectrum.

It is easy to transform (A) to the Fredholm type, when $k^2 < 0 = -a^2$. We first transform the origin, *i.e.*, put $l-x = x'$, etc, in the right side. (A) becomes

$$(l^2+m^2+n^2-k^2)M = -\int \frac{\lambda M(x'y'z')dx'\, dy'\, dz'}{[(l-x')^2+(m-y')^2+(n-z')^2]}\ ; \qquad \ldots \quad (2,32)$$

when $k^2 = -a^2$, we make a similarity transformation

$$l = l'a,\ x' = ax'', \text{ etc.},$$

and put

$$M(l, m, n) = M'(l', m', n')$$

whereby

$$dx'\, dy'\, dz' = a^3 dx''\, dy''\, dz''$$

and

$$(l^2+m^2+n^2+a^2) = a^2(l'^2+m'^2+n'^2+1), \text{ etc.}$$

The relation becomes

$$(l^2+m^2+n^2+1)M = -\frac{\lambda}{a}\int \frac{M'(x'y'z')dx'dy'dz'}{(l-x')^2+(m-y')^2+(n-z')^2.}$$

If further

$$\sqrt{(1+l^2+m^2+n^2)}M = \phi,$$

then

$$\phi(A) = \kappa \int \frac{\phi(P)dv_P}{[(l-x)^2+(m-y)^2+(n-z)^2]\sqrt{(1+l^2+m^2+n^2)}\sqrt{(1+x^2+y^2+z^2)}}\ ,$$

$$\ldots \quad (2,33)$$

where

$$\kappa = \frac{e^2}{\pi\hbar}\left(\frac{2m}{-E}\right)^{\frac{1}{2}}.$$

(2,33) is thus seen of the standard form

$$\phi(A) = -\lambda \int \phi(P)K(A, P)dv_p,$$

where the kernel is symmetrical in A and P. Such a transformation will however make the kernel imaginary within the domain, if $k^2 > 0$; to have a uniform treatment to cover both the cases, we will not utilise the transformation mentioned above.

III

We require the following simple result in our subsequent calculations. If r_1 and r_2 are the distances of a point $P(x, y, z)$ from two fixed points $A(a, b, c)$ and $F(f, g, h)$, the integral

$$I = \int \frac{dx\,dy\,dz}{r_1^2 r_2^2}$$

extended over the whole domain easily transforms to

$$\frac{2\pi}{c}\int_1^\infty \int_{-1}^{1} \frac{dX\,dY}{X^2-Y^2}$$

in bipolar co-ordinates with $AF = 2c$,

$$dx\,dy\,dz = c^3(\cosh^2\xi - \cos^2\eta)\sinh\xi\sin\eta\,d\xi d\eta d\phi,$$

and

$$X = \cosh\xi \quad \text{and} \quad Y = \cos\eta,$$

whence, by easy integration,

$$I = \frac{\pi^3}{2c} = \frac{\pi^3}{AF}. \qquad \ldots \quad (3,1)$$

Again, if

$$A(a, b, c) = \lambda \int \frac{M(a-x, b-y, c-z)dx\,dy\,dz}{x^2+y^2+z^2} = -\lambda \int \frac{M(x'y'z')dx'dy'dz'}{(a-x')^2+(b-y')^2+(c-z')^2} \qquad \ldots \quad (3,2)$$

we have, by multiplying the equation with

$$\frac{da\,db\,dc}{(f-a)^2+(g-b)^2+(h-c)^2}$$

and integrating over the whole domain,

$$\int \frac{A(a\,b\,c)da\,db\,dc}{[(f-a)^2+(g-b)^2+(h-c)^2]} = -\lambda \int \int \frac{M(x'y'z')dx'\,dy'dz'\,da\,db\,dc}{[(a-x')^2+\ldots][(f-a)^2+\ldots]}.$$

Changing the order of integration and utilising (3,1) we have

$$\int \frac{M(x'y'z')dx'dy'dz'}{[(f-x')^2+(g-y')^2+(h-z')^2]^{\frac{1}{2}}} = -\frac{1}{\pi^3\lambda}\int \frac{A(abc)da\,db\,dc}{[(f-a)^2+(g-b)^2+(h-c)^2]}$$

$$= \frac{1}{\pi^3\lambda}\int \frac{A(f-a',\ldots)da'db'dc'}{a'^2+b'^2+c'^2}\ ;$$

hence, operating with $\nabla^2_{(fgh)}$ on both sides, as

$$\nabla^2_{(fgh)}\int \frac{M(x'y'z')dx'dy'dz'}{[(f-x')^2+\ldots]^{\frac{1}{2}}} = -4\pi M(f,g,h)$$

from potential theory, and

$$\nabla^2_{(fgh)}\{A(f-a, g-b, h-c)\} = (\nabla^2 A)(f-a, g-b, h-c),$$

we have finally

$$M(f,g,h) = -\frac{1}{4\pi^4\lambda}\int \frac{(\nabla^2 A)(f-a, g-b, h-c)\,da\,db\,dc}{a^2+b^2+c^2}$$

$$= \frac{1}{4\pi^4\lambda}\int \frac{(\nabla^2 A')dx'dy'dz'}{[(f-x')^2+(g-y')^2+(h-z')^2]} \qquad \ldots \quad (3,3)$$

as a solution of (3,2).

This important result furnishing a solution of integral equation (3,2) with obvious restrictions about the nature of the function A, enables us to tackle our present problem.

IV

According to a well-known theorem due to Hobson, if an operator $S\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)$ is constructed by substituting $\frac{d}{dx}, \ldots$, etc. for x, y, z in a solid harmonic S_n of degree n (a positive integer),

$$S_n\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)F(r^2) = 2^n S_n(x,y,z)\left(\frac{d}{dr^2}\right)^n F. \qquad \ldots \quad (4,1)$$

Let us assume, with regard to the integral equation (A), that

$$M(x,y,z) = S_n(x,y,z)f(r)$$

or

$$M(x,y,z) = S_n\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)A(r^2) \qquad \ldots \quad (4,2)$$

in view of (4,1) ; also $(r^2-k^2)M(x,y,z)$ can be written as

$$S_n\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) B(r^2) \qquad \ldots \quad (4,3)$$

with the same operator, as the same surface-harmonic will occur in both. Also as

$$S_n\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) A = 2^n S_n(x, y, z) \left(\frac{d}{dr^2}\right)^n A = M,$$

and

$$S_n\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) B = 2^n S_n(x, y, z) \left(\frac{d}{dr^2}\right)^n B = (r^2-k^2)M,$$

we have

$$\left(\frac{d}{dr^2}\right)^n B = (r^2-k^2)\left(\frac{d}{dr^2}\right)^n A = \left(\frac{d}{dr^2}\right)^{n-1}[A(r^2-k^2)-nA],$$

implying

$$\frac{dB}{dr} = \frac{d}{dr}[A(r^2-k^2)]-2nAr. \qquad \ldots \quad (4,4)$$

We have also from (A)

$$S_n\left[\frac{d}{da}, \ \ldots\ \right] B = \lambda \int \frac{S_n\left[\dfrac{d}{d(a-x)}, \ \ldots\ \right] A \, dx\, dy\, dz}{x^2+y^2+z^2}$$

$$= \lambda S_n\left(\frac{d}{da}, \ \ldots\ \right) \int \frac{A[(a-x)^2+\ldots]dx\,dy\,dz}{x^2+y^2+z^2}.$$

Hence removing the operator S_n

$$B = \lambda \int \frac{A[a-x)^2+\ldots]dx\, dy\, dz,}{x^2+y^2+z^2}, \qquad \ldots \quad (4,51)$$

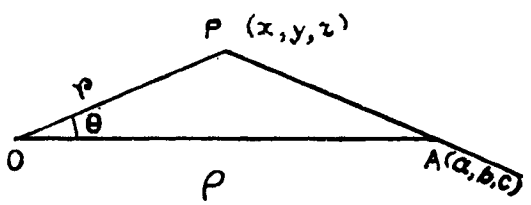
and hence on account of (3.3)

$$A = -\frac{1}{4\pi^4\lambda} \int \frac{(\nabla^2 B)(a-x, b-y, c-z)\, dx\, dy\, dz}{x^2+y^2+z^2}. \qquad \ldots \quad (4,52)$$

We now perform the integration ; assuming

$$(a-x)^2+(b-y)^2+(c-z)^2 = f^2 = r^2+\rho^2-2r\rho\cos\theta$$

according to the accompanying figure



we have, as $fdf = r\rho \sin\theta$,

$$\rho B(\rho^2) = 2\pi\lambda \int_0^\infty \frac{dr}{r} \int_{f^2=(\rho-r)^2}^{f^2=(\rho+r)^2} \tfrac{1}{2} A\,(f^2)\, df^2$$

$$= 2\pi\lambda \int_0^\infty \frac{dx}{x}[G_1(\rho+x) - G_1(\rho-x)],$$

where

$$\frac{1}{x}\frac{dG}{dx} = A(x^2)$$

and G_1, an even function of x. Similarly from (4,52)

$$\rho A = -\frac{1}{2\pi^3\lambda} \int_0^\infty \frac{dx}{x}\,[L(\rho+x) - L(\rho-x)],$$

where

$$\frac{dL}{xdx} = \nabla^2 B(x^2);$$

hence integrating and putting

$$\frac{dQ}{dx} = L, \text{ or } \frac{1}{x}\frac{d^2Q}{dx^2} = \nabla^2 B$$

we have

$$G_1(\rho) = -\frac{1}{2\pi^3\lambda} \int \frac{dx}{x}\,[Q(\rho+x) - Q(\rho-x)],$$

where

$$\frac{1}{x}\frac{d^2Q}{dx^2} = \nabla^2 B = \frac{1}{x}\frac{d^2}{dx^2}(xB),$$

$$\text{or} \quad Q = xB,$$

and therefore an odd function of x.

Putting finally

$$\frac{xB(x^2)}{2\pi^2\lambda} = G_2 = \frac{Q}{2\pi^2\lambda},$$

we have the following relations between G_1 and G_2 in the skew-reciprocal form :

$$G_2(\rho) = \frac{1}{\pi} \int_0^\infty \frac{G_1(\rho+x) - G_1(\rho-x)}{x}\, dx, \qquad \ldots \quad (4,53)$$

$$G_1(\rho) = -\frac{1}{\pi} \int_0^\infty \frac{G_2(\rho+x) - G_2(\rho-x)}{x}\, dx. \qquad \ldots \quad (4,54)$$

If the functions G_1 and G_2 are introduced in (4,4) we have

$$\frac{d}{dr}\left[\frac{G_2}{r}\right] = \frac{1}{2\pi^2\lambda}\left[\frac{d}{dr}\left(\frac{r^2-k^2}{r}\cdot\frac{dG_1}{dr}\right) - 2n\frac{dG_1}{dr}\right],$$

or

$$G_2 = \frac{1}{2\pi^2\lambda}\left[\ (x^2-k^2)\ \frac{dG_1}{dx} - 2nxG_1\ \right],$$

or

$$\frac{G_2}{(x^2-k^2)^{n+1}} = \frac{1}{2\pi^2\lambda}\frac{d}{dx}\left[\ \frac{G_1}{(x^2-k^2)^n}\ \right]. \qquad \ldots \quad (4,55)$$

The skew-reciprocal relations (4,53) and (4,54) then at once suggest that

$$\frac{G_1}{(x^2-k^2)^{n+1}} = -\frac{1}{2\pi^2\lambda}\frac{d}{dx}\left[\ \frac{G_2}{(x^2-k^2)^n}\ \right] \qquad \ldots \quad (4,56)$$

is true at the same time.

(4,55) and (4,56), lead at once, by integration, to

$$G_1 = \mathsf{A}(x^2-k^2)^n \cos\left(\ \frac{2\pi^2\lambda}{2k}\log\left|\frac{x-k}{x+k}\right|\ \right),$$

$$G_2 = -\mathsf{A}(x^2-k^2)^n \sin\left(\frac{2\pi^2\lambda}{2k}\log\left|\frac{x-k}{x+k}\right|\right), \qquad (4,57)$$

when $k^2 > 0$; when $k^2 = -a^2$, on the other hand,

$$G_1 = \mathsf{B}\,(x^2+a^2)^n \cos\left(\ \frac{2\pi^2\lambda}{a}\tan^{-1}\ \frac{x}{a}\right),$$

$$G_2 = -\mathsf{B}(x^2+a^2)^n \sin\left(\ \frac{2\pi^2\lambda}{a}\tan^{-1}\ \frac{x}{a}\right). \qquad \ldots \quad (4,58)$$

The two equivalent forms of M are

$$M(x,y,z) = \mathsf{A}\ S_n(x,y,z)\left(\frac{1}{r}\frac{d}{dr}\right)^{n+1}\left[\ (r^2-k^2)^n \cos\left(\ \frac{2\pi^2\lambda}{2k}\log\left|\frac{r-k}{r+k}\right|\ \right)\right]$$

or

$$= -2\pi^2\lambda\,\frac{\mathsf{A}S_n(x,y,z)}{r^2-k^2}\left(\ \frac{1}{r}\frac{d}{dr}\right)^n\left[\ \frac{(r^2-k^2)^n}{r}\sin\left(\ \frac{2\pi^2\lambda}{2k}\ \log\left|\frac{r-k}{r+k}\right|\ \right)\right] \qquad \ldots \quad (4,591)$$

for the case : $k^2 > 0$; while, if $k^2 = -a^2$,

$$M(x,y,z) = \mathsf{B}S_n(x,y,z)\left(\frac{1}{r}\frac{d}{dr}\right)^{n+1}\left[\ (r^2+a^2)^n\ \cos\left(\ \frac{2\pi^2\lambda}{a}\ \tan^{-1}\ \frac{r}{a}\right)\right]$$

or

$$= -2\pi^2\lambda\ \mathsf{B}\ \frac{S_n(x,y,z)}{r^2+a^2}\left(\ \frac{1}{r}\frac{d}{dr}\right)^n\left[\ \frac{(r^2+a^2)^n}{r}\ \sin\left(\ \frac{2\pi^2\lambda}{r}\ \tan^{-1}\frac{r}{a}\right)\right]. \quad \ldots \quad (4.592)$$

V

As M is to be single-valued, we see, in the case of $k^2\ (<0) = -a^2$, that a restriction comes in regarding the choice of constants of the problem.

$\tan^{-1}(r/a)$ is a multiple-valued function of the form $\theta+N\pi$; if therefore $\cos\left(\frac{2\pi^2\lambda}{a}\ \tan^{-1}\frac{r}{a}\right)$ and $\sin\left(\frac{2\pi^2\lambda}{a}\ \tan^{-1}\frac{r}{a}\right)$ are single-valued, it follows that $\pi^2\lambda/a$ must be an integer $= N$. This gives discrete energy-values :

$$k^2 = \frac{2mE}{\hbar^2} = -\frac{\pi^4\lambda^2}{N^2},$$

or

$$E = -\frac{2\pi^2e^4m}{\hbar^2N^2}$$

agreeing with the well-known result. Also as

$$2\tan^{-1}\frac{r}{a} = \cos^{-1}\frac{a^2-r^2}{a^2+r^2},$$

$$\cos\left(N\cos^{-1}\frac{a^2-r^2}{a^2+r^2}\right) = \frac{f_N(r^2)}{(a^2+r^2)^N}$$

we see that $M = 0$ unless $N \geqslant n+1$.

Again starting from the series

$$\frac{1}{1-2h\cos\theta+h^2} = \sum_1 h^{N-1}\frac{\sin N\theta}{\sin\theta},\quad \frac{1-h\cos\theta}{1-2h\cos\theta+h^2} = \sum_0 h^N\cos N\theta, \qquad (5,1)$$

and writing

$$\cos\theta = \frac{a^2-r^2}{a^2+r^2}\ \text{and}\ \sin\theta = \frac{2ar}{a^2+r^2}$$

we have

$$\frac{2a(a^2+r^2)}{\{(1-h)^2a^2+(1+h)^2r^2\}} = \Sigma\, h^{N-1}\,\frac{(a^2+r^2)}{r}\,\sin N\theta, \qquad \ldots\ (5,2)$$

$$\frac{a^2(1-h)+r^2(1+h)}{\{(1-h)^2a^2+(1+h)^2r^2\}} = \Sigma h^N\cos N\theta. \qquad \ldots\ (5,3)$$

Multiplying (5,2) with $(a^2+r^2)^{n-1}$ and differentiating n times with regard to r, we have

$$\frac{1}{[(1-h)^2a^2+(1+h)^2r^2]^{n+1}}$$

$$= \frac{(-1)^n}{n!2^{2n+1}a^{2n+1}} \Sigma h^{N-n-1} \left(\frac{d}{dr^2} \right)^n \left[\frac{(a^2+r^2)^n}{r} \sin \left(N \cos^{-1} \left(\frac{a^2-r^2}{a^2+r^2} \right) \right) \right]; \quad \ldots \quad (5,4)$$

from (5,3) we deduce similarly

$$\left(\frac{1-h^2}{[(1-h)^2a^2+(1+h)^2r^2]^{n+2}} \right)$$

$$= \frac{(-1)^{n+1}}{2^{2n+1}a^{2n+2}} \cdot \frac{1}{(n+1)!} \Sigma h^{N-n-1} \left(\frac{d}{dr^2} \right)^{n+1} \left[(a^2+r^2)^n \cos \left(N \cos^{-1} \frac{a^2-r^2}{a^2+r^2} \right) \right]. \quad (5,5)$$

Remembering the Gegenbauer expansion

$$\frac{1}{(\alpha^2-2\alpha h \cos\theta+h^2)^\nu} = \frac{1}{\alpha^{2\nu}} + \sum_\mu \frac{(2h)^\mu}{\alpha^{2\nu+\mu}} F^\nu_\mu (\cos\theta)$$

we can easily deduce, if $\nu = n+1$, $\alpha = 1$ and

$$\cos\theta = \frac{a^2-r^2}{a^2+r^2},$$

the following results :

$$\frac{1}{[a^2(1-h)^2+r^2(1+h)^2]^{n+1}} = \frac{1}{(a^2+r^2)^{n+1}} \left[1+\sum_1 (2h)^\lambda F^{n+1}_\lambda \left(\frac{a^2-r^2}{a^2+r^2} \right) \right], \quad \ldots \quad (5,6)$$

$$\frac{1-h^2}{[a^2(1-h)^2+r^2(1+h)^2]^{n+2}} = \frac{1}{(a^2+r^2)^{n+2}} \left[1+\sum_\lambda \frac{(n+1+\lambda)}{n+1} (2h)^\lambda F^{n+1}_\lambda \left(\frac{a^2-r^2}{a^2+r^2} \right) \right]. \quad (5,7)$$

Comparing (5.4) and (5.5) with (5.6) and (5.7) we have

$$\frac{1}{(a^2+r^2)} \left(\frac{d}{dr^2} \right)^n \left[\frac{(a^2+r^2)^n}{r} \sin N\theta \right] = \frac{(-1)^n 2^{N+n} a^{2n+1} n!}{(a^2+r^2)^{n+2}} F^{n+1}_{N-n-1} \left(\frac{a^2-r^2}{a^2+r^2} \right),$$

and

$$\left(\frac{d}{dr^2} \right)^{n+1} \left[(a^2+r^2)^n \cos N\theta \right] = \frac{(-1)^{n+1} 2^{N+n} a^{2n+2} N \cdot n!}{(a^2+r^2)^{n+2}} F^{n+1}_{N-n-1} \left(\frac{a^2-r^2}{a^2+r^2} \right).$$

Hence

$$M(x, y, z) = \mathbf{B} S_n(x, y, z) \left(\frac{1}{r} \frac{d}{dr} \right)^{n+1} \left[(r^2+a^2)^n \cos \left(N \cos^{-1} \frac{a^2-r^2}{a^2+r^2} \right) \right]$$

or

$$= -2Na\,\frac{\mathbf{B}S_n(x,y,z)}{a^2+r^2}\left(\frac{d}{rdr}\right)^n\left[\frac{(a^2+r^2)^n}{r}\sin\left(N\cos^{-1}\frac{a^2-r^2}{a^2+r^2}\right)\right]$$

$$= CS_n(x,y,z)\,F^{n+1}_{N-n-1}\left(\frac{a^2-r^2}{a^2+r^2}\right)$$

VI

When $k^2 < 0$, the problem can be transformed to a homogeneous integral equation of the Fredholm type, with a symmetrical kernel

$$\phi(A) = \lambda\int \phi(P)K(A,P)dv_P.$$

The results that we have obtained above enable us also to say that

$$\phi = C\sqrt{1+x^2+y^2+z^2}\;S_n(x,y,z)\left(\frac{1}{r}\frac{d}{dr}\right)^{n+1}\left[(1+x^2+y^2+z^2)^n\cos\left(N\cos^{-1}\frac{1-r^2}{1+r^2}\right)\right]$$

$$= D\frac{S_n(x,y,z)}{(1+x^2+y^2+z^2)^{n+3/2}}F^{n+1}_\lambda\left(\frac{1-r^2}{1+r^2}\right) \qquad \ldots\ (6{,}1)$$

are eigen-functions corresponding to the eigen-value $-(n+1+\lambda)/n^2$ for the kernel

$$K(A,P) = \frac{1}{(x-x')^2+(y-y')^2+(z-z')^2\sqrt{(1+x^2+y^2+z^2)}\sqrt{(1+x'^2+y'^2+z'^2)}}.$$

Writing

$$M(x,y,z) = \frac{\mathbf{C}S_n(x,y,z)}{(a^2+r^2)^{n+2}}F^{n+1}_\lambda\left(\frac{a^2-r^2}{a^2+r^2}\right) = \mathbf{C}\,Y_n(\theta,\phi)\frac{r^n}{(a^2+r^2)^{n+2}}F^{n+1}_\lambda\left(\frac{a^2-r^2}{a^2+r^2}\right),$$

also

$$\int M^2dxdydz = 1 = \mathbf{C}^2\int Y_n^2(\theta,\phi)d\omega\int\frac{r^{2n+2}}{(a^2+r^2)^{2n+4}}\left[F^{n+1}_\lambda\left(\frac{a^2-r^2}{a^2+r^2}\right)\right]^2dr.$$

To determine the radial integral, we use (5,7) and write

$$\mathrm{I} = \int\frac{(1-h^2)(1-t^2)r^{2n+2}dr}{[a^2(1-h)^2+r^2(1+h)^2]^{n+2}[a^2(1-t)^2+r^2(1+t)^2]^{n+2}}$$

$$= \sum_\lambda\sum_\sigma\frac{(n+1+\lambda)(n+1+\sigma)}{(n+1)^2}(2h)^\lambda\,(2t)^\sigma\int\int\frac{F^{n+1}_\lambda F^{n+1}_\sigma r^{2n+2}}{(a^2+r^2)^{2n+4}}dr,$$

which reduces to

$$I = \frac{(-1)^{n+1}}{(n+1)!}\cdot\frac{(1-h^2)(1-t^2)}{(1+h)^{2n+4}(1+t)^{2n+4}}\cdot\left\{\frac{d}{d(p+q)}\right\}^{n+1}\int_0^\infty\frac{dr}{[p+r^2][q+r^2]},$$

where

$$p = \frac{a^2(1-h)^2}{(1+h)^2},\ q = \frac{a^2(1-t)^2}{(1+t)^2},$$

when, by easy calculation,

$$I = \frac{\pi}{2^{4n+5}a^{2n+5}} \cdot \frac{(2n+1)!}{n!(n+1)!} \frac{(1+h)(1+t)}{(1-ht)^{2n+3}} . \qquad \ldots \quad (6.2)$$

Equating coefficient of $(ht)^\lambda$

$$\int \frac{r^{2n+2}}{(a^2+r^2)^{2n+4}} \left(F_\lambda^{n+1} \right)^2 dr = \frac{\pi(2n+1+\lambda)!}{2^{4n+2\lambda+5}\, a^{2n+5}(n+1+\lambda)\lambda!\{n!\}^2} ;$$

hence

$$\overline{M} \text{ (normalised)} = 2^{2n+\lambda+3}a^{n+3}\left(\frac{N}{2\pi a} \right)^{\frac{1}{2}} n! \left\{ \frac{\lambda!}{(N+n)!} \right\}^{\frac{1}{2}} \frac{F_\lambda^{n+1}\left(\frac{a^2-r^2}{a^2+r^2} \right)}{(a^2+r^2)^{n+2}} \overline{Y}_n(\theta, \phi), \qquad \ldots \quad (6.3]$$

where $\overline{Y}_n(\theta, \phi)$ is taken to be a normalised spherical harmonic.

We can normalise the eigen-functions of the homogeneous integral equation, for which similarly the value of

$$\int \frac{r^{2n+2}}{(1+r^2)^{2n+3}} \left(F_\lambda^{n+1} \right)^2 dr$$

will be necessary; for it, we utilise again (5.7) and write

$$\mathrm{I} = \int_0^\infty \frac{(1-h^2]r^{2n+2}dr}{[(1-h)^2+(1+h)^2r^2]^{n+2}[(1-t)^2+(1+t)^2r^2]^{n+1}}$$

$$= \sum_\lambda \sum_\sigma (2h)^\lambda (2t)^\sigma \frac{(n+1+\lambda)}{(n+1)} \int \frac{F_\lambda^{n+1} F_\sigma^{n+1} r^{2n+2} dr}{(1+r^2)^{2n+3}} ;$$

after easy integration

$$I = \frac{\pi}{2^{4n+4}} \cdot \frac{(2n+1)!}{n!\,(n+1)!} \cdot \frac{1}{(1-ht)^{2n+2}} . \qquad \ldots \quad (6.4)$$

Equating coefficient we have

$$\int \frac{\{F_\lambda^{n+1}\}^2 r^{2n+2} dr}{(1+r^2)^{2n+3}} = \frac{\pi}{2^{4n+2\lambda+4}(n!)^2(n+1+\lambda)} \cdot \frac{(2n+1+\lambda)!}{\lambda!} .$$

The normalised eigen-functions of the homogeneous equation is

$$\psi = \frac{2^{2n+\lambda+2}n!\,(n+1+\lambda)^{\frac{1}{2}}}{\cdot\ \sqrt{\pi}} \cdot \left(\frac{\lambda!}{(2n+1+\lambda)!} \right)^{\frac{1}{2}} \frac{Y_n(\theta, \phi)r^n}{(1+r^2)^{n+3/2}} F_\lambda^{n+1}\left(\frac{1-r^2}{1+r^2} \right). \ldots (6.5)$$

The completeness of the eigen-functions series will mean the equality

$$K(x, y) = \sum_n \frac{\pi^2 \psi_r^n(x)\, \psi_r^n(y)}{(n+1+\lambda)}$$

(multiple summations as there are repeated roots) leading to the relation

$$\frac{1}{[(x-x')^2+(y-y')^2+(z-z')^2][1+r^2]^{\frac{1}{2}}[1+r'^2]^{\frac{1}{2}}}$$

$$= \sum_1 \frac{\pi^2}{N} \sum_{n=0}^{n=N} \frac{2^{2N+2n+2}}{\pi} (n!)^2 N \left[\frac{(N-n-1)!}{(N+n)!} \right] \frac{r^n}{(1+r^2)^{3/2}} \cdot \frac{r'^n}{(1+r'^2)^{3/2}} F_\lambda^{n+1} F'^{n+1}_\lambda$$

$$\times \frac{(2n+1)}{4\pi} \left[\sum_1^n 2P_n^m(\mu)P_n^m(\mu') \frac{(n-m)!}{(n+m)!} \cos m(\phi-\phi') + P_n(\mu) P_n(\mu') \right]$$

and leading to an expansion of

$$\frac{1}{(x-x')^2+(y-y')^2+(z-z')^2} = \sum_N 2^{2N} \sum_0^{N-1} 2^{2n} F_{N-n-1}^{n+1} \left(\frac{1-r^2}{1+r^2} \right) F_{N-n-1}^{n+1}$$

$$\left(\frac{1-r'^2}{1+r'^2} \right) \frac{r^n r'^n}{(1+r^2)^{n+1}(1+r'^2)^{n+1}} \left[\frac{(N-n-1!)}{(N+n)!} \right] (n!)^2 \left[P_n(\mu)P_n(\mu') \right.$$

$$\left. + 2 \sum_1^n \frac{(n-m)!}{(n+m)!} P_n^m(\mu)P_n^m(\mu') \cos m(\phi-\phi') \right]; \qquad \ldots \quad (6.7)$$

by a change of axis making $(x'\ y'\ z')$ on the z axis

$$\frac{1}{r^2+r'^2-2rr'\mu} = \sum_N 2^{2N} \sum_n 2^{2n} F_{N-n-1}^{n+1} \left(\frac{1-r^2}{1+r^2} \right) \cdot F_{N-n-1}^{n+1} \left(\frac{1-r'^2}{1+r'^2} \right) \frac{r^n r'^n}{(1+r^2)^{n+1}\,(1+r'^2)^{n+1}}$$

$$\times (n!)^2 \left[\frac{(N-n-1)\,!}{(N+n)\,!} \right] (2n+1)\ P_n(\mu) \qquad (6.8)$$

from the well-known addition theorem of $P_n(\mu)$.

To verify the correctness of the result we note that

$$\frac{2}{2[r^2+r'^2-2rr'\mu]} = \frac{2}{(1+r^2)(r'^2+1)\left[\ 1-\frac{1-r^2}{1+r^2}\cdot\frac{1-r'^2}{1+r'^2}-\frac{4rr'}{(1+r^2)(1+r'^2)}\cos\phi\ \right]};$$

hence writing

$$\cos\theta = \frac{1-r^2}{1+r^2}, \quad \cos\theta' = \frac{1-r'^2}{1+r'^2},$$

we have

$$\frac{1}{r+r'^2-2rr'\mu^2} = \frac{2}{(1+r^2)(1+r'^2)[1-\cos\theta\cos\theta'-\sin\theta\sin\theta'\cos\phi]}$$

$$= \frac{4}{(1+r^2)(1+r'^2)}\sum_1^N \frac{\sin N\chi}{\sin\chi}, \tag{6·81}$$

if $\pi/2 > \chi > 0$ from (5.1), and $\cos\chi = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos\phi$; hence

$$\frac{\sin N\chi}{\sin\chi} = \sum_{n=0}^{N-1} 2^{2(N-1)}(2n+1)(n!)^2\left[\frac{(N-n-1)\,!}{(N+n)\,!}\right]$$

$$\times \sin^n\theta\sin^n\theta' F_{N-n-1}^{n+1}(\cos\theta)\cdot F_{N-n-1}^{n+1}(\cos\theta')\cdot P_n(\cos\phi), \tag{6.9}$$

a result which is a particular case of a general class of identity deduced by Gegenbauer (Whittaker, 1927).

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Letters to the Editor

GERMANIUM IN SPHALERITE FROM NEPAL

IN VIEW OF THE RECENT USE OF GERMANIUM crystal as a rectifier for ultra-short radio waves, it is worth while to make a thorough investigation of the possible sources of germanium in India. Recently, one of us made a search for germanium in coal ashes from different coalfields in India and a fair concentration of germanium was detected in a few samples[1,2].

Argyrodite, canfieldite and germanite are the only ores rich in germanium known at present. But none of them have been reported to occur in India. Zinc blende from different countries has been investigated by different workers[3] and in majority of samples germanium has been detected in traces.

Specimens of sphalerite collected from Nepal by one us have been spectroscopically examined. Two of these samples have been found to contain germanium (FIGS. 1-4). The following metals have also been detected in the mineral : Sn, Pb, Mg, Al, Si, Ga, Cd, Ge, Ag and Fe.

During our investigation, we found that germanium occurs concentrated in the magnetic portion of the mineral which seems to be almost pure iron oxide with traces of sulphur, lead, zinc, gallium and germanium. The non-magnetic portions were also examined spectroscopically and found to contain germanium only in traces.

Germanium was estimated colorimetrically according to the method by Marcel Orliac[4] slightly modified by us.

The percentage of germanium in the magnetic portion has been found to be about 0·24 while it varies in the non-magnetic portion from 0·0072 to 0·04. 300 mg. of pure GeO_2 has been extracted from 160 gm. of the magnetic portion of the mineral.

The most abundant source of germanium at present in U.S.A. is the concentrate from residues obtained during the melting of zinc ores. Buchanan reports as much as 0·25 per cent of GeO_2 from these residues. As the magnetic fraction from sphalerite contains about 0·24 per cent of germanium, it can be regarded as a good source of germanium.

It appears that germanium has not been detected in any iron ore so far ; thus this new mineral needs further study. We have spectroscopically examined a few haematite and pyrites samples but none of them contains germanium.

We take this opportunity to thank the Nepal Government and the Director of the Nepal Bureau of Mines for the samples of sphalerite. Our grateful thanks are due

The first reference on p. 271 makes a wrong ascription to Dutta and Sen instead of Dutta and Bose, in the original publication itself.

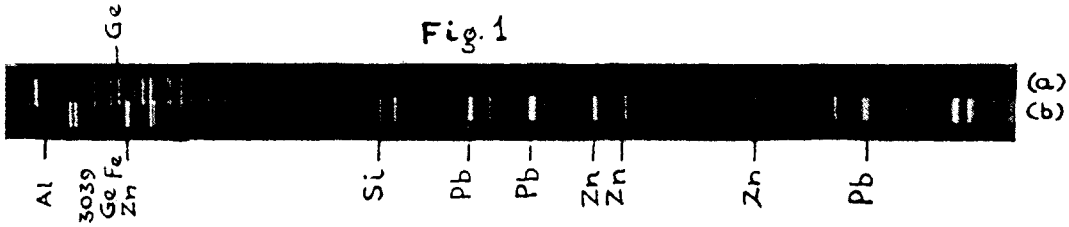


Fig. 1

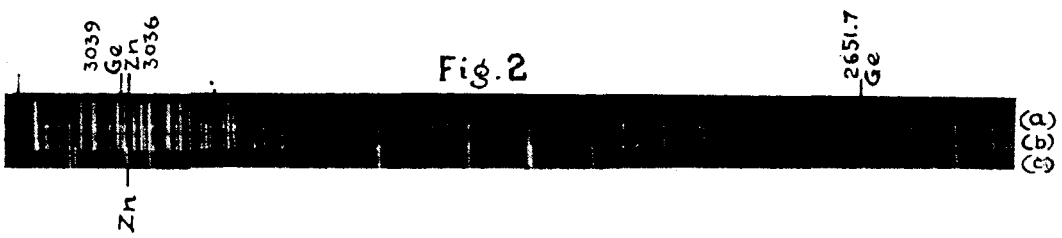


Fig. 2

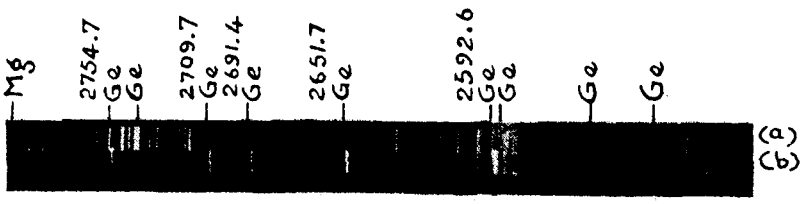


Fig. 3

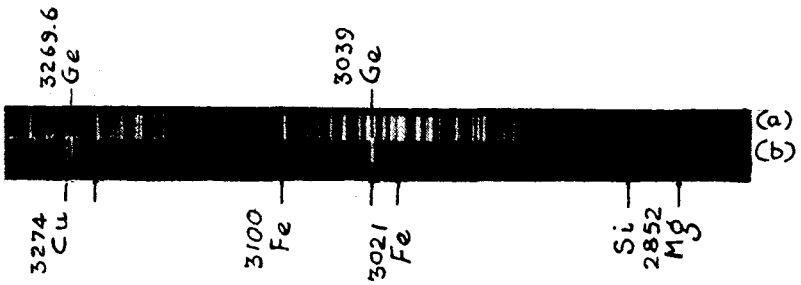


Fig. 4

FIG. 1 — (a) Fe, (b) SPHALERITE. FIG. 2 — (a) Fe, (b) MAGNETIC SAMPLE, (c) NON-MAGNETIC SAMPLE. FIG. 3 — (a) Fe, (b) Ge WITH TRACE OF Fe. FIG. 4 — (a) Fe, (b) Ge WITH TRACE OF Fe.

to Prof. P. B. Sarkar for his keen interest during the progress of the work. One of us (R.K.D.) is indebted to the *Council of Scientific & Industrial Research* for a research grant.

S. N. BOSE
R. K. DATTA

University College of Science & Technology
Calcutta
October 27, 1949

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Extraction of Germanium from Sphalerite Collected from Nepal—Part I

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A method for the extraction of germanium from the magnetic fraction of the mineral sphalerite has been described. The fraction which is shown to be magnetite contains 0·24 per cent germanium.

IN a previous communication[1], we described a magnetic fraction of the mineral sphalerite collected from Nepal. On chemical analysis, this fraction was found to be mainly ferric oxide with a coating of ferrous sulphide. It contains germanium to the extent of 0·24 per cent. That the mineral is magnetite has also been confirmed by X-ray analysis. Coal ash, euxenite and some specimens of zinc blende are known to contain germanium in small quantities, but its association with magnetite has not been reported.

Experimental

Analysis of the Mineral — The spectrogram revealed the existence of iron, silicon, lead (trace) and germanium.

Total Iron — A weighed amount of finely powdered mineral was dissolved in 1 : 1 hydrochloric acid and the total iron estimated by titration with potassium dichromate after reduction with stannous chloride.

Silica was estimated by the usual method, and the value verified by determining the loss in weight by treatment with hydrofluoric acid.

Sulphur — The mineral was decomposed with sodium peroxide and the sulphur estimated as barium sulphate.

Ferrous iron was estimated by decomposing the mineral with 5 c.c. of concentrated hydrochloric acid and 10 c.c. of hydrofluoric acid in an atmosphere of carbon dioxide, and subsequent titration with potassium dichromate using diphenylamine sulphonate as indicator.

Germanium — Of the various methods known for the estimation of germanium[3-5], Orliac's colorimetric method[5] with slight modification was found to be the most convenient. A weighed amount of the mineral was carefully roasted at 500°C. in an electric furnace and transferred to an all-glass distillation apparatus. 40 c.c. of 1 : 1 hydrochloric acid were added and the flask gently heated in a current of air. The distillate was collected in a flask containing dilute caustic soda solution and a drop of phenolphthalein indicator solution. After the distillation was complete, the solution was acidified with dilute hydrochloric acid and the volume made up to 100 c.c. To 10 c.c. of this solution, 1 c.c. ammonium molybdate solution (9 per cent) and 10 c.c. of alkaline potassium stannite solution containing 5 gm. of hydrated stannous chloride and 300 gm. of potassium hydroxide per litre were added and the total volume made up to 40 c.c. A series of standard solutions of germanium tetrachloride were

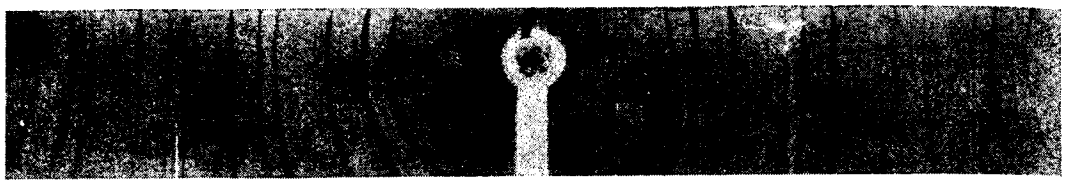


FIG. 1 — X-RAY POWDER DIAGRAM OF THE NON-MAGNETIC PORTION FROM THE MINERAL SPHALERITE.

prepared and the same volume of ammonium molybdate and alkaline stannite were added and the total volume in each case made up to 40 c.c. After an interval of 5 min. the transmittancies of the solutions were determined in a Lumetron photo-electric colorimeter, using 440 mμ filter, and a standard graph was constructed. From this graph germanium in the mineral was found to be 0·24 per cent.

The results of the analysis are tabulated below :

	%		
Total iron	69·5	Fe_2O_3	66·28
Ferrous iron	23·1	FeO	27·02
Sulphur	1·19	FeS	3·27
Silica	2·16	SiO_2	2·16
Germanium	0·24	GeO_2	0·34
			99·07

The mineral is apparently magnetite with a coating of ferrous sulphide. This has now been confirmed by X-ray powder diagram kindly supplied by Mons. Pierre Urbain, Director, Hydrological Institute, Sorbonne, Paris. The rontgenogram is reproduced in Fig. 1.

Extraction of Germanium Dioxide — None of the methods employed by Winkler for the extraction of germanium from Argyrodite was adopted, as the associated elements in this mineral are quite different from those in Argyrodite. After various trials the final procedure adopted was based upon the volatility of germanium tetrachloride and is an elaboration of the procedure described by Buchanan[2].

160 gm. of the finely powdered mineral were roasted in an electric muffle at about 550°C. for 12 hr. when all the sulphide was converted to sulphate and oxide. The roast ore was transferred to an all-glass Claisen flask. 400 c.c. of 1 : 1 hydrochloric acid were added and germanium tetrachloride distilled off in a current of hydrochloric acid gas till the volume is reduced to one-fourth. This distillation was repeated with further additions of hydrochloric acid. A rapid stream of hydrogen sulphide gas was then passed through the distillate for about 15 min. when a milky precipitate of germanium disulphide was obtained (for complete precipitation, the acid strength was maintained at 4N to 6N by the addition of concentrated hydrochloric acid). The mixture was set aside in a stoppered bottle for 48 hr., the sulphide separated by filtration and washed with 6N sulphuric acid saturated with hydrogen sulphide till it was free from chloride. The precipitate on the filter paper was treated with aqueous ammonia (about 8N) and the filtrate collected. The germanium disulphide was thus obtained as the thio-salt. The filtrate was evaporated, hydrogen peroxide added and again evaporated to dryness. The dried mass was carefully heated to remove ammonium sulphate completely, transferred to a crucible with water, evaporated and finally ignited to germanium dioxide at 600°C. The yield was 330 mg.

The spectrum of germanium dioxide thus obtained reveals the existence of traces of iron, lead, phosphorus (impurities from the hydrogen peroxide used).

Acknowledgement

Our grateful thanks are due to Prof. P. B. Sarkar for his keen interest in this investigation, and to the *Council of Scientific Industrial Research* for granting a scholarship to one of us (R. K. Dutta).

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Extraction of Germanium from Sphalerite Collected from Nepal—Part II

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A method for the extraction of germanium from the non-magnetic fraction of the mineral sphalerite has been described.

IN the previous communication[1] we have described the extraction of germanium dioxide from the magnetic fraction (mainly magnetite) of sphalerite. The non-magnetic portion left over was found to contain traces (0·006-0·04 per cent) of germanium.

The spectrogram of this portion revealed the presence of the following elements: lead, zinc, iron, silicon, germanium, tin, copper, cadmium, silver and antimony, of which the first two predominate. The ore was analysed as before and the germanium estimated according to the method already described.

The analytical results are as follows:

	%
Pb	14·2
Zn	48·4
S	27·3
SiO_2	5·6
Fe	2·3
Ge	0·0021

From the analytical data, this fraction of the mineral appears to be mainly a sulphate of zinc and lead.

Extraction of Germanium

A procedure different from the one adopted for magnetite had to be adopted for the extraction of germanium from this fraction. When the ore is treated with sulphuric acid the insoluble lead sulphate formed retains the major portion of the germanium but if the sulphuric acid is completely fumed off, most of the germanium goes into solution and only traces remain over with the precipitates of lead sulphate and sulphur.

The methods of Winkler, Buchanan[2], etc., were found to be unsuitable for the extraction of germanium. Tchakirian's method[3] was finally adopted. The insoluble residue was refluxed with a saturated solution of ammonium bioxalate and, curiously enough, not even a trace of germanium was found in the solution. A new method based on the fact that the insoluble residue containing free sulphur is totally converted to lead sulphide on treatment with concentrated sodium hydroxide solution was adopted. The dismutation of free sulphur in strongly alkaline medium takes place as follows:

$$4S + 6\bar{O}H = 2\bar{S}^{--} + \bar{S}_2\bar{O}_3 + 3H_2O$$

The lead sulphide so formed carried with it the whole of germanium as disulphide and the lead sulphide is separated by treatment with 2N hydrochloric acid.

Experimental

400 gm. of the ore were decomposed by an excess of 1 : 1 sulphuric acid, the resulting mixture heated to dryness and the excess acid fumed off. A large part of the free sulphur was removed by this treatment. The dried mass was extracted with hot water and filtered. Sodium sulphide, just sufficient to precipitate one-tenth of the total zinc, was added[4]. The filtrate was rejected as spectroscopic examination showed the absence of germanium. The residue was treated with 4N sulphuric acid and digested on a water bath for an hour and filtered. Germanium disulphide was left in the residue and most of the elements including zinc were thus removed. The residue was treated with sulphuric and nitric acids and dried. It was then distilled with 50 c.c. of 1 : 1 hydrochloric acid and the distillate treated according to the procedure already described. Germanium dioxide thus obtained was found from spectroscopic evidence to contain traces of antimony and tin.

When the residue was treated with concentrated sodium hydroxide solution, the whole of germanium was precipitated along with lead sulphide. This precipitate was digested with 2N hydrochloric acid, filtered and washed. Almost the whole of lead goes into solution leaving behind germanium

disulphide, silicon dioxide, titanium, silver and traces of lead. Silicon dioxide was the principal constituent. All the vapours issuing during the treatment were collected and found to be spectroscopically free from germanium showing that germanium disulphide is not decomposed by 2N hydrochloric acid. The residue was then treated with sulphuric and hydrofluoric acids and evaporated on a water bath, when all the silica was removed. The mass was then distilled with hydrochloric acid, as previously described, and the germanium dioxide separated. The resulting product contained traces of phosphorus derived from hydrogen peroxide. The yield was 240 mg. of germanium dioxide from 4 kg. of the non-magnetic fraction.

Our best thanks are due to Prof. P. B. Sarkar for his keen interest in the investigation and to the *Council of Scientific & Industrial Research* for financial help to one of us (R. K. Dutta).

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PHYSIQUE THÉORIQUE. — *Les identités de divergence dans la nouvelle théorie unitaire.* Note de M. **S. N. Bose**, présentée par M. Louis de Broglie.

Einstein (1) a déduit les quatre identités de divergence de sa nouvelle théorie des identités de Bianchi, en y levant l'ambiguïté de la différenciation covariante.

La méthode de Hilbert (2) légèrement modifiée fournit aussi immédiatement le résultat; il est peut-être intéressant d'indiquer le mode de calcul.

La relation entre les coefficients de connexion affine $\Gamma^{\alpha}_{\mu\nu}$ et $g^{\mu\nu}$ n'est pas simple dans la nouvelle théorie. On ne peut donc appliquer directement ici la méthode de Hilbert de variation d'une fonction invariante de $g^{\mu\nu}$ et de ses dérivées. Dans la nouvelle théorie, on ne suppose pas initialement cette relation entre Γ et g et la variation donne les équations ainsi que les équations de connexion de Γ et g. La nouvelle démonstration proposée tient compte de ceci. On comprendra la méthode d'après les indications suivantes.

Formons le tenseur $R_{\mu\nu}$, contracté du tenseur de Riemann généralisé. Son caractère covariant est évident d'après la loi de transformation de $\Gamma^{\alpha}_{\mu\nu}$ comme dans la théorie symétrique. $g^{\mu\nu}R_{\mu\nu}$ est donc un invariant dans les changements de coordonnées, de même que l'expression $|g|^{1/2}\,dx_1\,dx_2\,dx_3\,dx_4 = |g|^{1/2}\,dv_4$.

Si $R_{\mu\nu} = 0$ l'intégrale

$$\text{(1)} \qquad I = \int R_{\mu\nu}\,\delta g^{\mu\nu}\,\sqrt{g}\,dv_4 = 0$$

pour une variation quelconque des seuls $g^{\mu\nu}$; $R_{\mu\nu}$ étant fonction des coefficients de connexion affine n'est pas changé dans la variation.

Observons avec Hilbert (2) que

$$\text{(2)} \qquad p^{\mu\nu} = \frac{\delta g^{\mu\nu}}{\delta x^{\sigma}}\,\varphi^{\sigma} - g^{\mu\sigma}\,\varphi^{\nu}_{\sigma} - g^{\sigma\nu}\,\varphi^{\mu}_{\sigma}$$

construit à partir d'un vecteur contrevariant arbitraire φ^{σ} peut être pris pour $\delta g^{\mu\nu}$ dans (1). D'où

$$\text{(3)} \qquad 0 = \int R_{\mu\nu}\,\sqrt{|g|}\left(\frac{\partial g^{\mu\nu}}{\partial x^{\sigma}}\,\varphi^{\sigma} - g^{\mu\sigma}\,\varphi^{\nu}_{\sigma} - g^{\sigma\nu}\,\varphi^{\mu}_{\sigma}\right) dv_4.$$

---

(1) *Canad. J. Math.*, 2, 1950, p. 120 et suiv.

(2) *Math. Ann.*, 92, 1924, p. 1 et suiv.

L'intégration est étendue à une multiplicité à quatre dimensions limitée par une multiplicité à trois dimensions et l'on a partout $R_{\mu\nu} = 0$.

Nous observons que

$$(4) \qquad \frac{d}{dx^\sigma}[R_{\mu\nu}\, g^{\mu\nu}\, \varphi^\sigma \sqrt{|g|}] = \frac{\partial R_{\mu\nu}}{\partial x^\sigma}\, g^{\mu\nu} \sqrt{|g|}\, \varphi^\sigma + R_{\mu\nu} \frac{\partial g^{\mu\nu}}{\partial x^\sigma} \sqrt{|g|}\, \varphi^\sigma + R_{\mu\nu}\, g^{\mu\nu} \left[\sqrt{g} \frac{\partial \varphi^\sigma}{\partial x^\sigma} + \frac{\partial \sqrt{g}}{\partial x^\sigma} \varphi^\sigma \right]$$

Si nous considérons $\varphi^\sigma \equiv \Delta x^\sigma$ comme une variation convenable petite et arbitraire des coordonnées, variation qui s'annule à la frontière, nous observons que

$$(5) \qquad \left.\begin{aligned} &\frac{\partial \phi^\sigma}{\partial x^\sigma}\, dv_4 = \Delta dv_4 \quad \text{et} \quad \frac{\partial \sqrt{g}}{\partial x^\sigma} \varphi^\sigma = \Delta \sqrt{g}. \\ &\left[\sqrt{g}\, \frac{\delta \varphi^\sigma}{\delta x^\sigma} + \frac{\delta \sqrt{g}}{\delta x^\sigma} \varphi^\sigma \right] dv_4 = \Delta(\sqrt{g} dv_4) \end{aligned}\right\}$$

et par conséquent $= 0$, puisque $\sqrt{g}\, dv_4$ est invariant.

En multipliant (4) par dv_4 et en intégrant sur la multiplicité à quatre dimensions on a

$$(6) \qquad \int \left[\frac{\partial R_{\mu\nu}}{\partial x^\sigma}\, g^{\mu\nu} \sqrt{|g|}\, \varphi^\sigma + R_{\mu\nu} \frac{\partial g^{\mu\nu}}{\partial x^\sigma} \sqrt{|g|}\, \varphi^\sigma \right] dv_4 = \int \frac{\partial}{\partial x^\sigma} [R_{\mu\nu}\, g^{\mu\nu}\, \varphi^\sigma \sqrt{|g|}] dv_4,$$

$=$ une intégrale de surface $= 0$, d'après l'hypothèse de la variation.

On tire donc immédiatement de (3) et (6)

$$\int \left[R_{\mu\nu} (g^{\mu\sigma} \varphi^\nu_\sigma + g^{\sigma\nu} \varphi^\mu_\sigma) + \frac{\partial R^{\mu\nu}}{\partial x^\sigma} \varphi^\sigma \right] \sqrt{g} dv_4 = 0$$

Une intégration partielle pour éliminer les dérivées de φ^σ donne immédiatement les identités

$$g^{\mu\nu} \sqrt{g} \frac{\partial R_{\mu\nu}}{\partial x^\sigma} - \frac{d}{dx^\nu} [g^{\mu\nu} R_{\mu\sigma} + g^{\nu\mu} R_{\sigma\mu}] \sqrt{g} = 0$$

identiques au résultat d'Einstein ([3]). Toutefois, d'une certaine façon, notre résultat est légèrement plus général. On peut dire que le raisonnement d'Einstein est une méthode algébrique, où les connexions entre les coefficients affines et les $g^{\mu\nu}$ sont admises *a priori*, ainsi que les conditions d'intégrabilité et l'hypothèse particulière $\Gamma^\alpha_{\alpha\mu} - \Gamma^\alpha_{\mu\alpha} = 0$.

Notre démonstration est indépendante de ces hypothèses restrictives. En fait, en essayant de déduire les équations générales $R_{\mu\nu} = 0$ et les équations de connexion du principe de variation $\delta \int g^{\mu\nu} R_{\mu\nu} \sqrt{g}\, dv_4 = 0$, on est conduit à la connexion d'Einstein seulement si $\Gamma^\alpha_{\sigma\alpha} - \Gamma^\alpha_{\alpha\sigma} = 0$.

([3]) *Canad. J. Math.*, **2**, 1950, p. 125, équation (176).

D'autre part, en posant *a priori* la connexion d'Einstein, la compatibilité de la variation avec l'hypothèse initiale conduit à $\Gamma_\mu = \Gamma^\alpha_{\mu\alpha} - \Gamma^\alpha_{\alpha\mu} = 0$.

Puisque la démonstration donnée ici n'utilise pas les équations de connexion. elle montre donc nettement que les identités ci-dessus sont indépendantes des équations de connexion. En fait, les $R_{\mu\nu}$ sont fonctions des Γ seules, et la démonstration des identités de divergence suppose seulement l'existence des dérivées premières des $g^{\mu\nu}$. Elle subsistent même quand par suite des singularités du champ des $g^{\mu\nu}$, il n'existe pas de dérivées secondes ou d'équation de connexion intégrable.

Reprinted from *Comptes rendus de l'Academie des Sciences* **236**, pp. 1333-1335, 1953 (Gauthier – Villars Publishers, Paris).

The Identities of Divergence in the New Unitary Theory

Note of Mr. S. N. Bose, presented by Mr. Louis de Broglie

Einstein([1]) deduced the four identities of divergence from his new theory of Bianchi identities, clearing the ambiguity of covariant differentiation.

The method of Hilbert([2]), when slightly modified, also immediately gives the result. Perhaps it is interesting to indicate the mode of calculation.

The relation between the coefficients of affine connection $\Gamma^{\alpha}_{\mu\nu}$ and $g^{\mu\nu}$ is not simple in the new theory. Thus the method of Hilbert regarding the variation of an invariant function of $g^{\mu\nu}$ and its derivatives cannot be directly applied here. In the new theory, this relationship between Γ and g is initially not assumed and the variation gives rise to the equations as well as the equations of connection of Γ and g. The proposed new demonstration takes this into account. The method will be understood according to the following indications.

Let us form the tensor $R_{\mu\nu}$, contracted from the generalized Riemann tensor. Its covariant character is evident from the law of transformation of $\Gamma^{\alpha}_{\mu\nu}$ as in the symmetric theory. $g^{\mu\nu}\ R_{\mu\nu}$ is thus an invariant under changes of the coordinates, as in the expression $|g|^{1/2}\, dx_1\, dx_2\, dx_3\, dx_4 = |g|^{1/2}\, dv_4$.

If $R_{\mu\nu} = 0$ the integral

$$I = \int R_{\mu\nu}\, \delta g^{\mu\nu}\, \sqrt{g}\, dv_4 = 0 \tag{1}$$

for any variation of $g^{\mu\nu}$ alone ; $R_{\mu\nu}$ being a function of the coefficients of affine connection is not changed in the variation. Let us observe with Hilbert ([2]) that

$$p^{\mu\nu} = \frac{\delta g^{\mu\nu}}{\delta x^{\sigma}}\, \varphi^{\sigma} - g^{\mu\sigma}\, \varphi^{\nu}_{\sigma} - g^{\sigma\nu}\, \varphi^{\mu}_{\sigma} \tag{2}$$

obtained from an arbitrary contravariant vector φ^{σ} can be taken for $\delta g^{\mu\nu}$ in (1). Hence

$$0 = \int R_{\mu\nu} \sqrt{|g|} \left(\frac{\partial g^{\mu\nu}}{\partial x^{\sigma}}\, \varphi^{\sigma} - g^{\mu\sigma}\, \varphi^{\nu}_{\sigma} - g^{\sigma\nu}\, \varphi^{\mu}_{\sigma} \right) dv_4. \tag{3}$$

The integration is extended over a manifold of four dimensions limited by a manifold of three dimensions and one has everywhere $R_{\mu\nu} = 0$.

We observe that

([1]) *Canad. J. Math.*, **2**, 1950, p. 120 et seq.

([2]) *Math. Ann.* **92**, 1924, p. 1 et seq.

$$(4)\qquad \frac{d}{dx^\sigma}[R_{\mu\nu}\, g^{\mu\nu}\, \varphi^\sigma \sqrt{|g|}] = \frac{\partial R_{\mu\nu}}{\partial x^\sigma}\, g^{\mu\nu}\sqrt{|g|}\,\varphi^\sigma + R_{\mu\nu}\frac{\partial g^{\mu\nu}}{\partial x^\sigma}\sqrt{|g|}\,\varphi^\sigma + R_{\mu\nu}\, g^{\mu\nu}\left[\sqrt{g}\frac{\partial \varphi^\sigma}{\partial x^\sigma} + \frac{\partial \sqrt{g}}{\partial x^\sigma}\varphi^\sigma\right]$$

If we consider $\varphi^\sigma \equiv \Delta x^\sigma$ as a suitable small and arbitrary variation of the coordinates which vanishes on the boundary, we observe that

$$(5)\qquad \left.\begin{aligned} &\frac{\partial \phi^\sigma}{\partial x^\sigma}\, dv_4 = \Delta dv_4 \text{ and } \frac{\partial \sqrt{g}}{\partial x^\sigma}\varphi^\sigma = \Delta\sqrt{g}. \\ &\text{and} \\ &\left[\sqrt{g}\,\frac{\delta\varphi^\sigma}{\delta x^\sigma} + \frac{\delta\sqrt{g}}{\delta x^\sigma}\varphi^\sigma\right] dv_4 = \Delta(\sqrt{g}dv_4) \end{aligned}\right\}$$

and consequently = 0, since $\sqrt{g}\,dv_4$ is invariant. By multiplying (4) with dv_4 and integrating on the manifold with four dimensions we have

$$(6)\qquad \int\left[\frac{\partial R_{\mu\nu}}{\partial x^\sigma}\, g^{\mu\nu}\sqrt{|g|}\,\varphi^\sigma + R_{\mu\nu}\frac{\partial g^{\mu\nu}}{\partial x^\sigma}\sqrt{|g|}\,\varphi^\sigma\right] dv_4 = \int \frac{\partial}{\partial x^\sigma}[R_{\mu\nu}\, g^{\mu\nu}\,\varphi^\sigma\sqrt{|g|}]dv_4,$$

= a surface integral = 0, as per the hypothesis of variation.
We immediately obtain from (3) and (6)

$$\int\left[R_{\mu\nu}(g^{\mu\sigma}\varphi^\nu_\sigma + g^{\sigma\nu}\varphi^\mu_\sigma) + \frac{\partial R^{\mu\nu}}{\partial x^\sigma}\varphi^\sigma\right]\sqrt{g}dv_4 = 0$$

A partial integration for eliminating the derivatives of φ^σ immediately gives the identities

$$g^{\mu\nu}\sqrt{g}\frac{\partial R_{\mu\nu}}{\partial x^\sigma} - \frac{d}{dx^\nu}[g^{\mu\nu}R_{\mu\sigma} + g^{\nu\mu}R_{\sigma\mu}]\sqrt{g} = 0$$

identical to the result of Einstein ([3]). Nevertheless, in a certain way our result is slightly more general. We can say that the reasoning of Einstein is an algebraic method where the connections between the affine coefficients and the $g^{\mu\nu}$ as well as the conditions of integrability and a particular hypothesis $\Gamma^\alpha_{\alpha\mu} - \Gamma^\alpha_{\mu\alpha} = 0$ are assumed *a priori*.

Our demonstration is independent of these restrictive hypotheses. In fact, while trying to deduce the general equations $R_{\mu\nu} = 0$ and the equations of connection from the principle of variation $\delta\int g^{\mu\nu}\, R_{\mu\nu}\sqrt{g}\; dv_4 = 0$, we are led to the connection of Einstein only if $\Gamma^\alpha_{\sigma\alpha} - \Gamma^\alpha_{\alpha\sigma} = 0$.

On the other hand, by assuming *a priori* the connection of Einstein, the compatibility of the variation with the initial hypothesis leads to $\Gamma_\mu = \Gamma^\alpha_{\mu\alpha} - \Gamma^\alpha_{\alpha\mu} = 0$.

Since the demonstration given here does not use the equations of connection, it

([3]) *Canad. J. Math.*, **2**, 1950, p. 125, equation (176).

clearly shows that the above identities are independent of the equations of connection. In fact, the $R_{\mu\nu}$ are functions of Γ alone, and the demonstration of the identities of divergence only assumes the existence of the first derivatives of $g^{\mu\nu}$. It survives even when due to singularities of the field of $g^{\mu\nu}$, there are no second derivatives or integrable equations of connection.

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UNE THÉORIE DU CHAMP UNITAIRE AVEC $\Gamma_\mu \neq 0$

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Sommaire. — On établit des équations du champ unitaire en faisant varier une intégrale qui satisfait au postulat d'hermiticité. On obtient l'équivalence formelle avec les théories anciennes, soit avec $\Gamma_\mu = 0$, soit avec $\| a_{\lambda\mu} \| = 0$ quand $\Phi_\lambda = 0$; on peut interpréter Φ_λ comme la force de Lorentz si on prend les $a_{\lambda\mu}$ comme les composantes du champ électromagnétique.

Les équations approchées avec composantes antisymétriques prennent la forme maxwellienne pour une certaine valeur des constantes. Toutefois, pour une autre valeur, il est possible d'avoir le champ sans aucune singularité.

1. Introduction

On peut classer en trois groupes les équations caractéristiques de la théorie du champ d'Einstein :

I. Les équations dué champ qui intéressent uniquement les coefficients de connexion $\Gamma^\lambda_{\mu\nu}$ et leurs derivees.

II. Les équations de connexion qui relient le tenseur g aux coefficients de connexion affine.

III. Les quatre relations de caractére restrictif :

$$\Gamma^\lambda_{\mu\lambda} - \Gamma^\lambda_{\lambda\mu} = 2\Gamma_\mu = 0.$$

Notons que les relations (III) sont semblables aux 24 équations

$$\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = 0.$$

qui, dans la théorie relativiste de la gravitation sont les conséquences du postulat de la symmétrie de la connexion affine.

Dans la théorie du champ unitaire aussi bien que dans la théorie de la gravitation, les équations due champ (I) et les équations de connexion (II) peuvent se déduire simultanément d'un principe de variation où l'on égale à zéro la variation arbitraire d'une intégrale, la variation étant cepedant assujettie aux restrictions du type (III).

Néanmoins les conditions arbitraries sont bien moins nombreuses dans la théorie unitaire que dans la théorie de la gravitation.

Dans cet article, on essaye d'éliminer toutes les conditions restrictives et d'établir un système d'équations fondamentales en s'appuyant uniquement sur le principe de variation.

On a toutefois utilisé le postulat d'hermiticité récemment énoncé par Einstein pour construire l'intégrale dont la variation conduit à cette nouvelle généralisation.

On verra dans les résultats précisés ci-dessous que des termes additionnels contenant Γ_μ interviennent aussi bien dans les equations du champ que dans les equations de connexion.

Les équations de connexion qui concernent le tenseur g donnent lieu à quelques remarques. Elles preneent ici la forme

$$g^{\mu\nu}_{\lambda}+g^{\mu\alpha}\,\Gamma^{\nu}_{\lambda\alpha}+g^{\alpha\nu}\,\Gamma^{\mu}_{\alpha\lambda} = 3g^{\mu\nu}\,\Phi_\lambda,$$

tandis que dans les deux théories précédentes le premier membre de cette égalité est nul.

Φ_λ est defini ici par les équations suivantes :

$$\Phi_\lambda = \frac{a_{\lambda\beta}\,k^\beta}{\sqrt{|\,g\,|}}, \text{ avec } a_{\lambda\beta} = \frac{1}{2}\,(g_{\lambda\beta}-g_{\beta\lambda})$$

et

$$k^\beta = \frac{1}{3}\,\theta(g^{\beta\nu}+g^{\nu\beta})\Gamma_\nu .$$

Si $\Gamma_\nu = 0$, $k^\beta = 0$ et par conséquent on a aussi $\Phi = 0$ et l'équation reprend la forme ancienne.

Néanmoins il existe une autre possibilité : que $a_{\alpha\beta}$ soit la partie antisymétrique du tenseur covariant considérée comme liée au vecteur à six composantes ($\boldsymbol{E}$, $\boldsymbol{H}$) de la théorie électromagnétique.

Si le déterminant

$$\|\,a_{\lambda\beta}\,\| = (a_{12}a_{34}+a_{31}a_{24}+a_{23}a_{14})^2 = 0,$$

on peut avoir $\Phi_\lambda = 0$ même si k^μ et Γ_μ ne sont pas nuls.

Remarquons que puisqu'on suppose toujours

$$\|\,g^{\beta\nu}+g^{\nu\beta}\,\| \neq 0,$$

k^μ et Γ_μ sont nuls simultanemént.

La condition $\|\,a_{\alpha\beta}\,\| = 0$ combinée avec la règle habituelle de corrélation citée ci-dessus conduit immédiatement à ($\boldsymbol{EH}$) = 0, propriété classique du champ électromagnétique.

On est anisi tenté de poser une corrélation d'une part entre k^μ et le vecteur à quatre dimensions courant-charge (car $k^\mu_\mu = 0$ découle immédiatment des equations générales) et, d'autre part, entre Φ_λ et la force pondéromotrice de Lorentz, en raison de l'identification classique

$$(a_{23}a_{31}\,a_{12}) \rightarrow \vec{H}, \qquad (a_{41}\,a_{42}\,a_{43}) \rightarrow i\vec{E}.$$

On peut interpréter $\Phi_\lambda = 0$ comme déterminant une distribution stationnaire où la force de Lorentz est nulle, ce qui ramène l'equation de connexion à la forme qu'elle avait dans les théories précédentes.

Dans les équations de champ, l'élement symétrique $P^\lambda_{\mu\nu}$ et l'élément antisymétrique $G_{\mu\nu}$ ont des roles essentiellement différents.

Au moyen d'hypothéses sur les ordres de grandeurs relatifs des différents termes, nous avons pu ramener les equations fondamentales à une forme simple.

Physiquement, ces hypothéses impliquent qu'il est possible de négliger la gravitation dans l'étude des relations entre grandeurs électromagnétiques.

Les équations ainsi transformées ont une forme intéressante et l'on peut les interpréter comme de simples conséquences de la théorie de Maxwell si l'on attribue certaines valeurs particulières aux constantes arbitraries. Mais il est possible aussi d'obtenir un champ sans singularités pour d'autres valeurs particuliéres. Le champ n'est alors évidemment pas maxwellien.

Ces aspects encourageants du nouveau système d'équations m'ont amené à penser qu'il convenait de publier les premiers résultats pour provoquer les critiques et les observations.

2. Les Nouvelles Équations.

Suivant Einstein, toutes les relations de la théoire unitaire doivent satisfaire au postulat d'hermiticité, c'est-à-dire rester invariantes quand $g^{\mu\nu}, g_{\mu\nu}, \Gamma^{\lambda}_{\mu\nu}$, sontc hangés à la fois en $g^{\nu\mu}, g_{\nu\mu}, \Gamma^{\lambda}_{\nu\mu}$ par permutation des indices μ et ν.

On voit facilement que dans cette transformation le tenseur d'Einstein $\boldsymbol{E}$ de composantes

$$E_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\xi}_{\mu\nu}\,\Gamma^{\lambda}_{\xi\lambda} - \Gamma^{\xi}_{\lambda\mu}\,\Gamma^{\lambda}_{\xi\nu}$$

devient le tenseur $\boldsymbol{H}$ de composantes

$$H_{\nu\mu} = \Gamma^{\lambda}_{\nu\mu,\lambda} - \Gamma^{\lambda}_{\lambda\mu,\nu} + \Gamma^{\epsilon}_{\nu\mu}\,\Gamma^{\lambda}_{\lambda\epsilon} - \Gamma^{\epsilon}_{\lambda\mu}\,\Gamma^{\lambda}_{\nu\xi}.$$

Au lieu de l'expression

$$I = g^{\mu\nu} E_{\mu\nu} \sqrt{|g|} = g'^{\mu\nu} E_{\mu\nu}$$

qui intervient dans l'intégrale soumise à variation pour obtenir les équations fondamentales, nous adoptons une forme plus générale

$$I' = \frac{1}{2}(g'^{\mu\nu} E_{\mu\nu} + g'^{\nu\mu} H_{\nu\mu}) + a\gamma^{\mu\nu}\,\Gamma_{\mu}\Gamma_{\nu} + bA^{\mu\nu}\,(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu})$$

où

$$\gamma^{\mu\nu} = \frac{1}{2}(g'^{\mu\nu} + g'^{\nu\mu}) = \frac{1}{2}\sqrt{|g|}\;(g^{\mu\nu} + g^{\nu\mu})$$

$$A^{\mu\nu} = \frac{1}{2}(g'^{\mu\nu} - g'^{\nu\mu}) = \frac{1}{2}\sqrt{|g|}\,(g^{\mu\nu} - g^{\nu\mu}),$$

et

$$|\bar{g}| = |g|,$$

puispue la permutation de μ et ν ne modifie pas le déterminant. Les termes additionnels vérifient aussi la condition d'hermiticité puisque $\Gamma_{\mu} A^{\mu\nu}$ changent de signe et que $\gamma^{\mu\nu}$ conserve le sien avec la permutation.

Séparons dans les coefficients de connexion affine $\Gamma^{\lambda}_{\mu\nu}$ la partie symétrique $P^{\lambda}_{\mu\nu}$ et la partie antisymétrique $V^{\lambda}_{\mu\nu}$ et posons encore

$$\Gamma^{\lambda}_{\mu\nu} = G^{\lambda}_{\mu\nu} - \frac{1}{3}\,\delta^{\lambda}_{\mu}\Gamma_{\nu} + \frac{1}{3}\delta^{\lambda}_{\nu}\,\Gamma_{\mu},$$

avec

$$\Gamma_\mu = V^\lambda_{\mu\lambda},$$

on a

$$G^\lambda_{\mu\lambda} = 0.$$

En effectuant les substitutions dans I', tous les termes qui sont produits d'un facteur symétrique par un facteur antisymétrique disparaissent dans la sommation et nous avons finalement

$$I' = \gamma^{\mu\nu}(\bar{E}_{\mu\nu} - G^\lambda_{\mu\varepsilon} G^\varepsilon_{\lambda\nu} + x\Gamma_\mu \Gamma_\nu) + A^{\mu\nu}[G^\lambda_{\mu\nu;\lambda} - y(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu})]$$

où x, y sont des constantes arbitraires,

$$\bar{E}_{\mu\nu} = P^\lambda_{\mu\nu,\lambda} - \frac{1}{2}(P^\lambda_{\mu\lambda,\nu} + P^\lambda_{\nu\lambda,\mu}) + P^\varepsilon_{\mu\nu} P^\lambda_{\varepsilon\lambda} - P^\varepsilon_{\mu\lambda} P^\lambda_{\varepsilon\nu}$$

est la composante symétrisée du tenseur d'Einstein avec connexion affine symétrique et

$$G^\lambda_{\mu\nu;\lambda} = G^\lambda_{\mu\nu,\lambda} - G^\lambda_{\mu\varepsilon} P^\varepsilon_{\lambda\nu} - G^\lambda_{\varepsilon\nu} P^\varepsilon_{\mu\lambda} + G^\varepsilon_{\mu\varepsilon} P^\lambda_{\varepsilon\lambda}$$

est la dérivée covariante classique calculée aussi avec connexion symétrique.

Au cours de la variation arbitraire de

$$L = \int I' d\nu_4, \quad d\nu_4 = dx^1 dx^2 dx^3 dx^4.$$

$\gamma^{\mu\nu}$, $A^{\mu\nu}$ conservent automatiquement leur caractére symétrique. Les variations de $\gamma^{\mu\nu}$ et $A^{\mu\nu}$ peuvent donc etre considérées comme arbitraires, ce qui donne immédiatement les équations du champ

$$\frac{\partial I'}{\partial \gamma^{\mu\nu}} = \bar{E}_{\mu\nu} - G^\varepsilon_{\mu\lambda} G^\lambda_{\varepsilon\nu} + x\Gamma_\mu \Gamma_\nu = 0,$$

$$\frac{\partial I'}{\partial A^{\mu\nu}} = G^\lambda_{\mu\nu;\lambda} - y(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu}).$$

Pour obtenir les équations de connexion nous égalons d'abord I' à une divergence à quatre dimensions (H) et considérons H seulement puisque la divergence se transforme en une intégrale étendue à un domaine à trois dimensions où tous les coefficients des variations arbitraires s'annulent.

Rappelons-nous cependant que dans la variation des éléments Γ_μ, $P^\lambda_{\mu\nu}$, $G^\lambda_{\mu\nu}$ les quatre relations $G^\lambda_{\mu\lambda} = 0$ restent toujours valables; les 24 composants de $G^\lambda_{\mu\nu}$ ne sont pas arbitraires. Nous devons donc appliquer la méthode habituelle, employer des multiplicateurs indéterminés k^μ et varier la fonction

$$H - 2k^\mu G^\lambda_{\mu\lambda} = H'.$$

Ces coefficients k^μ seront détermines en dernier lieu à partir de l'équation finale.

Nous obtenons ainsi par un calcul facile le systéme d'équations

$$\gamma^{\mu\nu}_{,\lambda} + \gamma^{\mu\alpha} P^\nu_{\lambda\alpha} + \gamma^{\alpha\nu} P^\mu_{\alpha\lambda} - \gamma^{\mu\nu} P^\alpha_{\lambda\alpha} = -[A^{\mu\alpha} G^\nu_{\lambda\alpha} + A^{\alpha\nu} G^\mu_{\alpha\lambda}],$$

$$A^{\mu\nu}_{,\lambda} + A^{\mu\alpha} P^\nu_{\lambda\alpha} + A^{\alpha\nu} P^\mu_{\alpha\lambda} - A^{\mu\nu} P^\alpha_{\lambda\alpha} - k^\mu \delta^\nu_\lambda + k^\nu \delta^\mu_\alpha = -[\gamma^{\mu\alpha} G^\nu_{\lambda\alpha} + \gamma^{\alpha\nu} G^\mu_{\alpha\lambda}],$$

$$yA^{\mu\nu}_{,\nu} + x\gamma^{\mu\nu}\Gamma_\nu = 0,$$

et par conséquent

$$A^{\mu\nu}_{,\nu} = 3k^{\mu},$$

$$\gamma^{\mu\alpha}_{,\alpha} + \gamma^{\alpha\beta} P^{\mu}_{\alpha\beta} + A^{\alpha\beta} G^{\mu}_{\alpha\beta} = 0.$$

D'où on tire immédiatement

$$k^{\mu}_{,\mu} = 0$$

et

$$k^{\mu} = \theta \gamma^{\mu\nu} \Gamma_{\nu}, \quad \text{avec } \theta = -\frac{x}{3y} .$$

Nous pouvons maintenant donner aux équations de connexion la forme convenable : par addition et regroupement on a

$$g'_{\lambda}{}^{\mu\nu} + g'^{\mu\alpha} L^{\nu}_{\lambda\alpha} + g'^{\alpha\nu} L^{\mu}_{\lambda\alpha} - g'^{\mu\nu} L^{\alpha}_{\lambda\alpha} = k^{\mu}\delta^{\nu}_{\lambda} - k^{\nu}\delta^{\mu}_{\lambda},$$

où

$$L^{\nu}_{\lambda\alpha} = G^{\nu}_{\lambda\alpha} + P^{\nu}_{\lambda\alpha},$$

en multipliant par les coefficients $g'_{\mu\nu}$ définis par

$$g'^{\mu\nu} g'_{\mu\lambda} = \delta^{\nu}_{\lambda}, \quad g'^{\mu\nu} g'_{\lambda\nu} = \delta^{\mu}_{\lambda}.$$

nous avons d'abord

$$L'^{\alpha}_{\lambda\alpha} = \frac{1}{2} \frac{c, \lambda}{c} + a'_{\lambda\beta} k^{\beta},$$

$$c = |g|$$

et

$$a'_{\lambda\beta} = \frac{1}{2} (g'_{\lambda\beta} - g'_{\beta\lambda}),$$

puis en remplaçant $L^{\alpha}_{\lambda\alpha}$ par sa valeur et en divisant par $\sqrt{c}$, nous obtenons aprés une transformation facile

$$g^{\mu\nu}_{\lambda} + g^{\mu\alpha} L^{\nu}_{\lambda\alpha} + g^{\alpha\nu} L^{\mu}_{\alpha\lambda} - \frac{g^{\mu\nu} a_{\lambda\beta} k^{\beta}}{\sqrt{c}} = \frac{k^{\mu}\delta^{\nu}_{\lambda} - k^{\nu}\delta^{\mu}_{\lambda}}{\sqrt{c}} .$$

En observant que

$$g^{\mu\alpha} g_{\beta\alpha} k^{\beta} = k^{\mu}, \quad g^{\alpha\nu} g_{\alpha\beta} k^{\beta} = k^{\nu}$$

et en regroupant et additionnant certains termes, nous avons finalement

$$g^{\mu\alpha}_{\lambda} + g^{\mu\alpha} \Gamma'^{\nu}_{\lambda\alpha} + g^{\alpha\nu} \Gamma'^{\mu}_{\alpha\lambda} = 3g^{\mu\nu} \frac{a_{\lambda\beta} k^{\beta}}{\sqrt{c}} = 3g^{\mu\nu} \Phi_{\lambda} \ ;$$

où

$$\Gamma'^{\nu}_{\lambda\alpha} = L^{\nu}_{\lambda\alpha} - g_{\beta\alpha} \frac{k\beta}{\sqrt{c}} \delta^{\nu}_{\lambda} + g_{\lambda\beta} k^{\beta} \delta^{\nu}_{\alpha},$$

$$\Gamma'^{\mu}_{\alpha\lambda} = L^{\mu}_{\alpha\lambda} - g_{\beta\lambda} \frac{k^{\beta}}{\sqrt{c}} \delta^{\mu}_{\alpha} + g_{\alpha\beta} \frac{k^{\beta}}{\sqrt{c}} \delta^{\mu}_{\lambda}.$$

Les nouveaux coefficients affines sont

$$\Gamma'^{\lambda}_{\mu\nu} = Q^{\lambda}_{\mu\nu} + U^{\lambda}_{\mu\nu}$$

où

$$Q^{\lambda}_{\mu\nu} = P^{\lambda}_{\mu\nu} + \Phi_{\mu}\delta^{\lambda}_{\nu} + \Phi_{\nu}\delta^{\lambda}_{\mu}$$

et

$$U^\lambda_{\mu\nu} = G^\lambda_{\mu\nu} - \Psi_\nu \delta^\lambda_\mu + \Psi_\mu \delta^\lambda_\nu ;$$

$$\Psi_\nu = \frac{s_{\nu\beta} k^\beta}{\sqrt{c}} , \phi_\nu = \alpha_{\nu\beta} \frac{k^\beta}{\sqrt{c}} ;$$

En introduisant ces nouveaux coefficients affines dans l'équation fondamentale, nous pouvons écrire

$$\bar{E}'_{\mu\nu} + \frac{3}{2}[\Phi_{\mu;\nu} + \Phi_{\nu;\mu}] + 3\Phi_\mu \Phi_\nu + \frac{x}{\theta^2} b_{\mu\alpha} b_{\nu\beta} \frac{k^\alpha k^\beta}{c} - G^\lambda_{\mu\varepsilon} G^\varepsilon_{\lambda\nu} = 0,$$

$$(G^\lambda_{\mu\nu})_{;\lambda} - 3G^\varepsilon_{\mu\nu}\Phi_\varepsilon - \frac{y}{\theta\sqrt{c}}[(b'_{\mu\alpha}k^\alpha)_{,\nu} - (b'_{\nu\alpha}k^\alpha)_{,\mu}],$$

avec

$$\tfrac{1}{2} b_{\mu\alpha}(g^{\mu\nu} + g^{\nu\mu}) = \delta^\nu_\alpha.$$

3. Cas particular

Quand $\Phi_\lambda = 0$, mais Γ_μ, $k^\mu \neq 0$, on a

$$a_{12}a_{34} + a_{23}a_{14} + a_{31}a_{24} = 0.$$

Effectuons la transformation et observons que

$$\Gamma'_\mu = 3\theta\Gamma_\mu ;$$

$P^\lambda_{\mu\nu}$ et $G^\lambda_{\mu\nu}$ ne changent pas et nous obtenous

$$\bar{E}'_{\mu\nu} + t\Gamma'_\mu\Gamma'_\nu - G^\lambda_{\mu\varepsilon} G^\varepsilon_{\lambda\nu} = 0;$$

$$(G^\lambda_{\mu\nu})_{;\lambda} + t(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu}) = 0,$$

la seule modification est une réduction du nombre des constantes arbitraires.

L'équation de connexion est (puisque $\Phi_\lambda = 0$)

$$g^{\mu\lambda}_{;\lambda} + g^{\mu\alpha}\Gamma^\nu_{\lambda\alpha} + g^{\alpha\nu}\ \Gamma^\mu_{\alpha\lambda} = 0.$$

mais avec $\Gamma_\mu \neq 0$.

On montre alors facilement que

$$(A^{\mu\nu}\sqrt{\Delta})_\nu = \tfrac{1}{2}(g^{\mu\alpha} + g^{\alpha\mu})\Gamma_\alpha.$$

4. Approximation:

Pour trouver une approximation, nous mettons l'équation de connexion sous la forme

$$\gamma^{\mu\nu}_{;\lambda} + \gamma^{\mu\alpha}P^\nu_{\lambda\alpha} + \gamma^{\alpha\nu}P^\mu_{\alpha\lambda} - \gamma^{\mu\nu}P^\alpha_{\lambda\alpha} = -[A^{\mu\alpha}G^\nu_{\lambda\alpha} + A^{\alpha\nu}\ G^\mu_{\alpha\lambda}],$$

$$A^{\mu\nu}_{;\lambda} = A^{\mu\alpha}P^\nu_{\lambda\alpha} + A^{\alpha\nu}P^\mu_{\alpha\lambda} - A^{\mu\nu}P^\alpha_{\lambda\alpha} - k^\mu\delta^\nu_\lambda + k^\nu\delta^\mu_\lambda = -[\gamma^{\mu\alpha}G^\nu_{\lambda\alpha} + \gamma^{\alpha\nu}G^\mu_{\alpha\lambda}].$$

Nous supposons que les coefficients antisymétriques $A^{\mu\nu}$ êt $G^\lambda_{\mu\nu}$ sont petits, voisins de zero.

Alors les $P^\lambda_{\lambda\mu}$ peuvent aussi étre négligés.

$$\gamma^{\mu\nu} = \delta^\mu_\nu ,$$

la dérivation covariante est remplacée par la derivation ordinaire, et nous avons finalement en pr miére approximation

$$A^{\mu\nu}_{,\lambda} - k^\mu \delta^\nu_\lambda + k^\nu \delta^\mu_\lambda = -[G^\nu_{\lambda\mu} + G^\mu_{\nu\lambda}],$$

$$A^{\mu\nu}_{\nu} = 3k^\mu,$$

d'où l'on déduit facilement

$$(G^\lambda_{\mu\nu}) = A^{\mu\nu}_{,\lambda} - k^\mu \delta^\nu_\lambda + k^\nu \delta^\mu_\lambda - \tfrac{1}{2}[A^{\mu\nu}_{,\lambda} + A^{\lambda\mu}_{,\nu} + A^{\nu\lambda}_{,\mu}],$$

$$(G^\lambda_{\mu\nu})_\lambda = \frac{2}{3} A^{\mu\nu}_{,\lambda\lambda} - \frac{1}{6}[A^{\mu\nu}_{,\lambda} + A^{\nu\lambda}_{,\mu} + A^{\lambda\mu}_{,\nu}]_\lambda$$

et enfin comme

$$\Gamma_\mu = \frac{k^\mu}{\theta},$$

nous avons le système suivant :

$$A^{\mu\nu}_{\nu} = 3k^\mu \ ;$$

$$A^{\mu\nu}_{\lambda\lambda} - \frac{1}{4}[A^{\mu\nu}_{\lambda} + A^{\nu\lambda}_{\mu} + A^{\lambda\mu}_{\nu}]_\lambda = \frac{3y}{2\theta}[k^\mu_{,\nu} - k^\nu_{,\mu}].$$

En éliminant k^μ nous obtenons

$$A^{\mu\nu}_{\lambda\lambda}\left(1 - \frac{y}{2\theta}\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{y}{\theta}\right)(A^{\mu\nu}_{\lambda} + A^{\nu\lambda}_{\mu} + A^{\lambda\mu}_{\nu})_\lambda = 0.$$

Si $1 = \frac{y}{2\theta}$ nous pouvons poser

$$A^{\mu\nu}_{\lambda} + A^{\nu\lambda}_{\mu} + A^{\lambda\mu}_{\nu} = 0$$

et l'équation devient identique aux équations du champ électromagnétique de Maxwell.

Mais si $\frac{1}{2} - \frac{y}{\theta} = 0$, les équations deviennent

$$(A^{\mu\nu}_{\nu}) = \rho^\mu,$$

$$(A^{\mu\nu}_{\lambda} + \ldots)_\lambda = -(\rho^\mu_{,\nu} - \rho^\nu_{,\mu}),$$

avec

$$A^{\mu\nu}_{\lambda\lambda} = 0,$$

système different évidemment de celui de Maxwell, mais qui peut avoir une solntion sans singularitiés en auceun point.

II est peut être intéressant de remarquer que certaines recherches sur la quantification du champ électromagnétique ont amené de leur côté à poser $A^{\mu\nu}_{\lambda\lambda} = 0$, ce qui entrainait implicitement une modification des équations due champ.

Si $\| a_{\lambda\mu} \| = 0$ on obtient une approximation présentant les mêmes caractéres.

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A Unitary Field Theory with $\Gamma_\mu \neq 0$

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Summary — The equations of the unitary field are established by varying an integral which satisfies the postulate of hermiticity. One obtains an absolute equivalence with the old theories either with $\Gamma_\mu = 0$ or with $\| a_{\lambda\mu} \| = 0$ when $\Phi_\lambda = 0$; one can interpret Φ_λ as the Lorentz force if $a_{\lambda\mu}$ are taken as the components of the electromagnetic field.

The approximate equations with antisymmetric components take the Maxwellian form for a certain value of the constants. Nevertheless, for another value, it is possible to have a field without any singularity.

1. Introduction

The characteristic equations of Einstein's field theory can be classified into three groups :

I. The field equations which involve only the connection coefficients $\Gamma^\lambda_{\mu\nu}$ and their derivatives.

II. The connection equations which link the tensor g to the affine connection coefficients.

III. The four relations of restrictive character

$$\Gamma^\lambda_{\mu\lambda} - \Gamma^\lambda_{\lambda\mu} = 2\Gamma_\mu = 0.$$

Note that the relations in (III) resemble the 24 equations,

$$\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = 0.$$

which, in the relativistic theory of gravitation, are consequences of the postulate of symmetry of the affine connections.

In unitary field theory as well as in the theory of gravitation the equations of the field (I) and the equations of connection (II) can be simultaneously deduced from the principle of variation where the arbitrary variation of an integral is equal to zero, the variation being subjected to restrictions of type (III).

Nevertheless, the arbitrary conditions are much less numerous in the unitary theory than in the theory of gravitation.

In this article we are trying to eliminate all the restrictive conditions and establish a system of fundamental equations by relying only on the principle of variation.

We have used the postulate of hermiticity recently enunciated by Einstein to construct the integral, the variation of which leads to this new generalization.

The specific results given below will show that the additional terms containing Γ_μ occur as well in the field equations as in the equations of connection.

The equations of connection which concern the tensor g give rise to certain remarks. Here they take the form

$$g^{\mu\nu}_{\lambda}+g^{\mu\alpha}\,\Gamma^{\nu}_{\lambda\alpha}+g^{\alpha\nu}\,\Gamma^{\mu}_{\alpha\lambda}=3g^{\mu\nu}\,\Phi_{\lambda},$$

whereas in the two proceeding theories the first member of this equality is zero.

Here Φ_λ is defined by the following equation

$$\Phi_{\lambda}=\frac{a_{\lambda\beta}\,k\beta}{\sqrt{|\,g\,|}},\text{with } q_{\lambda\beta}=\frac{1}{2}\,(g_{\lambda\beta}-g_{\beta\lambda})$$

and

$$k^{\beta}=\frac{1}{3}\,\theta(g^{\beta\nu}+g^{\nu\beta})\Gamma_{\nu}.$$

If $\Gamma_\nu = 0$, $k^\beta = 0$ and consequently if we also have $\Phi = 0$, the equation again takes the old form.

However, there exits another possibility : that $a_{\alpha\beta}$ being the antisymmetric part of the covariant tensor, is considered as linked to a vector with the six components (**E**, **H**) of the electromagnetic theory.

If the determinant

$$\|\,a_{\lambda\beta}\,\|=(a_{12}a_{34}+a_{31}a_{24}+a_{23}a_{14})^2=0,$$

we can have $\Phi_\lambda = 0$ even if k^μ and Γ_μ are not zero.

Note that since we always suppose

$$\|\,g^{\beta\nu}+g^{\nu\beta}\,\|\neq 0,$$

k^μ and Γ_μ are simultaneously zero.

The condition $\|\,a_{\lambda\,\beta}\,\| = 0$ combined with the usual rule of correlation just mentioned immediately leads to (**E H**) = 0, a classical property of the electromagnetic field.

One is also tempted to put a correlation on the one hand between k^μ and the four dimensional charge current vector (because $k^{\mu}_{,\mu} = 0$ comes immediately from the general equations) and on the other hand, between Φ_λ and the ponderomotive force of Lorentz, due to the classical identifications

$$(a_{23}a_{31}\,a_{12})\rightarrow\vec{H},\qquad(a_{41}\,a_{42}\,a_{43})\rightarrow i\vec{E}.$$

We can interpret $\Phi_\lambda = 0$ as determining a stationary distribution, where the Lorentz force is zero. This brings back the equation of connection to the form which was found in previous theories.

In the field equations the symmetric element $P^{\lambda}_{\mu\,\nu}$ and the antisymmetric element $G^{\lambda}_{\mu\,\nu}$ have totally different roles.

By means of hypotheses on the orders of relative magnitude of the various terms we could bring back the fundamental equations to a simple form.

Physically, these hypotheses imply that it is possible to neglect gravitation in a study of relations between electromagnetic quantities.

The equations thus transformed have an interesting form and we can interpret them

as simple consequences of the Maxwell theory if we attribute particular values to the arbitrary constants.

But it is also possible to obtain a field without any singularities for other particular values. The field is then not Maxwellian.

These encouraging aspects from the new system of equations have made me think that it would be appropriate to publish the first findings in order to provoke criticisms and observations.

2. The new equations

Following Einstein, all the relations of unitary field theory must satisfy the postulate of hermiticity, i.e. remain unchanged when $g^{\mu\nu}$, $g_{\mu\nu}$, $\Gamma^{\lambda}_{\mu\nu}$ are changed to $g^{\nu\mu}$, $g_{\nu\mu}$, $\Gamma^{\lambda}_{\nu\mu}$ all together through permutations of the indices μ and ν.

We can easily see that in this transformation, the Einstein tensor **E** of components

$$E_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\xi}_{\mu\nu}\,\Gamma^{\lambda}_{\xi\lambda} - \Gamma^{\xi}_{\lambda\mu}\,\Gamma^{\lambda}_{\xi\nu}$$

become the tensor **H** of components

$$H_{\nu\mu} = \Gamma^{\lambda}_{\nu\mu,\lambda} - \Gamma^{\lambda}_{\lambda\mu,\nu} + \Gamma^{\varepsilon}_{\nu\mu}\,\Gamma^{\lambda}_{\lambda\varepsilon} - \Gamma^{\varepsilon}_{\lambda\mu}\,\Gamma^{\lambda}_{\nu\xi}.$$

Instead of the expression

$$I = g^{\mu\nu} E_{\mu\nu} \sqrt{|g|} = g'^{\mu\nu} E_{\mu\nu}$$

which occurs in the integral which is subjected to variation in order to obtain the fundamental equations, we adopt a form which is more general

$$I' = \frac{1}{2}(g'^{\mu\nu} E_{\mu\nu} + g'^{\nu\mu} H_{\nu\mu}) + a\gamma^{\mu\nu}\,\Gamma_\mu\Gamma_\nu + bA^{\mu\nu}\,(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu})$$

where

$$\gamma^{\mu\nu} = \frac{1}{2}(g'^{\mu\nu} + g'^{\nu\mu}) = \frac{1}{2}\sqrt{|\bar{g}|}\;(g^{\mu\nu} + g^{\nu\mu})$$

$$A^{\mu\nu} = \frac{1}{2}(g'^{\mu\nu} - g'^{\nu\mu}) = \frac{1}{2}\sqrt{|\bar{g}|}\;(g^{\mu\nu} - g^{\nu\mu}),$$

and

$$|\bar{g}| = |g|,$$

since a permutation of μ and ν does not modify the determinant. The additional terms also satisfy the condition of hermicity since $\Gamma_\mu\, A^{\mu\nu}$ changes sign and $\gamma^{\mu\nu}$ retains its sign with permutation. Let us separate the symmetric part $P^{\lambda}_{\mu\nu}$ and the antisymmetric part $V^{\lambda}_{\mu\nu}$ in the affine connection coefficients $\Gamma^{\lambda}_{\mu\nu}$ and let us also put

$$V_{\mu\nu} = G^{\lambda}_{\mu\nu} - \frac{1}{3}\,\delta^{\lambda}_{\mu}\Gamma_\nu + \frac{1}{3}\,\delta^{\lambda}_{\nu}\,\Gamma_\mu,$$

with

$$\Gamma_\mu = V^{\lambda}_{\mu\lambda},$$

one has

$$G^{\lambda}_{\mu\lambda} = 0.$$

Through substitution in I′, all the terms that are products of the symmetric factor and the antisymmetric factor disappear in the summation, and we finally have

$$I' = \gamma^{\mu\nu}(\bar{E}_{\mu\nu} - G^{\lambda}_{\mu\varepsilon} G^{\varepsilon}_{\lambda\nu} + x\Gamma_{\mu}\Gamma_{\nu}) + A^{\mu\nu}[G^{\lambda}_{\mu\nu;\lambda} - y(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu})]$$

where x, y are arbitrary constants,

$$\bar{E}_{\mu\nu} = P^{\lambda}_{\mu\nu,\lambda} - \frac{1}{2}(P^{\lambda}_{\mu\lambda,\nu} + P^{\lambda}_{\nu\lambda,\mu}) + P^{\varepsilon}_{\mu\nu} P^{\lambda}_{\varepsilon\lambda} - P^{\varepsilon}_{\mu\lambda} P^{\lambda}_{\varepsilon\nu}$$

is the symmetrized component of the Einstein tensor with symmetric affine connection and

$$G^{\lambda}_{\mu\nu;\lambda} = G^{\lambda}_{\mu\nu,\lambda} - G^{\lambda}_{\mu\varepsilon} P^{\varepsilon}_{\lambda\nu} - G^{\lambda}_{\varepsilon\nu} P^{\varepsilon}_{\mu\lambda} + G^{\varepsilon}_{\mu\nu} P^{\lambda}_{\varepsilon\lambda}$$

is the classical covariant derivative, also calculated with symmetric connection.

During the course of arbitrary variation of

$$L = \int I' d\nu_4, \quad d\nu_4 = dx^1 dx^2 dx^3 dx^4.$$

$\gamma^{\mu\nu}$, $A^{\mu\nu}$ automatically retain their symmetry character. The variations of $\gamma^{\mu\nu}$ and $A^{\mu\nu}$ can thus be considered as arbitrary, which immediately gives rise to the field equations

$$\frac{\partial I'}{\partial \gamma^{\mu\nu}} = \bar{E}_{\mu\nu} - G^{\varepsilon}_{\mu\lambda} G^{\lambda}_{\varepsilon\nu} + x\Gamma_{\mu}\Gamma_{\nu} = 0,$$

$$\frac{\partial I'}{\partial A^{\mu\nu}} = G^{\lambda}_{\mu\nu;\lambda} - y(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu}).$$

In order to obtain the equations of connection, let us first make I′ equal to a divergence in four dimensions (H) and let us consider only H since the divergence transforms itself to an integral over a three dimensional domain where all the coefficients of arbitrary variations cancel each other.

Let us recall the fact that in the variation of the elements Γ_{μ}, $P^{\lambda}_{\mu\nu}$, $G^{\lambda}_{\mu\nu}$ the four relations $G^{\lambda}_{\mu\lambda} = 0$ always remain valid ; the 24 component of $G^{\lambda}_{\mu\nu}$ are not arbitrary. We can thus apply the usual method of employing undermined multipliers k^{μ} and varying the function

$$H - 2k^{\mu} G^{\lambda}_{\mu\lambda} = H'.$$

These coefficients k^{μ} will be determined in the end from the final equation.

Thus through easy calculation we obtain a system of equations

$$\gamma^{\mu\nu}_{,\lambda} + \gamma^{\mu\alpha} P^{\nu}_{\lambda\alpha} + \gamma^{\alpha\nu} P^{\mu}_{\alpha\lambda} - \gamma^{\mu\nu} P^{\alpha}_{\lambda\alpha} = -[A^{\mu\alpha} G^{\nu}_{\lambda\alpha} + A^{\alpha\nu} G^{\mu}_{\alpha\lambda}],$$

$$A^{\mu\nu}_{,\lambda} + A^{\mu\alpha} P^{\nu}_{\lambda\alpha} + A^{\alpha\nu} P^{\mu}_{\alpha\lambda} - A^{\mu\nu} P^{\alpha}_{\lambda\alpha} - k^{\mu}\delta^{\nu}_{\lambda} + k^{\nu}\delta^{\mu}_{\alpha} = -[\gamma^{\mu\alpha} G^{\nu}_{\lambda\alpha} + \gamma^{\alpha\nu} G^{\mu}_{\alpha\lambda}],$$

$$yA^{\mu\nu}_{\nu} + x\gamma^{\mu\nu}\Gamma_{\nu} = 0,$$

and consequently

$$A^{\mu\nu}_{,\nu} = 3k^\mu,$$

$$\gamma^{\mu\alpha}_{,\alpha} + \gamma^{\alpha\beta} P^\mu_{\alpha\beta} + A^{\alpha\beta} G^\mu_{\alpha\beta} = 0.$$

From where we immediately obtain

$$k^\mu_{,\mu} = 0$$

and

$$k^\mu = \theta \gamma^{\mu\nu} \Gamma_\nu, \quad \text{avec } \theta = -\frac{x}{3y}.$$

We can now give a suitable form to the equations of connection : through addition and regrouping we have

$$g'_\lambda{}^{\mu\nu} + g'^{\mu\alpha} L^\nu_{\lambda\alpha} + g'^{\alpha\nu} L^\mu_{\lambda\alpha} - g'^{\mu\nu} L^\alpha_{\lambda\alpha} = k^\mu \delta^\nu_\lambda - k^\nu \delta^\mu_\lambda,$$

where

$$L^\nu_{\lambda\alpha} = G^\nu_{\lambda\alpha} + P^\nu_{\lambda\alpha},$$

by multiplying with the coefficient $g'_{\mu\nu}$ defined by

$$g'^{\mu\nu} g'_{\mu\lambda} = \delta^\nu_\lambda, \quad g'^{\mu\nu} g'_{\lambda\nu} = \delta^\mu_\lambda.$$

we first obtain

$$L'^\alpha_{\lambda\alpha} = \frac{1}{2} \frac{c, \lambda}{c} + a'_{\lambda\beta} k^\beta,$$

$$c = |g|$$

and

$$a'_{\lambda\beta} = \frac{1}{2} (g'_{\lambda\beta} - g'_{\beta\lambda}),$$

then, by replacing $L'^\alpha_{\lambda\alpha}$ by its value and dividing it by $\sqrt{c}$, we get an easy transformation

$$g^{\mu\nu}_\lambda + g^{\mu\alpha} L^\nu_{\lambda\alpha} + g^{\alpha\nu} L^\mu_{\alpha\lambda} - \frac{g^{\mu\nu} a_{\lambda\beta} k^\beta}{\sqrt{c}} = \frac{k^\mu \delta^\nu_\lambda - k^\nu \delta^\mu_\lambda}{\sqrt{c}}.$$

By observing that

$$g^{\mu\alpha} g_{\beta\alpha} k^\beta = k^\mu, \quad g^{\alpha\nu} g_{\alpha\beta} k^\beta = k^\nu$$

and by regrouping and adding certain terms, we finally obtain

$$g^{\mu\alpha}_{,\lambda} + g^{\mu\alpha} \Gamma'^\nu_{\lambda\alpha} + g^{\alpha\nu} \Gamma'^\mu_{\alpha\lambda} = 3g^{\mu\nu} \frac{a_{\lambda\beta} k^\beta}{\sqrt{c}} = 3g^{\mu\nu} \Phi_\lambda ;$$

where

$$\Gamma'^\nu_{\lambda\alpha} = L^\nu_{\lambda\alpha} - g_{\beta\alpha} \frac{k\beta}{\sqrt{c}} \delta^\nu_\lambda + g_{\lambda\beta} k^\beta \delta^\nu_\alpha,$$

$$\Gamma'^\mu_{\alpha\lambda} = L^\mu_{\alpha\lambda} - g_{\beta\lambda} \frac{k^\beta}{\sqrt{c}} \delta^\mu_\alpha + g_{\alpha\beta} \frac{k^\beta}{\sqrt{c}} \delta^\mu_\lambda.$$

The new affine coefficients are

$$\Gamma'^\lambda_{\mu_\nu} = Q^\lambda_{\mu_\nu} + U^{\cdot}_{\mu\nu}$$

where

$$Q^{\lambda}_{\mu\nu} = P^{\lambda}_{\mu\nu} + \Phi_{\mu}\delta^{\lambda}_{\nu} + \Phi_{\nu}\delta^{\lambda}_{\mu}$$

and

$$U^{\lambda}_{\mu\nu} = G^{\lambda}_{\mu\nu} - \Psi_{\nu}\delta^{\lambda}_{\mu} + \Psi_{\mu}\delta^{\lambda}_{\nu};$$

$$\Psi_{\nu} = -\frac{s_{\nu\beta}k^{\beta}}{\sqrt{c}}, \quad \phi_{\nu} = \alpha_{\nu\beta}\frac{k^{\beta}}{\sqrt{c}};$$

By introducing these new affine coefficients in the fundamental equation, we can write

$$\bar{E}'_{\mu\nu} + \frac{3}{2}[\Phi_{\mu;\nu} + \Phi_{\nu;\mu}] + 3\Phi_{\mu}\Phi_{\nu} + \frac{x}{\theta^2}b_{\mu\alpha}b_{\nu\beta}\frac{k^{\alpha}k^{\beta}}{c} - G^{\lambda}_{\mu\varepsilon}G^{\varepsilon}_{\lambda\nu} = 0,$$

$$(G^{\lambda}_{\mu\nu})_{;\lambda} - 3G^{\varepsilon}_{\mu\nu}\Phi_{\varepsilon} - \frac{y}{\theta\sqrt{c}}[(b'_{\mu\alpha}k^{\alpha})_{,\nu} - (b'_{\nu\alpha}k^{\alpha})_{,\mu}],$$

with

$$\tfrac{1}{2}b_{\mu\alpha}(g^{\mu\nu} + g^{\nu\mu}) = \delta^{\nu}_{\alpha}.$$

3. Particular case

When $\Phi_{\lambda} = 0$ but Γ_{μ}, $k^{\mu} \neq 0$ we have

$$a_{12}a_{34} + a_{23}a_{14} + a_{31}a_{24} = 0.$$

Let us make the transformation and observe that

$$\Gamma'_{\mu} = 3\theta\Gamma_{\mu};$$

$P^{\lambda}_{\mu\nu}$ and $G^{\lambda}_{\mu\nu}$ do not change and we obtain

$$E'_{\mu\nu} + t\Gamma'_{\mu}\Gamma'_{\nu} - G^{\lambda}_{\mu\varepsilon}G^{\varepsilon}_{\lambda\nu} = 0;$$
$$(G^{\lambda}_{\mu\nu})_{;\lambda} + t(\Gamma_{\mu,\nu} - \Gamma_{\nu,\mu}) = 0,$$

the only modification is a reduction of the number of arbitrary constants.

The equation of connection is (since $\Phi_{\lambda} = 0$)

$$g^{\mu\lambda}_{\lambda} + g^{\mu\alpha}\Gamma^{\nu}_{\lambda\alpha} + g^{\alpha\nu}\Gamma^{\mu}_{\alpha\lambda} = 0.$$

but with $\Gamma_{\mu} \neq 0$.

Thus it is easily shown that

$$(A^{\mu\nu}\sqrt{\Delta})_{\nu} = \tfrac{1}{2}(g^{\mu\alpha} + g^{\alpha\mu})\Gamma_{\alpha}.$$

4. Approximation

In order to find an approximation we put the equation of connection in the form

$$\gamma^{\mu\nu}_{,\lambda} + \gamma^{\mu\alpha}P^{\nu}_{\lambda\alpha} + \gamma^{\alpha\nu}P^{\mu}_{\alpha\lambda} - \gamma^{\mu\nu}P^{\alpha}_{\lambda\alpha} = -[A^{\mu\alpha}G^{\nu}_{\lambda\alpha} + A^{\alpha\nu}G^{\mu}_{\alpha\lambda}],$$

$$A^{\mu\nu}_{,\lambda} = A^{\mu\alpha}P^{\nu}_{\lambda\alpha} + A^{\alpha\nu}P^{\mu}_{\alpha\lambda} - A^{\mu\nu}P^{\alpha}_{\lambda\alpha} - k^{\mu}\delta^{\nu}_{\lambda} + k^{\nu}\delta^{\mu}_{\lambda} = -[\gamma^{\mu\alpha}G^{\nu}_{\lambda\alpha} + \gamma^{\alpha\nu}G^{\mu}_{\alpha\lambda}].$$

Let us suppose that the antisymmetric coefficients $A^{\mu\nu}$ and $G^{\lambda}_{\mu\nu}$ are small, close to zero.

Thus $P^{\nu}_{\lambda\mu}$ can also be neglected :

$$\gamma^{\mu\nu} = \delta^{\mu}_{\nu},$$

the covariant derivative is replaced by an ordinary derivative, and we finally obtain in the first approximation

$$A^{\mu\nu}_{,\lambda} - k^{\mu}\delta^{\nu}_{\lambda} + k^{\nu}\delta^{\mu}_{\lambda} = -[G^{\nu}_{\lambda\mu} + G^{\mu}_{\nu\lambda}],$$

$$A^{\mu\nu}_{\nu} = 3k^{\mu},$$

from which we can easily deduce

$$(G^{\lambda}_{\mu\nu}) = A^{\mu\nu}_{,\lambda} - k^{\mu}\delta^{\nu}_{\lambda} + k^{\nu}\delta^{\mu}_{\lambda} - \tfrac{1}{2}[A^{\mu\nu}_{,\lambda} + A^{\lambda\mu}_{,\nu} + A^{\nu\lambda}_{,\mu}],$$

and finally with

$$(G^{\lambda}_{\mu\nu})_{\lambda} = \frac{2}{3}A^{\mu\nu}_{,\lambda\lambda} - \frac{1}{6}[A^{\mu\nu}_{,\lambda} + A^{\nu\lambda}_{,\mu} + A^{\lambda\mu}_{,\nu}]_{\lambda}$$

$$\Gamma_{\mu} = \frac{k^{\mu}}{\theta},$$

we obtain the following system

$$A^{\mu\nu}_{\nu} = 3k^{\mu} \ ;$$

$$A^{\mu\nu}_{\lambda\lambda} - \frac{1}{4}[A^{\mu\nu}_{\lambda} + A^{\nu\lambda}_{\mu} + A^{\lambda\mu}_{\nu}]_{\lambda} = \frac{3\gamma}{2\theta}[k^{\mu}_{,\nu} - k^{\nu}_{,\mu}].$$

By eliminating k^{μ} we obtain

$$A^{\mu\nu}_{\lambda\lambda}\left(1 - \frac{\gamma}{2\theta}\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{\gamma}{\theta}\right)(A^{\mu\nu}_{\lambda} + A^{\nu\lambda}_{\mu} + A^{\lambda\mu}_{\nu})_{\lambda} = 0.$$

If $1 = \frac{\gamma}{2\theta}$ we can put

$$A^{\mu\nu}_{\lambda} + A^{\nu\lambda}_{\mu} + A^{\lambda\mu}_{\nu} = 0$$

and the equation becomes identical to Maxwell's equations of electromagnetic fields.

But if $\frac{1}{2} - \frac{\gamma}{\theta} = 0$, the equations become

$$(A^{\mu\nu}_{\nu}) = \rho^{\mu},$$

with

$$(A^{\mu\nu}_{\lambda} + \ldots)_{\lambda} = -(\rho^{\mu}_{,\nu} - \rho^{\nu}_{,\mu}),$$

$$A^{\mu\nu}_{\lambda\lambda} = 0,$$

the system being evidently different from that of Maxwell, but which can have a solution without singularities at any point.

It is perhaps interesting to note that certain researches on the quantization of the electromagnetic field have put the subsidiary condition $A^{\mu\ \nu}_{\lambda\ \lambda} = 0$, which would imply a modification of field equations.

If $\| a_{\lambda\mu} \| = 0$, we obtain an approximation presenting the same characteristics.

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CERTAINES CONSÉQUENCES DE L'EXISTENCE DU TENSEUR g DANS LE CHAMP AFFINE RELATIVISTE

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Sommaire. — On établit des relations où entrent seulement $\Gamma^{\lambda}_{\mu\nu}$ et leurs dérivées à partir des conditions d'intégrabilité des équations auxquelles satisfait le tenseur g des théories relativistes.
Le nombre de relations indépendantes entre éléments du champ est bien plus grand en théorie unitaire qu'en théorie de la gravitation.

1. INTRODUCTION

Le formalisme abstrait de la théorie de la relativité a actuellement abouti à la conception d'un champ de coefficients de connexion affine d'une variété 4-dimensionnelle.

Soient δx^{μ} les composantes d'un déplacement parallele et infiniment petit auquel un vecteur A de la variété considérée est assujetti: les variations des composantes A^{μ} qui en résultent sont données par la règle

$$\delta A^{\mu} = - A^{\alpha} \Gamma^{\mu}_{\alpha\beta} \, \delta x^{\beta} \qquad \ldots \quad (1.1)$$

Dans la théorie de la gravitation les termes $\Gamma^{\mu}_{\alpha\beta}$ sont symétriques, ou $\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\beta\alpha}$; donc, la loi de transport y est unique.

Au contraire, comme les coefficients sont dissymétriques dans la théorie unitaire, on peut y avoir aussi une deuxiéme loi de transport, savoir :

$$\bar{\delta} A^{\mu} = -\delta x^{\alpha} \, \Gamma^{\mu}_{\alpha\beta} A^{\beta}. \qquad \ldots \quad (1.2)$$

En s'appuyant sur la rele (1.1), on peut calculer facilement le tenseur de courbure $\boldsymbol{R}$ dont les composantes

$$\left.\begin{aligned} R^{\lambda}_{\alpha\mu\,\nu} &= \Gamma^{\lambda}_{\alpha\mu,\,\nu} - \Gamma^{\lambda}_{\alpha\nu,\,\mu} + \Gamma^{\varepsilon}_{\alpha\mu}\Gamma^{\lambda}_{\varepsilon\nu} - \Gamma^{\varepsilon}_{\alpha\nu}\Gamma^{\lambda}_{\varepsilon\mu} \\ &\left(\text{ou } \Gamma^{\lambda}_{\alpha\mu,\nu} = \frac{d}{dx^{\nu}} \Gamma^{\lambda}_{\alpha\mu}, \ldots \right) \end{aligned}\right\} \qquad (1.3)$$

sont antisymétriques en indices μ et ν : et ensuite, par voie de contraction, on a facilement le tenseur d'Einstein $\boldsymbol{E}$, avec les composantes

$$E_{\alpha\mu} = \Gamma^{\lambda}_{\alpha\mu,\lambda} - \Gamma^{\lambda}_{\alpha\lambda,\,\mu} + \Gamma^{\varepsilon}_{\alpha\mu}\Gamma^{\lambda}_{\varepsilon\lambda} - \Gamma^{\varepsilon}_{\alpha\lambda}\Gamma^{\lambda}_{\varepsilon\mu} \qquad (1.4)$$

Mais la théorie unitaire adment aussi (1.2) comme loi du transport; donc, le meme raisonnement que ci-dessus nous donnera, à partire de (1.2) un autre transport S (substitué à $\boldsymbol{R}$) avec des composantes

$$S^{\lambda}_{\mu\alpha\,\nu} = \Gamma^{\lambda}_{\mu\alpha,\,\nu} - \Gamma^{\lambda}_{\nu\alpha,\,\mu} + \Gamma^{\varepsilon}_{\mu\alpha}\Gamma^{\lambda}_{\nu\varepsilon} - \Gamma^{\varepsilon}_{\nu\alpha}\Gamma^{\lambda}_{\mu\varepsilon} \qquad (1.5)$$

et finalement aussi le tenseur $\boldsymbol{H}$ (substitute à $\boldsymbol{E}$ Einstein) avec des composantes

$$H_{\mu\alpha} = \Gamma^{\lambda}_{\mu\alpha,\lambda} - \Gamma^{\lambda}_{\lambda\alpha,\mu} \mp \Gamma^{\varepsilon}_{\mu\alpha}\Gamma^{\lambda}_{\lambda\varepsilon} - \Gamma^{\varepsilon}_{\lambda\alpha}\Gamma^{\lambda}_{\mu\varepsilon}. \tag{1.6}$$

Les équations $E_{\alpha\mu} = 0$ sont celles du champ dans la théori de la gravitation.

L'ambiguité de la loi du transport dans la théorie unitaire opposeapparement des difficulties au choix de $E_{\alpha\mu} = 0$ comme équations fonndamentales.

Mais celles-ci ont été tournées par Einstein de la manière suivante : il impose au champ affine quatre conditions restrictives savoir

$$\Gamma^{\lambda}_{\mu\lambda} - \Gamma^{\lambda}_{\lambda\mu} = 2\Gamma_{\mu} = 0. \tag{1.7}$$

Bein qu'arbitraire, cette hypothèse donne facilement

$$\boldsymbol{E} = \boldsymbol{H} ;$$

les deux règles conduisent ainsi au même tenseur; donc les equations du champ peuvent rester les mêmes dans lés deux théories.

A côté du systeme

$$E_{\mu\nu} = 0 \tag{A}$$

qui dépend seulement des coefficients $\Gamma^{\lambda}_{\mu\nu}$ et de leurs dérivées premieres, il y a aussi un second système dans lequel entrent également les composantes du potentiel $g^{\mu\nu}$ et leurs dérivées : ce sont en effet les équations

$$g^{\mu\nu}_{,k} + g^{\mu\lambda}\Gamma^{\nu}_{k\lambda} + g^{\lambda\nu}\Gamma^{\mu}_{\lambda k} = 0. \tag{B}$$

On a démontré que (A) et (B) se laissent déduire d'un principe variationnel.

On forme l'integrale invariante

$$I = \int \sqrt{|g|}\, g^{\mu\nu}E_{\mu\nu}dV_4$$

avec des composantes du potentiel et du tenseur d'Einstein (supposé unique à cause de $\Gamma_{\mu} = 0$) et l'on pose

$$\delta I = 0$$

pour toutes variations arbitraries de $g^{\mu\nu}$ et $\Gamma^{\lambda}_{\mu\nu}$; on obtient immediatement les équations cherchées.

II est évient que l'existence du tenseur de potentiel dans le champ affine donnerait à celui-ci un caractere tout à fait particulier.

En effet (B), regardée comme un système d'équations différentielles, permettra de déterminer $g^{\mu\nu}$ seulement quand les conditions d'intégrabilité seront satisfaites.

Ainsi, les relations qui suivent comme conséquences de cette hypothéses d'intégrabilité entre les coefficients $\Gamma^{\lambda}_{\mu\nu}$ et leurs dérivées exprimerontcelles des propriétés caractéristiques du champ, qui sont postulées implicitement par la théorie dé la relativité comme par la théorie actuelle du champ unitaire.

Les obtenir sous une forme explicite est précisément le but de ce travail.

2. Cas I. Quand les coefficients $\Gamma^{\lambda}_{\mu\nu}$ sont symétriques, les conditions d'intégrailité des équations (B) savoir :

$$g^{\mu\nu}_{,k} + g^{\mu\lambda}\Gamma^{\nu}_{k\lambda} + g^{\lambda\nu}\Gamma^{\mu}_{\lambda k} = 0$$

sont plus facilement exprimables par l'intermédiaire du tenseur de courbure $\boldsymbol{R}$.

Si les composantes $R_{lmn}^{\ \ \ \lambda}$ sont exprimées par les multiples $g^{\mu\nu}$ et des parenthèses à quatre indices comme

$$R_{lmn}^{\ \ \ \lambda} = g^{\lambda k}\,(klmn), \qquad \ldots \quad (2.1)$$

les conditions d'intégrabilité seront bien exprimées par les propriétés suivantes et bien connues de ces parenthèses, savoir :

$$(klmn) = -(lkmn) = (mnkl) = -(nmkl). \qquad \ldots \quad (2.2)$$

A partir de celles-ci, on peut èliminer simultanément $g^{\mu\nu}$ et les parenthèses et obtenir des relations entre les composantes de R seulement.

(i) Comme

$$R_{lmn}^{\ \ \ \lambda} = g^{\lambda k}\,(klmn),$$

on pose $\lambda = l$ et l'on contracte. On observe que les parenthèses sont antisymétriques en indices (k et l) à côté de g^{lk} qui sont symétriques.

On a alors automatiquement :

$$R_{lmn}^{\ \ \ l} = 0 \qquad \ldots \quad (2.3)$$

(ii) De même, comme

$$R_{lmn}^{\ \ \ \lambda} R_{l'm'\lambda}^{\ \ \ \ n} = g^{\lambda k}\,(klmn)\,g^{nk'}\,(k'l'm'\lambda),$$

et

$$R_{ml}^{\ \ k'} R_{m'l'k'}^{\ \ \ \ k} = g^{k'n}\,(nmlk)\,g^{k\lambda}\,(\lambda\, m'l'k'),$$

nous avons

$$R_{lmn}^{\ \ \ \lambda} R_{l'm'\lambda}^{\ \ \ \ n} = R_{ml}^{\ \ k'} R_{m'l'k'}^{\ \ \ \ k} \qquad \ldots \quad (2.4)$$

à cause des relations (2.2) (en ce qui concerne les parentheses).

(iii) On peut ecrire les 96 composantes du tenseur $R_{klm}^{\ \ \ \ \lambda}$ explicitement en fonction de Γ^{λ}_{mn} et de leurs dérivées, à savoir (1.3), d'où découlent aisément les relations

$$R_{lmn}^{\ \ \ \lambda} + R_{mnl}^{\ \ \ \lambda} + R_{nlm}^{\ \ \ \lambda} = 0. \qquad \ldots \quad (2.5)$$

(2.3), (2.4), (2.5) sont des propriétés caractéristiques du champ symétrique, exprimées par les composantes du tenseur de courbure $\boldsymbol{R}$.

3. Cas II. Quand le champ est dissymétrique, les conditions d'intégrabilité des équations (B) peuvent aussi se déduire avec facilité.

Avec (B) on considère aussi le système de leurs dérivées premières (B_1); entre (B) et (B_1) on élimine toutes les dérivées premiéres de $g^{\mu\nu}$ et en ajoutant les conditions

$$g_{,l,k}^{\mu\nu} - g_{,k,l}^{\mu\nu} = 0,$$

on arrive immédiatement aux conditions d'intégrabilité suivantes :

$$0 = g^{a\lambda} R^{\mu}_{a m\, n} + g^{\mu d} S^{\lambda}_{m d\, n}, \qquad \ldots \quad (3.1)$$

d'où l'on peut déduire immédiatement celles-ci :

(i) En multipliant (3.1) par $g_{\mu\lambda}$ et en sommant ensuite (comme $g^{a\lambda} g_{\mu\lambda} = \delta^a_\mu$, $g^{\mu d} g_{\mu\lambda} = \delta^d_\lambda$ et δ' sont les symboles de Kronecker), on a

$$R^{a}_{a m\, n} + S^{d}_{m d\, n} = 0$$

$$\frac{d}{dx^m}(\Gamma^a_{na} + \Gamma^a_{an}) = \frac{d}{dx^n}(\Gamma^a_{ma} + \Gamma^a_{am}). \qquad \ldots \quad (3.2)$$

(ii) Comme m et μ sont des indices antisymétriaues, (3.1) peut aussi s'écrire comme ci-dessous

$$g^{a\lambda} R^{\mu}_{a m\, n} = g^{\mu d} S^{\lambda}_{n d\, m}$$

en posant $\lambda = m$ et $\mu = n$, et aprés les contractions indiquées, on obtient la relation

$$g^{\lambda\mu}(E_{\lambda\mu} - H_{\lambda\mu}) = 0. \qquad \ldots \quad (3.3)$$

Pour obtenir des relations plus générales, nous commencons par un changement de notations pour les composantes des tenseurs R et S.

Comme il y a seulement six paires de (m, n), c'est-à-dire d'indices antisymmetriques, nous remplacons chaque $\binom{m}{n}$ dans les composantes par un des nombres $r = (r = 1, \ldots, 6)$ qui est transféré sous le symbole alphabétique R ou S du tenseur.

Ainsi (3.1) prennent la forme suivante =

$$g^{a\lambda} R^{\mu}_{r\, a} + g^{\mu d} S^{\lambda}_{r\, d} = 0. \qquad (3.4)$$

Suivent le nombre r les 96 équations s' arrangement maintenant en six groups comportant chacun 16 relations.

Mais les $g^{a\lambda} = g^a_\lambda$, R^{μ}_{a}, S^{λ}_{d}, etc., se laissent interpréter comme les éléments de certaines matrices carrées G, R et S (les indices indiquant maintenant les lignes et les colonnes suivant des conventions bien connues), et par coséquent les relations (3.4) peuvant être représentées commue une seule équation de matrices :

$$RG + G\tilde{S} = 0 \qquad (3.5)$$

où $\overline{S}$ représente la matrice transposée de S avec

$$\overline{S}^{\lambda}_{\sigma} = S^{\sigma}_{\lambda}.$$

En prenant les formes transposeés, on a aussi

$$\underset{r}{SG} + \underset{r}{\overline{G}}\underset{r}{\overline{R}} = 0. \tag{3.6}$$

Les épuations (3.5) et (3.6) jouissent de propriétés d'hermiticité, ainsi qu'il a été postulé par Einstein pour toutes les relations de la théoire unitaire.

En effet, on passe de (3.5) à (3.6) en transposant G et en changeant le tenseur R en S et vice versa, ce qui implique que les dites relations restent inaltérées comme un tout par les changements simultanés des $g^{\mu\nu}$ en $g^{\nu\mu}$ et des $\Gamma^{\alpha}_{\mu\nu}$ en $\Gamma^{\alpha}_{\nu\mu}$: exactement comme l'exige le principe d'hermiticité.

Multipliant finalement les six équations (3.6) par des nombres arbitraries λ_r, et faisant la somme, on a, aprés une transformation facile :

$$G^{-1}RG = -\bar{S} \quad (\text{où } R = \lambda_r R_r \ldots). \tag{3.7}$$

Les matrices R et $\bar{S}$ sont donc liées par une transformation qui laisse intacts leurs invariants, d'où

$$I_r(R) - (-1)^r l_r(S) = 0 \quad (r = 1, 2, 3, 4). \tag{3.8}$$

Comme les λ_r sont tout à fait arbitraries, on peut admettre que chaque coefficient de polynome en λ_r dans (3.8) pris séparément est égal à zéro.

Dans toutes les relations qui s'ensuivent entrent seulement $\Gamma^{\lambda}_{\mu\nu}$ et leurs dérivées, chacune d'elles exprimant les propriétés caractéristiques du champ unitaire conditionneés par l'hypothése de l'existence du tenseur de potentiel.

Si l'on adopte pour les produits de la chaîne des $\underset{r}{R}$ ou des $\underset{r}{S}$ les notations suivantes :

$$\underset{r}{R^{\lambda}_{\mu}}\underset{s}{R^{\mu}_{\lambda}} = R(r, s); \quad \underset{r}{S^{\lambda}_{\mu}}\underset{s}{S^{\mu}_{\nu}}\underset{t}{S^{\nu}_{\lambda}} = S(rst), \quad \ldots;$$

$$\underset{r}{R^{\lambda}_{\lambda}} = R(r), \qquad \underset{r}{S^{\lambda}_{\lambda}} = S(r), \tag{3.9}$$

les relations obtenues ci-dessus pourront s'écrire aussi comme suit

$$\left.\begin{aligned} &R(r) = -S(r), \\ &R(r, s) = \quad S(rs) = S(sr), \\ &R(rst) = -S(str) = -S(rts) = -S(srt), \\ &R(rstu) = S(utsr) = S(ruts) = \ldots \end{aligned}\right\} \tag{3.10}$$

4. *a.* Comme conséauences de l'hypothése de l'existence du tenseur de potentiel, dans le champ des connexions symmétriques, on a les relations suivantes :

16 relations :

(i) $$\underset{n}{R^{\lambda}_{lm}} + \underset{l}{R^{\lambda}_{mn}} + \underset{m}{R^{\lambda}_{nl}} = 0;$$

6 relations :

(ii) $$R^{l}_{lm} = 0;$$

45 relations :

(iii) $$R^{\lambda}_{mk} R^{n}_{n\,rl} = R^{\lambda}_{lm} R^{n}_{l\,rn}.$$

Compte tenu des 10 équations du champ, on a en tout 77 relations homogènes entre les composantes $\Gamma^{\lambda}_{\mu\nu}$ et leurs dériveés premières.

b. **Dans le champ unitaire, on a :**

6 relations :

$$R(r) = -S(r);$$

21 relations :

$$R(rs) = S(sr);$$

56 relations :

$$R(rst) = -S(tsr);$$

126 relations :

$$R(rstu) = S(utsr).$$

Couplées avec les équations du champ, le total peut aller jusqu'à 225 relations entre les mêmes éléments du champ.

Ainsi les conditions qui sont imposées aux éléments affines, dans la théoire unitaire, sont bien plus restrictives que celles qui sont imposées aux éléments du champ de la gravitation.

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Certain Consequences of the Existence of the Tensor g in the Affine Relativistic Field

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Summary — Relations are established where only $\Gamma^{\lambda}_{\mu\nu}$ and their derivatives enter from conditions of integrability of the equations which the tensor g of the relativistic theory satisfies.

The number of independent relations between the elements of the field is much larger in the unitary theory than in the theory of gravitation.

1. Introduction

The abstract formalism of the theory of relativity developed from the concept of the field of coefficients of affine connection of a four dimensional variety.

If δx^{μ} are the components of a parallel and infinitely small displacement to which a vector A of the chosen variety is subjected, the variations of the components A^{μ} which result are obtained by the rule

$$\delta A^{\mu} = - A^{\alpha} \Gamma^{\mu}_{\alpha\beta} \, \delta x^{\beta} \tag{1.1}$$

In the theory of gravitation the terms $\Gamma^{\mu}_{\alpha\beta}$ are symmetric, or $\Gamma^{\mu}_{\alpha\beta} = \Gamma^{\mu}_{\beta\alpha}$; thus the rule of transport there is unique. On the contrary, as the coefficients are not symmetric in the unitary theory, one has here a second rule of transport :

$$\bar{\delta} A^{\mu} = -\delta x^{\alpha} \, \Gamma^{\mu}_{\alpha\beta} A^{\beta}. \tag{1.2}$$

Based on relation (1.1) we can easily calculate the tensor of curvature **R**, the components of which

$$\left.\begin{aligned} R^{\lambda}_{\alpha\mu,\nu} &= \Gamma^{\lambda}_{\alpha\mu,\nu} - \Gamma^{\lambda}_{\alpha\nu,\mu} + \Gamma^{\varepsilon}_{\alpha\mu}\Gamma^{\lambda}_{\varepsilon\nu} - \Gamma^{\varepsilon}_{\alpha\nu}\Gamma^{\lambda}_{\varepsilon\mu} \\ &\left(\text{ou } \Gamma^{\lambda}_{\alpha\mu,\nu} = \frac{d}{dx^{\nu}} \Gamma^{\lambda}_{\alpha\mu}, \dots \right) \end{aligned}\right\} \tag{1.3}$$

are antisymmetric in the indices μ and ν ; and then by contraction we obtain the Einstein tensor **E** with the components

$$E_{\alpha\mu} = \Gamma^{\lambda}_{\alpha\mu,\lambda} - \Gamma^{\lambda}_{\alpha\lambda,\mu} + \Gamma^{\varepsilon}_{\alpha\mu}\Gamma^{\lambda}_{\varepsilon\lambda} - \Gamma^{\varepsilon}_{\alpha\lambda}\Gamma^{\lambda}_{\varepsilon\mu} \tag{1.4}$$

But the unitary theory also admits (1.2) as a rule of transport. Thus the same reasoning mentioned above will give us from (1.2) another transport S (replacing **R**) with its components

$$S^{\lambda}_{\underset{\nu}{\mu\alpha}} = \Gamma^{\lambda}_{\mu\alpha,\nu} - \Gamma^{\lambda}_{\nu\alpha,\mu} + \Gamma^{\varepsilon}_{\mu\alpha}\Gamma^{\lambda}_{\nu\varepsilon} - \Gamma^{\varepsilon}_{\nu\alpha}\Gamma^{\lambda}_{\mu\varepsilon} \tag{1.5}$$

and finally also the tensor **H** (replacing **E** of Einstein) with the components

$$H_{\mu\alpha} = \Gamma^{\lambda}_{\mu\alpha,\lambda} - \Gamma^{\lambda}_{\lambda\alpha,\mu} \mp \Gamma^{\varepsilon}_{\mu\alpha}\Gamma^{\lambda}_{\lambda\varepsilon} - \Gamma^{\varepsilon}_{\lambda\alpha}\Gamma^{\lambda}_{\mu\varepsilon}. \tag{1.6}$$

The equations $E_{\alpha\mu=0}$ are those of the field in the theory of gravitation.

The ambiguity of the law of transport in the unitary theory seems to make it difficult to choose $E_{\alpha\mu} = 0$ as fundamental equations. But the difficulty has been solved by Einstein in the following manner : it imposes four restrictive conditions on the affine field, namely

$$\Gamma^{\lambda}_{\mu\lambda} - \Gamma^{\lambda}_{\lambda\mu} = 2\Gamma_{\mu} = 0. \tag{1.7}$$

Though arbitrary, this hypothesis easily gives

$$\mathbf{E} = \mathbf{H} \ ;$$

the two laws thus lead to the same tensor. Therefore the field equations can remain the same in both the theories.

Besides the system

$$E_{\mu\nu} = 0 \tag{A}$$

which depends only on the coefficients $\Gamma^{\lambda}_{\mu\nu}$ and their first derivatives, there is also a second system where the components of the potential $g^{\mu\nu}$ and their derivatives are also included ; these are indeed the equations

$$g^{\mu\nu}_{,k} + g^{\mu\lambda}\Gamma^{\nu}_{k\lambda} + g^{\lambda\nu}\Gamma^{\mu}_{\lambda k} = 0. \tag{B}$$

We have demonstrated that (A) and (B) are obtained from one variational principle. We form the invariant integral

$$I = \int \sqrt{|g|}\, g^{\mu\nu}E_{\mu\nu}dV_4$$

with some components of the potential and the Einstein tensor (supposed to be unique due to $\Gamma_{\mu} = 0$) and we hold

$$\delta I = 0$$

for all arbitrary variations of $g^{\mu\nu}$ and $\Gamma^{\lambda}_{\mu\nu}$; we immediately obtain the required equations.

It is evident that the existence of a potential tensor in the affine field would give it a very special character.

Indeed (B), regarded as a system of differential equations, can determine $g^{\mu\nu}$ only when the conditions of integratibility are satisfied.

Hence the relations, which follow as consequences of this hypothesis of integrability between the coefficients $\Gamma^{\lambda}_{\mu\nu}$ and their derivatives will express those relations of the

characteristic properties of the field which are implicitly postulated by the theory of relativity as well as the theory of the unitary field itself.

To obtain them in an explicit form is precisely the aim of this work.

2. Case I. — When the coefficients $\Gamma^{\lambda}_{\mu\nu}$ are symmetric, the conditions of integrability of the equations (B), namely :

$$g^{\mu\nu}_{,k} + g^{\mu\lambda}\Gamma^{\nu}_{k\lambda} + g^{\lambda\nu}\Gamma^{\mu}_{\lambda k} = 0$$

can more easily be expressed in terms of the curvature tensor **R**.

If the components $R^{\lambda}_{lm\,n}$ are expressed by the multiples $g^{\mu\nu}$ and brackets with four indices as

$$R^{\lambda}_{lm\,n} = g^{\lambda k}\,(klmn), \tag{2.1}$$

the conditions of integrability will be well expressed by the following properties of those brackets which are well known :

$$(klmn) = -(lkmn) = (mnkl) = -(nmkl). \tag{2.2}$$

From these we simultaneously eliminate $g^{\mu\nu}$ and the brackets and obtain relations between the components of **R** only.

(i) As $$R^{\lambda}_{lm\,n} = g^{\lambda k}\,(klmn),$$

we hold $\lambda = l$ and contract. We now observe that the brackets are antisymmetrical in the indices (k and l) while g^{lk} are symmetrical.

We then automatically have :

$$R^{l}_{lm\,n} = 0 \tag{2.3}$$

(ii) Similarly, since

$$R^{\lambda}_{lm\,n}R^{n}_{l'm'\,\lambda} = g^{\lambda k}\,(klmn)\,g^{nk'}\,(k'l'm'\lambda),$$

and

$$R^{k'}_{ml\,k}\,R^{k}_{m'l'\,k'} = g^{k'n}\,(nmlk)\,g^{\kappa\lambda}\,(\lambda\, m'l'k'),$$

we have

$$R^{\lambda}_{lm\,n}R^{n}_{l'm'\,\lambda} = R^{k'}_{ml\,k}\,R^{k}_{m'l'\,k'} \tag{2.4}$$

due to the relations (2.2) (with regard to the brackets).

(iii) We can write the 96 components of the tensor $R^{\lambda}_{kl\,m}$ explicitly as functions of Γ^{λ}_{mn} and their derivatives, that is to say (1.3), which easily gives the relations

$$R^{\lambda}_{lm\,n} + R^{\lambda}_{mn\,l} + R^{\lambda}_{nl\,m} = 0. \tag{2.5}$$

(2.3), (2.4), (2.5) are characteristic properties of the symmetrric field, expressed by the components of the curvature tensor **R**.

3. Case II. — When the field is non-symmetric, the conditions of integrability of the equations (B) can also be easily deduced.

With (B) we also consider the system of their first derivatives (B_1). We eliminate all the first derivatives of $g^{\mu\nu}$ between (B) and (B_1) and using the conditions

$$g^{\mu\nu}_{,l,k} - g^{\mu\nu}_{,k,l} = 0,$$

we immediately come to the following conditions of integrability :

$$0 = g^{a\lambda} R^{\mu}_{am\,n} + g^{\mu d} S^{\lambda}_{md\,n}, \tag{3.1}$$

from which we can immediately deduce these :

(i) By multiplying (3.1) with $g_{\mu\lambda}$ and then summing (since $g^{a\lambda} g_{\mu\lambda} = \delta^{a}_{\mu}$, $g^{\mu d} g_{\mu\lambda} = \delta^{d}_{\lambda}$ and δ 's are the symbols of Kronecker) we get

$$R^{a}_{am\,n} + S^{d}_{md\,n} = 0$$

or

$$\frac{d}{dx^m}(\Gamma^{a}_{na} + \Gamma^{a}_{an}) = \frac{d}{dx^n}(\Gamma^{a}_{ma} + \Gamma^{a}_{am}). \tag{3.2}$$

(ii) As m and μ are the antisymmetric indices, (3.1) can also be written as

$$g^{a\lambda} R^{\mu}_{am\,n} = g^{\mu d} S^{\lambda}_{nd\,m}$$

by putting $\lambda = m$ and $\mu = n$, and then after the contraction indicated, we obtain the relation

$$g^{\lambda\mu}(E_{\lambda\mu} - H_{\lambda\mu}) = 0. \tag{3.3}$$

In order to obtain more general relations, we begin with a change of notations for the components of the tensors R and S.

As there are only six pairs of (m, n), i.e. antisymmetric indices, each $\binom{m}{n}$ in the component is replaced by one of the numbers r (r=1, ... , 6) which replaces the symbol **R** or **S** of the tensor.

Thus (3.1) take the following form :

$$g^{a\lambda} R^{\mu}_{a\,r} + g^{\mu d} S^{\lambda}_{d\,r} = 0. \tag{3.4}$$

Following the numbers r, the 96 equations arrange themselves into six groups each having 16 relations.

But $g^{\alpha\lambda} = g^{\alpha}_{\lambda}$, R^{μ}_{α}, S^{λ}_{α}, etc can be interpretated as the elements of certain square matrices G, R and S (the indices now indicate the rows and columns as per the well known conventions), and consequently the relations (3.4) can be represented as a single equation of the matrices :

$$\underset{r}{RG} + G\underset{r}{\bar{S}} = 0 \tag{3.5}$$

where $\bar{\mathbf{S}}$ represents the transposed matrix of $\mathbf{S}$ with

$$\bar{S}^{\lambda}_{\alpha} = S^{\alpha}_{\lambda}.$$

By considering the transposed forms, we also have

$$\underset{r}{SG} + \bar{G}\underset{r}{\bar{R}} = 0. \tag{3.6}$$

The equations (3.5) and (3.6) have the properties of hermiticity as it was postulated by Einstein for all the relations of the unitary theory.

In fact, we pass from (3.5) to (3.6) by transposing G and changing the tensor $\mathbf{R}$ and $\mathbf{S}$ and vice versa, which implies that the said relations remain unaltered as a whole by the simultaneous changes of $g^{\mu\nu}$ into $g^{\nu\mu}$ and of $\Gamma^{\alpha}_{\mu\nu}$ into $\Gamma^{\alpha}_{\nu\mu}$, exactly as per the principle of hermiticity.

Finally, by multiplying the six equations (3.6) by arbitrary numbers λ_r and summing up, we get after an easy transformation :

$$G^{-1} R G = -\bar{S} \quad (\text{where } R = \lambda_r R_r \cdots). \tag{3.7}$$

The matrices R and $\bar{S}$ are therefore linked by a transformation which leaves their invariants intact, from which

$$I_r(R) - (-1)^r I_r(S) = 0 \quad (r = 1, 2, 3, 4). \tag{3.8}$$

As the λ_r are completely arbitrary, it can be said that each coefficient of the polynomial in λ_r in (3.8), taken separately, is equal to zero.

In all the relations that arise only $\Gamma^{\lambda}_{\mu\nu}$ and their derivatives enter, each of them expressing the characteristic properties of the unitary field conditioned by the hypothesis of the existence of the potential tensor.

If the products of the chain of the $\underset{r}{R}$ or the $\underset{r}{S}$ are given the following notations :

$$\underset{r}{R^{\lambda}_{\mu}}\underset{s}{R^{\mu}_{\lambda}} = R(r, s); \quad \underset{r}{S^{\lambda}_{\mu}}\underset{s}{S^{\mu}_{\nu}}\underset{t}{S^{\nu}_{\lambda}} = S(rst), \quad \ldots ; \tag{3.9}$$

$$\underset{r}{R^{\lambda}_{\lambda}} = R(r), \qquad \underset{r}{S^{\lambda}_{\lambda}} = S(r),$$

the relations obtained above can also be written as

$$\left.\begin{aligned} R(r) &= -S(r), \\ R(r, s) &= \quad S(rs) = S(sr), \\ R(rst) &= -S(str) = -S(rts) = -S(srt), \\ R(rstu) &= S(utsr) = S(ruts) = \ldots \end{aligned}\right\} \tag{3.10}$$

4.a. As consequences of the hypothesis of the existence of the potential tensor in the field of symmetric connections, the following relations are obtained :

16 relations :

(i) $$R^{\lambda}_{nlm} + R^{\lambda}_{lmn} + R^{\lambda}_{mnl} = 0;$$

6 relations :

(ii) $$R^{l}_{lmn} = 0;$$

45 relations :

(iii) $$R^{\lambda}_{mnl} R^{\mu}_{m'l'\lambda} = R^{\lambda}_{lm\mu} R^{\mu}_{l'm'\lambda}.$$

Taking into account the 10 field equations, we have a total of 77 homogenous relations between the components $\Gamma^{\lambda}_{\mu\nu}$ and their first derivatives.

b. In the unitary field, we have :

6 relations :

$$R(r) = -S(r);$$

21 relations :

$$R(rs) = S(sr);$$

56 relations :

$$R(rst) = -S(tsr);$$

126 relations :

$$R(rstu) = S(utsr).$$

Coupled with the field equations the total can go upto 225 relations between the same elements of the field.

Thus the conditions which are imposed on the affine elements in the unitary theory are more restrictive than those that are imposed on the elements of the gravitational field.

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THE AFFINE CONNECTION IN EINSTEIN'S NEW UNITARY FIELD THEORY

By S. N. Bose

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The non-symmetric $\Gamma^{\alpha}_{\mu\nu}$'s are defined by the 64 equations

$$\frac{\partial g_{\mu\nu}}{\partial x_\sigma} = g_{\mu\lambda}\Gamma^{\lambda}_{\sigma\nu} + g_{\lambda\nu}\Gamma^{\lambda}_{\mu\sigma} \tag{1}$$

in the new theory. A method of obtaining explicit expressions of Γ's in terms of $g_{\mu\nu}$ and its derivatives is sketched in this note. Following the usual convention of the relativity theory, the summation symbols are suppressed throughout, it being understood that summation is always implied whenever dummy indices occur.

I. The generalized Christoffel bracket $\left(\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right)$ is defined by the equation

$$\left(\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right) = \tfrac{1}{2}\left(\frac{\partial g_{\mu\sigma}}{\partial x_\nu} + \frac{\partial g_{\sigma\nu}}{\partial x_\mu} - \frac{\partial g_{\nu\mu}}{\partial x_\sigma}\right); \tag{2}$$

(the subscripts are written in a cyclic order (Schrödinger)). Also,

$$\left(\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right) = s_{\sigma\lambda}\Gamma^{\lambda}_{\mu\nu} + a_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} + a_{\lambda\nu}\Gamma^{\lambda}_{\sigma\mu} \tag{3}$$

where s, a, denote symmetric and anti-symmetric components of g. If $\Gamma^{\alpha}_{\mu\nu}$ is similarly decomposed into

$$\Sigma^{\alpha}_{\mu\nu} = \tfrac{1}{2}(\Gamma^{\alpha}_{\mu\nu} + \Gamma^{\alpha}_{\nu\mu})$$

(4) and

$$V^{\alpha}_{\mu\nu} = \tfrac{1}{2}(\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}),$$

and the bracket $\left(\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right)$ into

$$\left[\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right] = \tfrac{1}{2}\left[\left(\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right) + \left(\begin{smallmatrix}\nu\mu\\ \sigma\end{smallmatrix}\right)\right]$$

(5) and

$$\left\{\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right\} = \tfrac{1}{2}\cdot\left[\left(\begin{smallmatrix}\mu\nu\\ \sigma\end{smallmatrix}\right) - \left(\begin{smallmatrix}\nu\mu\\ \sigma\end{smallmatrix}\right)\right]$$

the following relations follow immediately

$$\text{(A)} \qquad \begin{cases} s_{\sigma\lambda}\Sigma^{\lambda}_{\mu\nu} + a_{\mu\lambda}V^{\lambda}_{\nu\sigma} + a_{\lambda\nu}V^{\lambda}_{\sigma\mu} = \left[{}^{\mu\nu}_{\sigma}\right] \\ s_{\sigma\lambda}V^{\lambda}_{\mu\nu} + a_{\mu\lambda}\Sigma^{\lambda}_{\nu\sigma} + a_{\lambda\nu}\Sigma^{\lambda}_{\sigma\mu} = \left\{{}^{\mu\nu}_{\sigma}\right\}. \end{cases}$$

Again if

$$\text{(6)} \qquad \begin{aligned} s_{\sigma\lambda}\Sigma^{\lambda}_{\mu\nu} &= P_{\sigma\mu\nu}, \text{ or } \Sigma^{\lambda}_{\mu\nu} = s^{\lambda k}P_{k\mu\nu} \\ s_{\sigma\lambda}V^{\lambda}_{\mu\nu} &= T_{\sigma\mu\nu}\ , \text{ or } V^{\lambda}_{\mu\nu} = s^{\lambda k}T_{k\mu\nu}\ , \end{aligned}$$

we can express Σ and V in terms of P and T with the help of $s^{\mu\nu}$, the contravariant component of $s_{\mu\nu}$ obtained in the usual manner; also taking

$$C^{\sigma}_{\mu} = s^{\sigma\lambda}a_{\lambda\mu}$$

as the components of the matrix product of s and a, we can rewrite the equations in the form

$$\text{(A')} \qquad \begin{aligned} \left[{}^{\mu\nu}_{\sigma}\right] &= P_{\sigma\mu\nu} - C^{k}_{\mu}T_{k\nu\sigma} + C^{k}_{\nu}T_{k\sigma\mu} \\ \left\{{}^{\mu\nu}_{\sigma}\right\} &= T_{\sigma\mu\nu} - C^{k}_{\mu}P_{k\nu\sigma} + C^{k}_{\nu}P_{k\sigma\mu}\ . \end{aligned}$$

P's can now be easily eliminated, and a tensor equation, involving only T's is obtained in the form

$$\text{(B)} \qquad \begin{aligned} &\left\{{}^{\mu\nu}_{\sigma}\right\} + C^{k}_{\mu}\left[{}^{\nu\sigma}_{k}\right] - C^{k}_{\nu}\left[{}^{\sigma\mu}_{k}\right] \\ &\qquad = T_{\sigma\mu\nu} + C^{\lambda}_{\sigma}(T_{\lambda k\nu}C^{k}_{\mu} + T_{\lambda\mu k}C^{k}_{\nu}) - C^{\lambda}_{\mu}C^{k}_{\nu}(T_{\lambda k\sigma} + T_{k\sigma\lambda}). \end{aligned}$$

This tensor equation may be regarded as characterizing the continuum of the Unitary Theory.

When (B) is solved, yielding T in terms of known functions of g and its derivatives, and also C^{λ}_{k}'s, the substitution in (A) of T immediately gives the P's and hence also the Γ's.

To solve this central equation we first observe that a cyclic addition easily yields

$$\text{(7)} \qquad \begin{aligned} T_{\sigma\mu\nu} + T_{\mu\nu\sigma} + T_{\nu\sigma\mu} = T(\mu\nu\sigma) &= \left\{{}^{\mu\nu}_{\sigma}\right\} + \left\{{}^{\nu\sigma}_{\mu}\right\} + \left\{{}^{\sigma\mu}_{\nu}\right\} = \{\sigma\mu\nu\} \\ \{\sigma\mu\nu\} &= \tfrac{1}{2}\left(\frac{\partial a_{\mu\sigma}}{\partial x_{\nu}} + \frac{\partial a_{\sigma\nu}}{\partial x_{\mu}} + \frac{\partial a_{\nu\mu}}{\partial x_{\sigma}}\right) \end{aligned}$$

which is easily recognized as a tensor.

Also if Christoffel co-efficients $\gamma^{\alpha}_{\mu\nu} = s^{\alpha t}\left[{}^{\mu\nu}_{t}\right]$ (the usual symmetric γ's) are used,

the left-hand side of (B) is changed to

$$\{\mu\nu\sigma\} + a_{\mu\nu\sigma}$$

where

$$a_{\mu\nu\sigma} = \frac{\partial a_{\mu\nu}}{\partial x_\sigma} - a_{\mu\lambda}\gamma^{\lambda}_{\sigma\nu} - a_{\lambda\nu}\gamma^{\lambda}_{\mu\sigma} \tag{8}$$

is the familiar covariant derivative with symmetric $\gamma^{\alpha}_{\mu\nu}$'s as in Riemann-Geometry; also observing that

$$T_{\lambda k\sigma} + T_{k\sigma\lambda} = T(\sigma\lambda k) - T_{\sigma\lambda k} = \{\sigma\lambda k\} - T_{\sigma\lambda k}\,, \tag{9}$$

the equation (B) can be transformed into the form

$$\text{(C)} \quad \begin{cases} \{\sigma\mu\nu\} + \{\sigma k\lambda\}C^{k}_{\mu}C^{\lambda}_{\nu} + a_{\mu\nu\sigma} \equiv U_{\sigma\mu\nu} \\ U_{\sigma\mu\nu} \equiv T_{\sigma\mu\nu} + T_{\sigma k\lambda}C^{k}_{\mu}C^{\lambda}_{\nu} + C^{\lambda}_{\sigma}[T_{\lambda k\nu}C^{k}_{\mu} + T_{\lambda\mu k}C^{k}_{\nu}] \end{cases}$$

where the anti-symmetric suffixes are put in the end on both sides in the tensors with three-index symbols.

II. To solve the equation (C) we remark that the co-efficients C^{k}_{μ} that occur in the equation are the components of the matrix

$$C = sa = \| C^{k}_{\mu} \| \text{ or } C^{k}_{\mu} = s^{kt}a_{t\mu}\,.$$

The general equation for the eigen vector M of the matrix C can be written in the form

$$| C - pE | M = 0 \text{ or } C^{k}_{\mu}M^{\mu}_{p} = pM^{k}_{p}\,; \tag{10}$$

in the vector $M = M^{\sigma}_{p}$ the superscript σ indicates the component, and the subscript p the associated eigen-value.

The characteristic eigen values are the roots of the quartic

$$x^4 + I_2 x^2 + I_4 = 0 \tag{11}$$

as the first and third invariants vanish (the matrix being the product of a symmetric matrix).

A few well-known properties of matrix equations are here re- capitulated for ready reference.

1. A matrix C, and its transposition $\overline{C}$ have the same eigen values, but different eigen vectors ; and the eigen vectors, written with two suffixes, (representing the component and the eigen value) satisfy the following relations :

$$\overline{M}^{\sigma}_{k}M^{\sigma}_{k'} = \delta^{k}_{k'}\,,\ \overline{M}^{\sigma}_{k}M^{\sigma'}_{k} = \delta^{\sigma}_{\sigma'}\,.$$

1 (a). The theorem is true even in the case of repeated roots of the characteristic equation of C, if the eigen vectors are suitably constructed.

2. Taking the elements of the matrix C, as C^k_μ, where k denotes the rows and μ the columns of the matrix, a new compound matrix $C \times C$, can be constructed in the following manner :

$$(C \times C)\binom{kl}{\mu\nu} = C^k_\mu C^l_\nu - C^k_\nu C^l_\mu \tag{12}$$

obtained by the usual cross-multiplication rule. The elements can be arranged in a six × six matrix in any proper order ; e.g.,

$$(23) \to 1,\ (31) \to 2,\ (12) \to 3,\ (14) \to 4,\ (24) \to 5,\ (34) \to 6.$$

3. The eigen vectors of the matrixes $C \times C$ (arranged in any assumed manner) may be obtained by taking the vector product of the eigen vectors M^σ_p of C. $M_a \times M_b$ is thus the eigen six-vector of the six matrix, $C \times C$ corresponding to the eigen value $p_a p_b$.

4. The eigen vector of $\overline{C} \times \overline{C}$ is similarly obtained as $\overline{M}_a \times \overline{M}_b$ corresponding to the same eigen value $p_a p_b$.

5. The eigen vectors of the compound matrices have corresponding orthogonal properties.

The components of the eigen vectors can be written by the notation used before as $M\binom{\sigma\sigma'}{ab}$ and $\bar{M}\binom{\epsilon\epsilon'}{cd}$.

The orthogonality relation is written as

$$\begin{cases} \bar{M}\binom{\sigma\sigma'}{ab} M\binom{\sigma\sigma'}{cd} = \delta^a_c \delta^b_d \\ \bar{M}\binom{\sigma\sigma'}{ab} M\binom{\epsilon\epsilon'}{ab} = \delta^\sigma_\epsilon \delta^{\sigma'}_{\epsilon'} . \end{cases} \tag{13}$$

III. Keeping the above well-known properties of matrices in mind we can tackle the problem of solving the general equation. We multiply the equation (C') by $M^\sigma_a M^\mu_b M^\nu_c$ formed from the eigen vectors of C, corresponding to three eigen values a, b, and c, where $b \neq c$; then remembering the equations

$$C^k_\mu M^\mu_b = p_b M^k_l , \qquad C^k_\nu M^\nu_c = p_c M^k_c , \text{ etc.,} \qquad (b, c \text{ not summed})$$

we get easily, by suitably changing the dummies

$$\begin{cases} M^\sigma_a M^\mu_b M^\nu_c U_{\sigma\mu\nu} \equiv \{hkl\} M^h_a M^k_b M^l_c (1 + p_b p_c) + a_{klh} M^h_a M^k_b M^l_c ; \\ \text{also} \\ M^\sigma_a M^\mu_b M^\nu_c U_{\sigma\mu\nu} = T_{hkl} M^h_a M^k_b M^l_c (1 + p_a p_b + p_b p_c + p_c p_a). \end{cases} \tag{14}$$

Or remembering the anti-symmetric nature of the suffixes k, l we write the same equation with k, l in the reversed order and add, using the symbol

$$M\binom{kl}{bc} = M^k_a M^l_c - M^k_c M^l_b$$

easily getting the result in the form

$$(1 + p_a p_b + p_b p_c + p_c p_a)^{-1}[(1 + p_b p_c)\{hkl\}M^h_a M\binom{kl}{bc} + a_{kch}M^h_a M\binom{kl}{bc}] = T_{hkl}M^h_a M\binom{kl}{bc}. \tag{15}$$

Remembering the orthogonal properties of the eigen vectors, namely equations (12) and (13), and multiplying by $\bar{M}^\sigma_a \bar{M}\binom{\mu\nu}{bc}$ on both sides followed by summation on a, b, c, we have

$$T_{\sigma\mu\nu} = (1 + p_a p_b + p_b p_c + p_c p_a)^{-1}[(1 + p_b p_c)\{hkl\}\bar{M}^\sigma_a \bar{M}\binom{\mu\nu}{bc}M^h_a M\binom{kl}{bc} + a_{klh}\bar{M}^\sigma_a \bar{M}\binom{\mu\nu}{bc}M^h_a M\binom{kl}{bc}]. \tag{16}$$

An explicit form of $T_{\sigma\mu\nu}$ is thus obtained in terms of the eigen vectors of C and $\bar{C}$ and the covariant derivatives of a_{kl} calculated with the symmetric Christoffel co-efficients, γ's obtained from the symmetric parts of $g_{\mu\nu}$, namely $s_{\mu\nu}$. The discriminant-determinant of the equation (C) is obviously

$$D_{24} = \prod (1 + p_a p_b + p_b p_c + p_c p_a)(\text{product with roots of (11) with } b \neq c). \tag{17}$$

The value of the determinant can be easily calculated in terms of the invariants of C; in fact,

$$D_{24} \equiv (1 + I_2 + I_4)^8[(1 + 5I_4 - I_2)^2 - 4I_4(2 - I_4)^2]^2. \tag{18}$$

The determinant D always vanish when $1 + I_2 + I_4 = 0$; but this happens when the determinant of $g = \| g_{\mu\nu} \| = 0$.

This is proved as follows:

Any transformation $s(\ )s^{-1}$, where $s^{-1} = \bar{s}$, preserves the symmetric or/and the anti-symmetric nature of a matrix; $s_{\mu\nu}$ part of $g_{\mu\nu}$ may be diagonalized, and at the same time the anti-symmetric tensor $a_{\mu\nu}$ preserves its anti-symmetric character. Hence g can be reduced to the form

$$\begin{vmatrix} s_{11}\ a_{12} \cdots \\ \\ a_{41} \cdots s_{44} \end{vmatrix} \text{ and } a_{\mu\nu} + a_{\nu\mu} = 0$$

by such a transformation.

The values of all invariants remain obviously unaltered. In this special co-ordinates

$$\Delta = \| g_{\mu\nu} \| = s_{11}s_{22}s_{33}s_{44} + \Sigma s_{11}s_{22}a_{34}^2 + (a_{12}a_{34} + \cdots).^2 \tag{19}$$

The invariants of the matrix C can be easily calculated which gives

$$I_2 = (s_{11}s_{22}s_{33}s_{44})^{-1}\Sigma s_{11}s_{22}a_{34}^2$$

$$I_4 = (s_{11}s_{22}s_{33}s_{44})^{-1}(a_{12}a_{34} + \cdots)^2$$

hence

$$\Delta = s_1s_2s_3s_4(1 + I_2 + I_4). \tag{20}$$

Hence if $(1 + I_2 + I_4) = 0$

$$\| g_{\mu\nu} \| = 0 \text{ also.}$$

That when T is determined Γ's are also determined follow simply as a consequence of equation (A).

IV. In later developments of his theory Einstein has introduced the additional condition

$$\Gamma_l \equiv \Gamma^\lambda_{l\lambda} - \Gamma^\lambda_{\lambda l} = 0;$$

however, from the contravariant form of equation (1), viz.:

$$-\frac{\partial g^{\mu\nu}}{\partial x_\sigma} = -g^{\mu\nu}_\sigma = g^{\mu\lambda}\Gamma^\nu_{\sigma\lambda} + g^{\lambda\nu}\Gamma^\mu_{\lambda\sigma} \tag{21}$$

the following relation

$$[|\, g \,|^{\frac{1}{2}}(g^{\mu\lambda} - g^{\nu\mu})]_\mu = \tfrac{1}{2}\,|\, g \,|^{\frac{1}{2}}(g^{\nu\lambda} + g^{\lambda\nu})(\Gamma^\mu_{\lambda\mu} - \Gamma^\mu_{\mu\nu}) \tag{22}$$

follows easily. The condition

$$\Gamma^\lambda_{\lambda l} - \Gamma^\lambda_{\lambda l} = 0$$

means four relations involving the differential co-efficients of $a_{\mu\nu}$ and $g_{\mu\nu}$. In the equation (C) for the determination of T, all first order differential co-efficients occur in the left side of the in-homogeneous equation, whereas the homogeneous

part involves only C^{λ}_{k}, composed of $s^{\mu\nu}$ and $a_{\mu\nu}$ and no differential co-efficients occur here. Hence the additional conditions do not affect the general nature of the solution.

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XVIII. A REPORT ON THE STUDY OF THERMOLUMINESCENCE

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(*Received for publication on February 9, 1955*)

When some solids are irradiated with ionizing radiations like X-ray or cathode rays, they often exhibit fluorescence followed by a long period phosphorescent afterglow. Again, if the irradiated sample is heated, the stored energy is released in the form of radiation, emitted at various temperatures characteristic of the sample. This thermo-stimulated release of energy is commonly known as thermoluminescence. The capacity of substances to store energy is large at low temperatures, so thermoluminescence is pronounced if the irradiation is carried out at low temperature (*e.g.*, at −184°C). We may plot in a thermoluminescence curve the total intensity obtained from the sample at various temperatures during heating. From the curve, the trap depths of light-storing states below the conduction band, can be evaluated [1]. Shallow traps yield glow peaks at comparatively low temperatures, while deeper traps are released at higher temperatures. Quartz, alkali halides, calcite, glasses and many other substances show this kind of thermoluminescence. Of late, thermoluminescence has been a useful tool in many problems of research. Dosimetry of radiations, study of heterogeneous catalysts, ionic nature of elements in glasses, and identification of clay minerals are some of the uses to which thermoluminescence has been put.

Study of thermoluminescence of alkali halides was initiated in this laboratory in 1950, as a part of a larger programme of investigating solid state and of co-ordinating fluorescence study with the results of soft X-ray spectroscopy. The fluorescence spectra of alkali halides both at room and low temperatures had already been studied [2, 3, 4]. Measurements of the afterglow [5] decay rates had also been done, so thermoluminescence study of alkali halides and also of some organic substance was undertaken, with a view to clarify the processes involved during energy storage in these phosphors.

In this laboratory, thermoluminescence has been studied by exciting the sample at liquid oxygen temperature, in a demountable cathode-ray tube fitted with a window of quartz. The sample is mounted as a fine coating on a hollow bulb made of thin silver. The bulb can be filled with liquid oxygen to keep the sample at low temperature and it may be rapidly heated with an electric heater immersed in the bath. A thermocouple on the surface of the bulb, connected to a Moll galvanometer, gives temperature of the sample. Another galvanometer connected to the photomultplier tube, placed facing the specimen, gives luminescence intensity. The movements of the two galvanometers during thermoluminescence are recorded on a slow rotating photographic drum. A typical example of thermoluminescence curve is shown in Fig. 1.

For recording thermoluminescence, different photomultiplier tubes (931A, 1P28, 1P22) have been employed to suit the emissions of the different samples. Fluorescence spectra of the substance being known from a previous study, there was no difficulty in selecting the photomultipliers. Although low rate of heating (0·25°C/sec.) is usually preferred, yet it has been observed that, in the case of many phosphors, the weaker peaks get flattened out unless the rate of heating is high (6–10°C/sec.). A high rate of heating

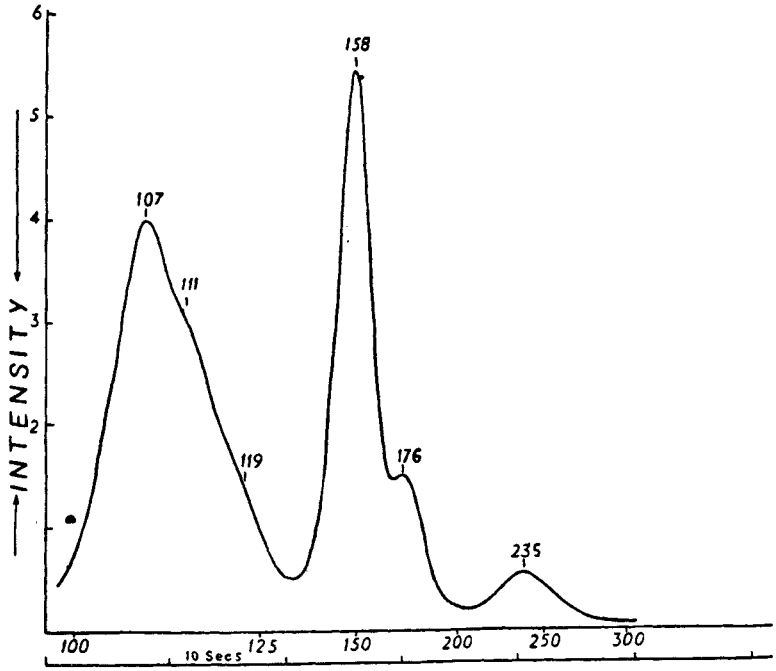


FIG. 1. Thermoluminescence of KI.

was often maintained and a fine film of powdered sample was found more suitable than a single crystal. For recording the spectral nature of the different glow peaks, two photomultipliers, sensitive in different regions, were made to record thermoluminescence simultaneously during the same run of the experiment, and optical filters were used to separate out the emissions.

Quite a number of alkali halides, both pure and activated, has been studied and some of the results have been published[6, 7]. These halides yield a number of thermoluminescence peaks between 90°K and 600°K. It has been possible to explain some of the results of afterglow decay rates in the light of information obtained from thermoluminescence study. The effects of thallium activation of samples on thermoluminescence have been investigated and it has been found that in some samples thallium produces a new peak of its own, without affecting the peaks of the parent lattice. But in some samples, like KCl and KI, it completely changes the thermoluminescence curve. From the comparative method of study with optical filters, it has been found that the different peaks emitted by a specimen are not spectrally identical. In general the peaks emitted at high temperatures are rich in emissions of ultra-violet and blue regions. But thorough study of any peak with an ordinary spectrograph is not possible, because the individual peaks disappear within a few seconds. To study the spectral nature of individual peaks, construction of a rapid spectrophotometer was deemed necessary and has been recently completed. With the help of this spectrophotometer some interesting results have been recently obtained (see Figs. 2, 3, 4 and 5).

The automatic rapid scanning spectrophotometer employs two concave mirrors of 25 cm. focal length and large focal ratios of $f/4{\cdot}2$ constructed from stainless steel and a

large prism of quartz. Light from the first slit is collimated by the first concave mirror, and, after dispersion by the prism, the beam is focussed by means of the other concave mirror. But before falling on the exit slit, the spectrum is reflected by another plane mirror. The latter is mounted on a turntable, which can be made to oscillate to and fro with the help of an induction motor and a cam. The entire spectrum (ultra-violet and visible) sweeps before the slit in about 0·95 second and flies back instantaneously to the original position by a cam arrangement. Just behind the exit slit is placed the photomultiplier tube which receives the light. The output of the photomultiplier tube passing through a pre-amplifier is amplified and displayed on a Dumont 304H cathode-ray tube with long time. A micro-switch attached to the cam triggers the sweep circuit every time the ultra-violet end of the spectrum just comes near the slit so that the same part of the spectrum is given at the same position. During each sweep, a specially adopted synchronous 35 mm. camera arrangement photographs the intensity distribution curve displayed on the cathode-ray screen. With the help of this spectrophotometer, the spectral intensity distribution of the emission can be photographed in less than one second and the apparatus is capable of giving all the necessary information during the entire thermoluminescence experiment. Not only the spectral composition and the intensity of the different peaks can be obtained, but also the changes in emissions in subsequent periods of a single thermoluminescence can be easily and accurately studied. Commercial types of photomultipliers were found to be not sensitive enough for this work, so a special photomultiplier with 19 dynodes was procured from France.

It is well-known that most of the solids (like alkali halides, quartz, calcite), on irradiation with ionizing radiations, develop colour centres. From simultaneous changes in colour centres and by recording diffuse reflectivity during thermoluminescence, it has been shown that some of the luminescence peaks correspond with thermally actuated transformation in colour centres. It has been found that thermoluminescence-behaviour of alklali halides is very susceptible to previous history of heat treatment of the sample. Other properties have also been studied in this laboratory.

Thermoluminescence of some aromatic hydrocarbons both solids and liquids between 90°K and 250°K has also been investigated. This work gives information regarding the metastable states of the molecules excited by cathode-rays at low temperatures. The depth of the metastable states below the upper excited states, as measured from the thermoluminescence, agrees fairly with that obtained from the study of fluorescence and phosphorescence spectra. Thermoluminescence opens a new method of investigation of the metastable states of the molecules.

This article would not be complete unless another interesting application of thermoluminescence is mentioned. We know that many naturally occurring minerals exhibit thermoluminescence. Recently a number of samples of sands procured from various parts of this country was studied. All sands show feeble thermoluminescence, but those from Bargarh and Mangalhat showed pronounced one. Measurement of γ-ray counts in the case of the latter showed slightly extra counts over the background. This leads to the possibility that thermoluminescence might be developed as a preliminary method of detecting radioactive minerals. Many minerals (like bentonite) are often found to yield thermoluminescent radiation. The ultimate reason for the occurrence of thermoluminescence in natural minerals has not yet been found. Radioactivity may play some part in this occurrence, but it is yet too early to make any definite statement on this matter.

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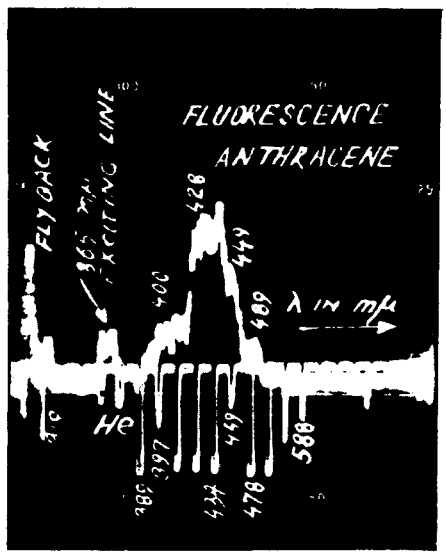


FIG. 2. Shows the fluorescent spectra of anthracene.

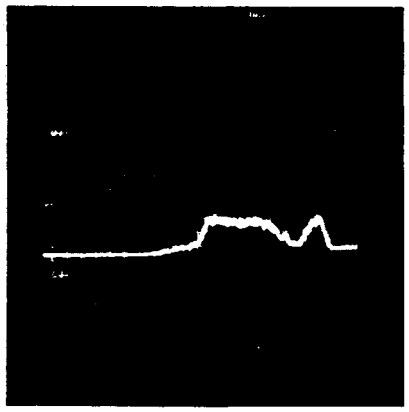


FIG. 3.

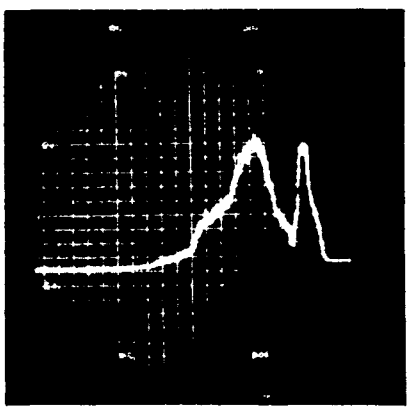


FIG. 4.

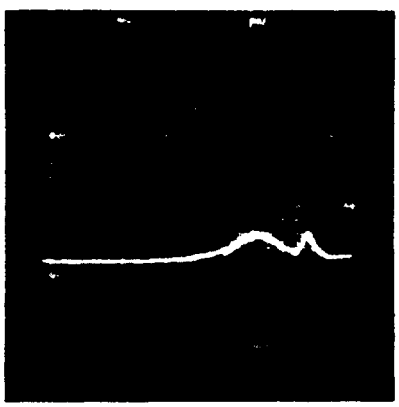


FIG. 5.

FIGS. 3, 4 and 5 give the radiation as observed from a sample of irradiated NaCl during heating. (Note change of character with time.) Figs. 3, 4 and 5 are photographs taken at 3, 7 and 17 secs. respectively after the commencement of heating.

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SOLUTION D'UNE ÉQUATION TENSORIELLE INTERVENANT DANS LA THÉORIE DU CHAMP UNITAIRE;

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Résumé.—On exprime explicitement en fonction des C et de leurs invariants la solution de l'équation tensorielle d'où dépend la determination des $\Gamma^{\lambda}_{\mu\nu}$.

On remarque que l'itération fournit une solution rigoureuse quand $\det C = 0$

1. Dans un précédent article [1], on a montré qu'il est possible de ramener le calcul des coefficients affines $\Gamma^{\lambda}_{\mu\nu}$ en fonction des $g_{\mu\nu}$ et de leurs dérivées à la solution de l'équation tensorielle

$$U_{a\mu\nu} = T_{a\mu\nu} + T_{akl} C^k_\mu C^l_\nu + C^t_a (T_{tk\nu} C^k_\mu + T_{t\mu l} C^l_\nu). \tag{1.1}$$

Les tenseurs U et T sont antisymétriques par rapport aux indices μ et ν, C^k_μ sont les éléments d'une matrice C formée à partir des parties symmétriques et antisymétriques du tenseur g :

$$\begin{cases} C = Sa, \quad a_{\mu\nu} = \dfrac{1}{2}(g_{\mu\nu} - g_{\nu\mu}), \quad s_{tk} = \dfrac{1}{2}(g_{tk} + g_{kt}), \\ S^{\mu t} S_{tk} = \delta^\mu_k \end{cases} \tag{1.2}$$

Les valeurs propers x de l matrice C vérifient l'équation

$$x^4 + I_2 x^2 + I_4 = 0, \tag{1.3}$$

I_2 et I_4 sont les invariants d'ordre pair de C; de l'équation (1.2) il résulte immédiatement que les invariants impairs sont nuls.

Le vecteur propre M^k_a de C correspondant à la valeur propre a vérifie l'équation

$$C^\mu_k M^k_a = a M^\mu_a. \tag{1.4}$$

La matrice transposée $\bar{C}(\bar{C}^\mu_k = C^k_\mu)$ a les mêmes valeurs propres que C mais ses vecteurs propres $\bar{M}^k_a$ sont différents.

Si l'équation caractéristique a toutes ses racines distinctes, il existe une série complète de vecteurs propres M et $\bar{M}$ de C et $\bar{C}$ et l'on peut exprimer les éléments de C en fonction des vecteurs propres : Savoir

$$C^k_\mu = \Sigma \lambda \bar{M}^\mu_\lambda M^k_\lambda. \tag{1.5}$$

On calcule facilement la solution de (1.1) :

$$(1.6)\qquad \begin{cases} T_{\alpha\mu\nu} = \sum\limits_{abc} (1+ab+bc+ca)^{-1} \overline{M}^{\alpha}_{a} M^{x}_{a} \overline{M} \begin{pmatrix} \mu\nu \\ bc \end{pmatrix} M \begin{pmatrix} yz \\ bc \end{pmatrix} U_{xyz} \quad (b \neq c), \\ \\ M \begin{pmatrix} yz \\ bc \end{pmatrix} = M^{y}_{b} M^{z}_{c} - M^{y}_{c} M^{z}_{b} \ ; \end{cases}$$

et la sommation porte sur les 24 combinaisons possibles des (*abc*).

On peut écrire la solution (1.6) sous la forme d'une équation tensorielle

$$T = BU,$$

le tenseur B ayant pour composantes :

$$(1.7)\qquad B^{xyz}_{\alpha\mu\nu} = \Sigma \frac{\overline{M}^{\alpha}_{a} \overline{M} \begin{pmatrix} \mu\nu \\ bc \end{pmatrix} M^{x}_{a} M \begin{pmatrix} yz \\ bc \end{pmatrix}}{1+bc+ca+ab}$$

(1.7) est identique à (1,5).

Je me propose de faire la sommation dans (1.7) et d'obtenir les composantes de B en fonction des éléments de C.

2. Au moyen de C et de la matrice unitaire E nous pouvons former une algèbre $\{C_4\}$ avec addition et multiplication matricielle.

La formule générale d'une matrice A dans $\{C_4\}$ est

$$(2.1)\qquad \begin{cases} A = \lambda_0 + \mu_1 C_1 + \mu_2 C_2 + \mu_3 C_3; \\ C_1 = C, \quad C_2 = CC, \quad C_3 = CCC. \end{cases}$$

Toutes les matrices A commutent et ont donc les mêmes vecteurs propres M que C.

Les matrices $\overline{A}$ de l'algèbre transposée ont aussi les mêmes vecteurs propres $\overline{M}$ que $\overline{C}$.

Si $a, \overline{a}, b, \overline{b}$ sont les racines de l'équation caractéristique (1.3), on voit facilement que les matrices de base se développent ainsi :

$$(2.2)\qquad \begin{cases} E = f(a)+f(\bar{a})+f(b)+f(\bar{b}): & C_2 = a^2[f(a)+f(\bar{a})]+b^2[f(b)+f(\bar{b})]: \\ C_1 = a[f(a)-f(\bar{a})+b[f(b)+f(\bar{b})]; & C_3 = a^3[(fa)-f(\bar{a})]+b^3[f(b)-f(\bar{b})]; \end{cases}$$

où $f(a) = \overline{M}_a M_a, \ldots$ en ajoutant les indices inférieurs et supérieurs aux formules ci-dessus, onobtient

$$C^{\lambda}_{\mu} = \left(\overline{M}^{\mu}_{a} M^{\lambda}_{a} - \overline{M}^{\mu}_{\bar{a}} M^{\lambda}_{\bar{a}} \right) a + b \left(\overline{M}^{\mu}_{b} M^{\lambda}_{b} - \overline{M}^{\mu}_{\bar{b}} M^{\lambda}_{\bar{b}} \right)$$

Déduisons maintenant une nouvelle algèbre $\{V_6\}$: où l'on forme ainsi les matrices V à partir de deux éléments quelconques A, B, de $\{C_4\}$:

$$V = (AB),$$

avec

$$V^{\mu\nu}_{rs} = \frac{1}{2}\,(A^{\mu}_{r}\,B^{\nu}_{s} + A^{\nu}_{s}\,B^{\mu}_{r} - A^{\mu}_{s}\,B^{\nu}_{r} - A^{\nu}_{r}\,B^{\mu}_{s}) \quad (\mu \neq \nu \text{ et } r \neq s). \tag{2.3}$$

La règle de composition (2.3) montre que toutes les matrices V de $\{V_6\}$ commutent entre elles.

Toutes les matrices V ont les mêmes vecteurs propres à six composantes que (CC); on voit facilement que les six valeurs propres de (CC) s'obtiennent en multipliant deux à deux les valeurs propres de C. Ce sont

$$a\bar{a},\quad b\bar{b},\quad ab,\quad \bar{a}\bar{b},\quad a\bar{b},\quad \bar{a}b; \tag{2.4}$$

et les vecteurs propres à six composantes se déduisent des vecteurs propres de C par une règle analogue à (2.3) par exemple :

$$M^{\mu\nu}_{ab} = M^{\mu}_{a}\,M^{\nu}_{b} - M^{\mu}_{b}M^{\nu}_{a}, \tag{2.5}$$

sont des composantes de $M\,(ab)$ le vecteur qui correspond à la valeur propre ab de (CC).

Les matrices $\bar{V}$ de l'algèbre transposée $\{\bar{V}_6\}$ ont aussi les mêmes vecteurs propres à six composantes que $(\overline{CC})$: on voit facilement que les vecteurs $\bar{M}(ab)$ se déduisent des vecteurs propres de $\bar{C}$ par la règle savoir.

$$\bar{M}^{\mu\nu}_{ab} = \bar{M}^{\mu}_{a}\,\bar{M}^{\nu}_{b} - \bar{M}^{\mu}_{b}\,\bar{M}^{\nu}_{a}, \quad \ldots$$

Comme bases de l'algèbre $\{V_6\}$, nous pouvons prendre six matrices quelconques de V_6 linéairement indépendantes.

Nous choisissons les six matrices suivantes dont nous donnons le développement en fonction des vecteurs propres :

$$\left\{\begin{aligned}
&(EE) = V(a\bar{a}) + V(b\bar{b}) + V(ab) + V(\bar{a}\bar{b}) + V(a\bar{b}) + V(\bar{a}b),\\
&(CC) = -a^2V(a\bar{a}) - b^2V(b\bar{b}) + ab[V(ab) + V(\bar{a}\bar{b})] - ab[V(a\bar{b}) + V(\bar{a}b)]\\
&(C_2C_2) = a^4V(a\bar{a}) + b^4V(b\bar{b}) + a^2b^2[V(ab) + V(\bar{a}\bar{b})] + a^2b^2[V(a\bar{b}) + V(\bar{a}b)],\\
&2(EC_2) = -2a^2V(a\bar{a}) - 2b^2V(b\bar{b}) + (a^2+b^2)[V(ab + V(\bar{a}\bar{b}) + V(a\bar{b}) + V(\bar{a}b)],\\
&2(EC) = -(a+b)[V(ab) - V(\bar{a}\bar{b})] + (a-b)[V(a\bar{b}) - V(\bar{a}b)],\\
&2(CC_2) = ab(a+b)[V(ab) - V(\bar{a}\bar{b})] - ab(a-b)[V(a\bar{b}) - V(\bar{a}b)],\\
&V(ab) = \bar{M}(ab)\,M(ab), \quad \ldots
\end{aligned}\right. \tag{2.6}$$

Let formules (2.6) sont obtenues d'une manière tout à fait analogue à la manière dont on a obtenu les formules (2.2).

Les quatre autres combinaisons qui peuvent être formées à partir des éléments

de base de $\{C_4\}$ dépendent des six ci-dessus en vertu des quatre identités ci-dessous :

$$(2.7)\qquad \begin{cases} (EC_3)+(C\,C_2)+I_2(EC) & = 0, \\ 2(CC_3)+(C_2C_2)+I_2(CC)-I_1(EE) & = 0, \\ (C_2C_3)-I_4(EC) & = 0, \\ (C_3C_3)-I_2(C_2C_2)-I_1[(CC)+2(EC_2)] & = 0. \end{cases}$$

On montre aussi facilement qu'en multipliant V_1 et V_2 dans $\{V_6\}$ avec

$$(2.8)\qquad V_1 = (AB), \quad V_2 = (CD),$$

on a

$$V_1V_2 = \tfrac{1}{2}[(AC\cdot BD)+(AD\cdot BC)].$$

Avec (2.8), les identités (2.7) et l'identité générale de Cayley-Hamilton :

$$(2.9)\qquad C_4+I_2C_2+I_4E = 0$$

nous pouvons réduire toute matrice de forme (C_rC_s) à une fonction linéaire des six éléments de la base adoptée en (2.6).

On peut choisir des bases et démontrer des identitiés analogues pour toute algèbre $V_{\frac{n(n-1)}{2}}$ composée à partir de $\{C_n\}$ [par la manière analogue à (2.3)].

Avec les mineurs m_3 d'une matrice C_4 quelconque, nous pouvons former la matrice conjuguée D.

Dans le cas particulier de C ici étudié, où $C = Sa$ est le produit de composantes symétriques S par les antisymétriques a, on voit facilement que D est donné par

$$D = A's',$$

où A' et s' sont formés de même à partir des mineurs m_s de a et S.

Admetton que toujours

$$\| S \| = \Delta \quad \text{et} \quad S \neq 0.$$

On peut cependant avoir

$$\| C \| = 0 \quad \text{si} \quad \| a \| = 0.$$

Le cas $\| a \| = 0$ est intéressant pour la théorie du champ unitaire. Nous discuterons donc les propriétés particulières de V_6 quand $\| C \| = 0$ par suite de $\| a \| = 0$.

a est une matrice antisymétrique d'ordere 4. Donc Δ déterminant de a est un carré parfait et $\sqrt{\Delta}$ est un facteur de tous les mineurs m_3 de a_4.

Si $\| a \| = 0$, A et D mineurs conjugués de C, sont identiquement nuls.

Pour la matrice conjuguée, nous avons une identité analogue à (2.9) ;

$$(2.10)\qquad C_3+I_2C_1+D = 0 \quad (C_3 = CCC).$$

Quand

$$(2.11)\qquad \| a \| = 0, \quad I_4 = 0 \quad \text{et} \quad D = 0,$$

nous avons

$$C_3+I_2C_1=0$$

et des identités (2.7) nous tirons immédiatement

$$(2.12)\qquad (CC_2)=0,\ (C_2C_2)=1_2(C_1C_1),\ (C_3C_3)=1_2(C_2C_2).$$

Nous allons utiliser ces relations pour déduire la solution de (1.1) dans le cas particulier $I_4=0$.

3. Nous effectuons maintenant la sommation indiquée en (1.6) et (1.7).

Notons quelques propriétés simples du denominatéur $D\ (abc)$:

$$(3.1)\qquad \begin{cases} D(a\bar{a}c)=D(ca\bar{a})=1-a^2, \\ D(abc)=D(\bar{a}\bar{b}\bar{c}), \\ D(aab)=\lambda_+-b^2, \qquad \lambda_+=1+(a+b)^2, \\ D(aa\bar{b})=\lambda_--b^2, \qquad \lambda_-=1+(a-b)^2, \end{cases}$$

A l'aide de ces relations simples, nous pouvons regrouper facilement les termes à sommer :

On utilise les notations ci-après pour écrire le résultat final :

$$(3.2)\qquad \begin{cases} V(ab)+V(\bar{a}\bar{b})=X, \qquad V(a\bar{b})+V(\bar{a}b)=Y\ ; \\ V(ab)-V(\bar{a}\bar{b})=U, \qquad V(a\bar{b})-V(\bar{a}b)=V\ ; \\ \bar{M}_aM_a=f(a), \qquad \bar{M}(bc)M(bc),=V(bc), \quad \ldots\ldots \end{cases}$$

La sommation (1.7) est notée

$$(3.3)\qquad B=\Sigma\frac{f(a)V(bc)}{D(abc)}\ .$$

Après regroupement, il vient

$$B=\Sigma f(\lambda)\Big[\frac{V(a\bar{a})}{1-a^2}+\frac{V(b\bar{b})}{1-b^2}\Big]+\frac{1}{2}(X+Y)\Big[\frac{f(a)+f(\bar{a})}{1-a^2}+\frac{f(b)+f(\bar{b})}{1-b^2}\Big]$$

$$+\frac{1}{2}X\Big[\frac{f(a)+f(\bar{a})}{\lambda_+-b^2}+\frac{f(b)+f(\bar{b})}{\lambda_+-a^2}\Big]+\frac{1}{2}Y\Big[\frac{f(a)+f(\bar{a})}{\lambda_--b^2}+\frac{f(b)+f(\bar{b})}{\lambda_--a^2}\Big]$$

$$(3.4)\qquad -\frac{1}{2}U\Big[\frac{f(a)-f(\bar{a})}{1-a^2}+\frac{f(b)-f(\bar{b})}{1-b^2}\Big]-\frac{1}{2}V\Big[\frac{f(a)-f(\bar{a})}{1-a^2}-\frac{f(b)-f(\bar{b})}{1-b^2}\Big]$$

$$-\frac{1}{2}U\left[\frac{f(a)-f(\bar{a})}{\lambda_+-b^2}+\frac{f(b)-f(\bar{b})}{\lambda_+-a^2}\right]+\frac{1}{2}V\left[\frac{f(a)-f(\bar{a})}{\lambda_--b^2}-\frac{f(b)-f(\bar{b})}{\lambda_--a^2}\right].$$

Servons-nous de (2.2) et (2.6) pour exprimer X, Y, U, V et $f(a)\pm f(\bar{a})$, ...; X, Y, U, V en fonction de matrices de $\{V_6\}$ et $\{C_4\}$, et nous obtenons après un calcul simple le résultat sous sa forme définitive

$$\begin{aligned}(3.5)\quad B = {} & E[d_0(EE)+d_1(CC)+d_2(C_2C_2)+2\mu(EC_2)]\\ & +C_2[g_0(EE)+g_1(CC)+g_2(C_2C_2)+2\nu(EC_2)]\\ & +C_1[2f_1(EC)+2h_1(CC_2)]+C_3[2f_3(EC)+2h_3(CC_2)],\end{aligned}$$

où les coefficients d, g, μ, ν, f, h sont fonctions des seuls invariants de C. Si

$$t_0\equiv 1+I_2+I_4,\quad t_+\equiv\lambda_+^2+I_2\lambda_++I_4,\quad t_-=\lambda_-^2+I_2\lambda_-+I_4;$$

$$\frac{t_+-t_-}{2}\equiv x=1+5I_4-I_2,\ \frac{t_+-t_-}{4ab}\equiv y=2-I_2$$

et

$$t_+t_-=(1-I_2+5I_4)^2-4I_4(2-I_2)^2$$

sont fonctions des invariants.

Les coefficients de (3.5) sont

$$(3.6)\quad\begin{cases} d_0=\dfrac{1+I_2}{t_0}-\dfrac{I_4y}{t_+t_-}-\dfrac{I_4(x-2y)}{t_0t_+t_-},\\ d_1=\dfrac{x-y(y-1)}{t_+t_-},\ d_2=-\dfrac{y}{t_+t_-}+\dfrac{2y-x}{t_0t_+t_-};\\ g_0=\dfrac{1_4(x-2y)}{t_0t_+t_-},\ g_1=\dfrac{y}{t_+t_-},\ g_2=\dfrac{x-2y}{t_0t_+t_-},\ 2f_3=\dfrac{2[2x+y(2I_2-t_0)]}{t_0t_+t_-};\\ 2h_3=-\dfrac{4(x+yI_2)}{t_0t_+t_-},\ 2\mu=\dfrac{I}{t_0}+\dfrac{x}{t_+t_-}-\dfrac{2(x-2yI_4)}{t_0t_+t_-},\ 2\nu=\dfrac{2(x-2I_4y)}{t_0t_+t_-};\\ 2f_1=\dfrac{2[f_3I_2+41_4y-x(1+I_4)]}{t_0t_+t_-}\quad\text{et}\quad 2h_1=-\dfrac{4[I_2h_3+x-y(1+I_4)]}{t_0t_+t_-}.\end{cases}$$

4. Nous avons exprimé le résultat final (3.6) en fonction de matrices déduites de C suivant des lois déterminées, avec des coefficients qui sont fonctions des invariants de C. Ce résultat est valuable dans tous les cas, même quand pour des matrices parti-

culieres, il n'existe pas un système complet de vecteurs propres et l'on ne peut plus développer suivant (3.3) la matrice résolvante B mais le résultat général (3.5) reste toujours vrai. L'expression (3.6) donne donc la solution générale du problème posé en (1.1).

Néanmoins, il est intéressant d'observer que, dans les cas particuliers $I_4 = 0$ ou $I_2 = 0$, $I_4 = 0$, on peut facilement mettre la solution sous forme d'une somme d'un petit nombre de termes, grâce à une méthode d'itération.

Discutons brièvement le cas $I_4 = 0$.

On a déjà déduit les propriétés spéciales de la matrice C en (2.11) et (2.12) : à savoir

$$(CC_2) = 0, \quad (C_2C_2) = I_2(C_1C_1), \quad (C_3C_3) = I_2(C_2C_2),$$
$$C_3+I_2C_1 = 0, \qquad C_4+I_2C_2 = 0.$$

Nous écrivons (1.1) dans la forme

$$U = AT = [E\{(EE) + (CC)\}+2C(EC)]T$$

ou

$$U = T+[E(CC)+2C(EC)]\,T = T+\delta T; \tag{4.1}$$

et cherchons une solution en posant

$$T = U+\Phi_1.$$

L'équation ayant la forme

$$U \equiv T+\delta T.$$

On déduit facilement pour Φ, l'equation

$$-\delta U = \Phi_1+\delta\Phi_1.$$

En répétant l'opération, nous obtenons successivement

$$\delta\delta U = \Phi_2+\delta\Phi_1, \quad -\delta\delta\delta U = \Phi_3+\delta\Phi_3. \tag{4.2}$$

En raison des propriétés spéciales de l'opérateur [(2.11)—(2.12)] nous déduisons facilment

$$\delta\delta\delta = I_2^2\delta. \tag{4.3}$$

En posant $\delta = \Delta-E$, où E est l'opérateur-unité, l'équation (4.2) s'écrit

$$\Delta\Phi_3 = -I_2^2(\Delta-E)U$$

ou

$$\Delta(\Phi_3+I_2^2U) = I_2^2U. \tag{4.4}$$

L'équation originale est

$$\Delta T = U.$$

Comme dét $\Delta \neq 0$, la solution est unique et l'on voit facilement que la solution de (4.4) est

$$\Phi_3 + I_2^2 U = I_2^2 T.$$

En écrivant

$$\Phi_3 = T - U + \delta U - \delta\delta U = I_2^2(T-U),$$

nous avons facilement

$$T = U + \frac{\delta U - \delta\delta U}{I_2^2 - 1} \quad . \tag{4.5}$$

En développant, nous avons le résultat suivant :

$$T_{\sigma\mu\nu} = U_{\sigma\mu\nu} - (1+I_2)^{-1} U_{\sigma kl} C_\mu^k C_\nu^l + (I_2^2-1)^{-1}[C_\sigma^t (C_\mu^k U_{tk\nu} + C_\nu^l U_{t\mu l})]$$

$$-(I_2^2-1)^{-1} C_{2,\sigma}^t (U_{tk\nu} C_{2,\mu}^k + U_{t\mu l} C_{2,\nu}^l) - 2(I_2^2-1)^{-1} C_{2,\sigma}^t C_\mu^k C_\nu^l U_{tkl}.$$

La solution générale nous donne le même résultat en posant $I_4 = 0$ et en utilisant les relations particulières (2.11) et (2.12).

On peut aussi traiter de même les autres cas $I_2 = 0$, $I_4 = 0$.

Post-Scriptum.

Si les coefficients λ de

$$N \equiv \lambda_0 + \lambda_1 C_1 + \lambda_2 C_2 + \lambda_3 C_3 \text{ (nombre quelconque dans } [C]_4)$$

sont choisis dans l'algèbre $[V]_6$, ces nombres forment une nouvele algèbre $[cV]_{24}$.

Résoudre

$$U = [e(EE) + e(cc) + 2c(Ec)]\, T \equiv BT$$

est donc équivalent à trouber l'inverse de B en tant que nombre algébrique. L'inverse est aussi un nomber dans $[cV]$ et on peut le trouver facilement de la facon suivante.

Nous vérifions d'abord les résultats suivants à partir de la règle de multiplication et des identités pour l'algèbre $[V]_6$.

Si

$$L = (C_2 C_2) - I_4(EE), \qquad M = 2(EC_2) + I_2(CC),$$

$$R = 2L + I_2 M, \qquad X = 4(EC)^2 = 2(CC) + 2(EC_2),$$

et $D \equiv I_2^2 - 4I_4$, le discriminant des équations

$$x^2 + I_2 x + I_4 = 0,$$

on a alors

$$L(EC) = M(EC) = 0; \quad d'où\ LX = MX = RX = 0$$

$$RL = DL, \quad RM = DM, \quad RR = DR, \quad M^2 = R$$

et

$$XX + 2I_2X + D = R.$$

Dans ce qui suit, nous employons les abréviations suivantes : $s = 2+I_2$, $t_0 = 1+I_2+I_4$, $x = 1+5I_4-I_2$, et $y = 2-I_2$.

Comme $B = (EE)+(CC)+2C(EC)$, en le multipliant par son conjugé

$$B^* = (EE)+(CC)-2C(EC),$$

nous avons

$$BB^* = (EE+CC)^2-4C_2(EC)^2$$

ou

$$BB^* = \left(1+\frac{R-sM}{t_0}\right)(t_0+X-C_2X)$$

après factorisation;

Comme $$\left(1+\frac{R-sM}{t_0}\right)\left(1+\frac{R+sM}{t_0}\right) \equiv 1,$$

$$\frac{1}{BB^*} = \frac{\left[1+\frac{R+sM}{2t_0}\right][t_0+X(1+I_2)+C_2X]}{[t_0+X-C_2X][t_0+X(1+I_2)+C_2X]}$$

après l'addition du même facteur dans le numérateur et le dénominateur de l'expression de droite.

En faisant la multiplication, on obtient pour le dénominateur

$$t_0[x+y(X+I_2)+R].$$

Nous observons que

$$[x+y(X+I_2)+R][x-y(X+I_2)-\nu R] = \Omega = t_+t_- = x^2-4I_4y^2$$

si $$\nu = \frac{x-2y}{t_0}.$$

On peut donc exprimer facilement l'inverse par l'équation suivante :

$$B^{-1} = \frac{B^*\left[1+\frac{R+sM}{2t_0}\right][t_0+X(1+I_2)+C_2X][x-y(X+I_2)-\nu R]}{t_0t_+t_-}.$$

En supprimant les crochets, on obtient un résultat équivalent à celui de l'article principal.

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Solution of a Tensor Equation Occurring in the Unitary Field Theory

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Summary — The solution to the tensor equation on which the determination of $\Gamma^{\lambda}_{\mu\nu}$ depends, is explicitly expressed in terms of the C and their invariants.

Note that the iteration gives a rigorous solution when

$$\det C = 0.$$

1. In a preceding article ([1]) it has been shown that it is possible to reduce the calculation of the affine coefficients $\Gamma^{\lambda}_{\mu\nu}$ in terms of $g^{\mu\nu}$ and their derivatives to the solution of the tensor equation

$$U_{a\mu\nu} = T_{a\mu\nu} + T_{akl} C^k_\mu C^l_\nu + C^t_a (T_{tk\nu} C^k_\mu + T_{t\mu l} C^l_\nu). \tag{1.1}$$

The tensors U and T are antisymmetric with regard to the indices μ and ν, $C^{\,k}_{\mu}$ are the elements of a matrix C formed from the symmetric and antisymmetric parts of the tensor g :

$$\begin{cases} C = Sa, \quad a_{\mu\nu} = \dfrac{1}{2}(g_{\mu\nu} - g_{\nu\mu}), \quad s_{tk} = \dfrac{1}{2}(g_{tk} + g_{kt}), \\ S^{\mu t} S_{tk} = \delta^{\mu}_{k}. \end{cases} \tag{1.2}$$

The eigenvalues x of the matrix C satisfy the equation

$$x^4 + I_2 x^2 + I_4 = 0, \tag{1.3}$$

I_2 and I_4 are even invariants of C ; from the equation (1.2) it immediately follows that the odd invariants are zero.

The eigenvector $M^{\,k}_{a}$ of C corresponding to the eigenvalue a satisfies the equation

$$C^{\mu}_{k} M^{k}_{a} = a M^{\mu}_{a}. \tag{1.4}$$

The transposed matrix $\overline{C}$ ($\overline{C}^{\,\mu}_{k} = C^{\,k}_{\mu}$) has the same eigenvalues as C but its eigenvectors $\overline{M}^{\,k}_{a}$ are different.

If the characteristic equation has all its roots distinct, then there exists a complete series of eigenvectors M and $\overline{M}$ of C and $\overline{C}$ and we can express the elements of C in terms of the eigenvectors : that is

$$C^{k}_{\mu} = \Sigma \lambda \overline{M}^{\mu}_{\lambda} M^{k}_{\lambda}. \tag{1.5}$$

The solution is easily calculated from (1.1) :

$$(1.6)\qquad \begin{cases} T_{\sigma\mu\nu} = \sum\limits_{abc} (1+ab+bc+ca)^{-1} \overline{M}^{\sigma}_{a} M^{x}_{a} \overline{M}\begin{pmatrix}\mu\nu\\ bc\end{pmatrix} M\begin{pmatrix}yz\\ bc\end{pmatrix} U_{xyz} \quad (b \neq c), \\ \\ M\begin{pmatrix}yz\\ bc\end{pmatrix} = M^{y}_{b} M^{z}_{c} - M^{y}_{c} M^{z}_{b} \ ; \end{cases}$$

and the summation is over the 24 possible combinations of (abc).

The solution (1.6) can be written in the form of a tensor equation

the components of the tensor B being

$$(1.7)\qquad B^{xyz}_{\sigma\mu\nu} = \Sigma \frac{\overline{M}^{\sigma}_{a} \overline{M}\begin{pmatrix}\mu\nu\\ bc\end{pmatrix} M^{x}_{a} M\begin{pmatrix}yz\\ bc\end{pmatrix}}{1+bc+ca+ab}$$

(1.7) is identical to (1.5).

I intend to carry out the summation in (1.7) and obtain the components of B in terms of the elements of C.

2. By means of C and of the unitary matrix E we can form an algebra $\{C_4\}$ with addition and matrix multiplication.

The general formula of a matrix A in $\{C_4\}$ is

$$(2.1)\qquad \begin{cases} A = \lambda_0 + \mu_1 C_1 + \mu_2 C_2 + \mu_3 C_3; \\ C_1 = C, \quad C_2 = CC, \quad C_3 = CCC. \end{cases}$$

All the matrices A commute and therefore have the same eigenvector M as C.

The matrices $\overline{A}$ of the transposed algebra also have the same eigenvector $\overline{M}$ as $\overline{C}$.

If a , $\bar{a}$, b , $\bar{b}$ are the roots of the characteristic equation (1.3), one can easily see that the basic matrices develop in the following way :

$$(2.2)\qquad \begin{cases} E = f(a)+f(\bar{a})+f(b)+f(\bar{b}): & C_2 = a^2[f(a)+f(\bar{a})]+b^2[f(b)+f(\bar{b})]; \\ C_1 = a[f(a)-f(\bar{a})+b[f(b)+f(\bar{b})]: & C_3 = a^3[(fa)-f(\bar{a})]+b^3[f(b)-f(\bar{b})]; \end{cases}$$

where $f(a) = \overline{M}_a \ M_a$,. . and by adding the lower and upper indices to the above formulae, one gets

$$C^{\lambda}_{\mu} = \left(\overline{M}^{\mu}_{a} M^{\lambda}_{a} - \overline{M}^{\mu}_{\bar{a}} M^{\lambda}_{\bar{a}} \right) a + b \left(\overline{M}^{\mu}_{b} M^{\lambda}_{b} - \overline{M}^{\mu}_{\bar{b}} M^{\lambda}_{\bar{b}} \right)$$

Let us now deduce a new algebra $\{V_6\}$, where the matrix V is formed from any two elements A, B of $\{C_4\}$:

$$V = (AB),$$

with

$$(2.3)\qquad V^{\mu\nu}_{rs} = \frac{1}{2} \left(A^{\mu}_{r} B^{\nu}_{s} + A^{\nu}_{s} B^{\mu}_{r} - A^{\mu}_{s} B^{\nu}_{r} - A^{\nu}_{r} B^{\mu}_{s} \right) \quad (\mu \neq \nu \text{ et } r \neq s).$$

The rule of composition (2.3) shows that all the matrices V of $\{V_6\}$ commute among themselves.

All the matrices V have the same eigenvectors with six components as (CC) ; it is easily seen that the six eigenvalues of (CC) are obtained by multiplying the eigenvalues of C two by two. These are

$$(2.4) \qquad a\bar{a},\quad b\bar{b},\quad ab,\quad \bar{a}\bar{b},\quad a\bar{b},\quad \bar{a}b;$$

and the eigenvectors with six components are deduced from the eigenvectors of C through a similar rule to (2.3) ; for example :

$$(2.5) \qquad M^{\mu\nu}_{ab} = M^{\mu}_{a}\, M^{\nu}_{b} - M^{\mu}_{b} M^{\nu}_{a},$$

are the components of Mab, the vector which corresponds to the eigenvalue ab of (CC).

The matrices $\overline{V}$ of the transposed algebra $\{\overline{V}_6\}$, also have the same eigenvectors with six components as ($\overline{C}\,\overline{C}$) ; it is easily noticed that the vectors $\overline{M}\,(ab)$ are deduced from the eigenvectors of $\overline{C}$ by the following rule :

$$\overline{M}^{\mu\nu}_{ab} = \overline{M}^{\mu}_{a}\, \overline{M}^{\nu}_{b} - \overline{M}^{\mu}_{b}\, \overline{M}^{\nu}_{a}, \quad \ldots$$

As the bases of the algebra $\{V_6\}$, we can take any six linearly independent matrices of V_6.

We choose the following six matrices whose expansion we give in terms of the eigenvectors :

$$(2.6) \qquad \begin{cases} (EE) = V(a\bar{a}) + V(b\bar{b}) + V(ab) + V(\bar{a}\bar{b}) + V(a\bar{b}) + V(\bar{a}b), \\ (CC) = -a^2 V(a\bar{a}) - b^2 V(b\bar{b}) + ab[V(ab) + V(\bar{a}\bar{b})] - ab[V(a\bar{b}) + V(\bar{a}b)] \\ (C_2C_2) = a^4 V(a\bar{a}) + b^4 V(b\bar{b}) + a^2b^2[V(ab) + V(\bar{a}\bar{b})] + a^2b^2[V(a\bar{b}) + V(\bar{a}b)], \\ 2(EC_2) = -2a^2 V(a\bar{a}) - 2b^2 V(b\bar{b}) + (a^2+b^2)[V(ab + V(\bar{a}\bar{b}) + V(a\bar{b}) + V(\bar{a}b)], \\ 2(EC) = -(a+b)[V(ab) - V(\bar{a}\bar{b})] + (a-b)[V(a\bar{b}) - V(\bar{a}b)], \\ 2(CC_2) = ab(a+b)[V(ab) - V(\bar{a}\bar{b})] - ab(a-b)[V(a\bar{b}) - V(\bar{a}b)], \\ V(ab) = \overline{M}(ab)\, M(ab), \quad \ldots \end{cases}$$

The formulae (2.6) are obtained in a manner completely similar to the manner used to obtain the formulas (2.2).

The four other combinations which can be formed from the basic elements of $\{C_4\}$ depend on the six above mentioned equations by virtue of the four identities mentioned below :

$$(2.7) \qquad \begin{cases} (EC_3) + (C\,C_2) + I_2(EC) = 0, \\ 2(CC_3) + (C_2C_2) + I_2(CC) - I_1(EE) = 0, \\ (C_2C_3) - I_4(EC) = 0, \\ (C_3C_3) - I_2(C_2C_2) - I_4[(CC) + 2(EC_2)] = 0. \end{cases}$$

It is also easily shown that by multiplying V_1 and V_2 in $\{V_6\}$ with

we have

$$V_1 = (AB), \quad V_2 = (CD),$$

(2.8)
$$V_1 V_2 = \tfrac{1}{2}[(AC \cdot BD)+(AD \cdot BC)].$$

With (2.8), the identities in (2.7) and the general identity of Cayley - Hamilton :

(2.9)
$$C_4+I_2C_2+I_4E = 0$$

each matrix of the form (C_r C_s) can be reduced to a linear function of the six elements of the basis adopted in (2.6).

We can choose the bases and demonstrate the analogous identities for all algebras $V_{\frac{n(n-1)}{2}}$ composed from $\{C_n\}$ [by a process similar to (2.3)].

With the minors m_3 of any matrix C_4, a conjugate matrix D can be formed.

In the case of C studied here, where C = Sa is the product of the symmetric component S by the antisymmetric a, it is easily seen that D is given by

$$D = A's',$$

where A' and s' are formed similarly from the minors m_3 of a and S.

We take for granted that always

$$\| S \| = \Delta \quad \text{et} \quad S \neq 0.$$

One can nevertheless have

$$\| C \| = 0 \quad \text{si} \quad \| a \| = 0.$$

The case $\| a \| = 0$ is interesting for the unitary field theory. Therefore we will discuss the particular properties of V_6 when $\| C \| = 0$ as a consequence of $\| a \| = 0$.

a is an antisymmetric matrix of order 4. Therefore Δ which is the determinant of a is a perfect sequare and $\sqrt{\Delta}$ is a factor of all the minors m_3 of a.

If $\| a \| = 0$, A and D, conjugate minors of C, are identically zero. For the conjugate matrix, we have an identity analogous to (2.9) ;

(2.10)
$$C_3+I_2C_1+D = 0 \quad (C_3 = CCC).$$

When

$$\| a \| = 0, \quad I_4 = 0 \quad \text{et} \quad D = 0,$$

we have

(2.11)
$$C_3+I_2C_1 = 0$$

and from the identities (2.7) we immediately obtain

(2.12)
$$(CC_2) = 0, \quad (C_2C_2) = 1_2(C_1C_1), \quad (C_3C_3) = 1_2(C_2C_2).$$

We will use these relations to deduce the solution of (1.1) in the particular case $I_4 = 0$.

3. Now we carry out the summation indicated in (1.6) and (1.7). Let us note some simple properties of the denominator D (abc) :

$$(3.1)\qquad \begin{cases} D(a\bar{a}c) = D(ca\bar{a}) = 1-a^2, \\ D(abc) = D(\bar{a}\bar{b}\bar{c}), \\ D(aab) = \lambda_+ - b^2, \qquad \lambda_+ = 1+(a+b)^2, \\ D(aa\bar{b}) = \lambda_- - b^2, \qquad \lambda_- = 1+(a-b)^2, \end{cases}$$

With the help of these simple relations, we can easily regroup the terms to be summed up :

We use the following notations to write the final results :

$$(3.2)\qquad \begin{cases} V(ab)+V(\bar{a}\bar{b}) = X, \qquad V(a\bar{b})+V(\bar{a}b) = Y\ ; \\ V(ab)-V(\bar{a}\bar{b}) = U, \qquad V(a\bar{b})-V(\bar{a}b) = V\ ; \\ \bar{M}_a M_a = f(a), \qquad \bar{M}(bc)M(bc), = V(bc), \quad \ldots\ldots \end{cases}$$

The summation (1.7) is written as

$$(3.3)\qquad B = \Sigma \frac{f(a)V(bc)}{D(abc)}\ .$$

After regrouping we get

$$(3.4)\qquad B = \Sigma f(\lambda)\left[\frac{V(a\bar{a})}{1-a^2}+\frac{V(b\bar{b})}{1-b^2}\right]+\frac{1}{2}(X+Y)\left[\frac{f(a)+f(\bar{a})}{1-a^2}+\frac{f(b)+f(b)}{1-b^2}\right]$$

$$+\frac{1}{2}X\left[\frac{f(a)+f(\bar{a})}{\lambda_+-b^2}+\frac{f(b)+f(\bar{b})}{\lambda_+-a^2}\right]+\frac{1}{2}Y\left[\frac{f(a)+f(\bar{a})}{\lambda_--b^2}+\frac{f(b)+f(\bar{b})}{\lambda_--a^2}\right]$$

$$-\frac{1}{2}U\left[\frac{f(a)-f(\bar{a})}{1-a^2}+\frac{f(b)-f(\bar{b})}{1-b^2}\right]-\frac{1}{2}V\left[\frac{f(a)-f(\bar{a})}{1-a^2}-\frac{f(b)-f(\bar{b})}{1-b^2}\right]$$

$$-\frac{1}{2}U\left[\frac{f(a)-f(\bar{a})}{\lambda_+-b^2}+\frac{f(b)-f(\bar{b})}{\lambda_+-a^2}\right]+\frac{1}{2}V\left[\frac{f(a)-f(\bar{a})}{\lambda_--b^2}-\frac{f(b)-f(\bar{b})}{\lambda_--a^2}\right].$$

Let us make use of (2.2) and (2.6) to express X, Y, U, V and $f(a) \pm f(\bar{a})$, . . . ; X, Y, U,V are functions of matrices of {V6} and $\{C_4\}$, and we obtainn after an easy calculation the result in its final form

$$
\begin{aligned}
B = {} & E[d_0(EE)+d_1(CC)+d_3(C_2C_2)+2\mu(EC_2)] \\
& +C_2[g_0(EE)+g_1(CC)+g_2(C_2C_2)+2\nu(EC_2)] \\
& +C_1[2f_1(EC)+2h_1(CC_2)]+C_3[2f_3(EC)+2h_3(CC_2)],
\end{aligned} \tag{3.5}
$$

where the coefficient d, g, μ, ν, f, h are functions only of the invariants of C. If

$$
t_0 \equiv 1+I_2+I_4, \quad t_+ \equiv \lambda_+^2+I_2\lambda_++I_4, \quad t_- = \lambda_-^2+I_2\lambda_-+I_4;
$$

$$
\frac{t_+-t_-}{2} \equiv x = 1+5I_4-I_2, \quad \frac{t_+-t_-}{4ab} \equiv y = 2-I_2
$$

and

$$
t_+t_- = (1-I_2+5I_4)^2-4I_4(2-I_2)^2
$$

are functions of the invariants

The coefficients of (3.5) are

$$
\left\{
\begin{aligned}
& d_0 = \frac{1+I_2}{t_0} - \frac{I_4 y}{t_+t_-} - \frac{I_4(x-2y)}{t_0t_+t_-}, \\
& d_1 = \frac{x-y(y-1)}{t_+t_-}, \; d_2 = -\frac{y}{t_+t_-} + \frac{2y-x}{t_0t_+t_-}; \\
& g_0 = \frac{1_4(x-2y)}{t_0t_+t_-}, \; g_1 = \frac{y}{t_+t_-}, \; g_2 = \frac{x-2y}{t_0t_+t_-}, \; 2f_3 = \frac{2[2x+y(2I_2-t_0)]}{t_0t_+t_-}; \\
& 2h_3 = -\frac{4(x+yI_2)}{t_0t_+t_-}, \; 2\mu = \frac{I}{t_0} + \frac{x}{t_+t_-} - \frac{2(x-2yI_4)}{t_0t_+t_-}, \; 2\nu = \frac{2(x-2I_4y)}{t_0t_+t_-}; \\
& 2f_1 = \frac{2[f_3I_2+41_4y-x(1+I_4)]}{t_0t_+t_-} \quad \text{et} \quad 2h_1 = -\frac{4[I_2h_3+x-y(1+I_4)]}{t_0t_+t_-}.
\end{aligned}
\right. \tag{3.6}
$$

4. We have expressed the final result (3.6) in terms of the matrices deduced from C following fixed rules, with coefficients which are functions of the invariants of C. This result is valid in all the cases, even when for particular matrices there is no complete system of eigenvectors and we can no longer develop the resolvent matrix B as per (3.3) but the general result (3.5) always remains true. The expression (3.6) thus gives the general solution of the problem posed in (1.1).

Nevertheless it is interesting to observe that in the particular cases $I_4 = 0$ or $I_2 = 0$, $I_4 = 0$, we can easily put the solution in the form of a total sum of a small number of terms, thanks to a method of iteration.

Let us briefly discuss the case $I_4 = 0$.

We have already deduced the special properties of the matrix C in (2.11) and (2.12) : to wit

$$(CC_2) = 0, \quad (C_2C_2) = I_2(C_1C_1), \quad (C_3C_3) = I_2(C_2C_2),$$
$$C_3+I_2C_1 = 0, \qquad C_4+I_2C_2 = 0.$$

We write (1.[illegible] the form

$$U = AT = [E\{(EE) + (CC)\}+2C\ EC)]T$$

or

$$U = T+[E(CC)+2C(EC)]\ T = T+\delta T; \tag{4.1}$$

and look for a solution by putting

$$T = U+\Phi_1.$$

The equation having the form

$$U \equiv T+\delta T.$$

we easily deduce for Φ the equation

$$-\delta U = \Phi_1+\delta\Phi_1.$$

By repeating the operation, we successively obtain

$$\delta\delta U = \Phi_2+\delta\Phi_1, \quad -\delta\delta\delta U = \Phi_3+\delta\Phi_3. \tag{4.2}$$

Beacause of special properties of the operator [(2.11) - (2.12)] we easily deduce

$$\delta\delta\delta = I_2^2\delta. \tag{4.3}$$

By putting $\delta = \Delta - E$, where E is the unit operator, the equation (4.2) is written as

$$\Delta\Phi_3 = -I_2^2(\Delta-E)U$$

or

$$\Delta(\Phi_3+I_2^2U) = I_2^2U. \tag{4.4}$$

The original equation is

$$\Delta T = U.$$

With $\det \Delta \neq 0$, the solution is unique and one easily sees that the solution of (4.4) is

$$\Phi_3+I_2^2U = I_2^2T.$$

By writing

$$\Phi_3 = T-U+\delta U-\delta\delta U = I_2^2(T-U),.$$

we easily have

$$T = U+\frac{\delta U-\delta\delta U}{I_2^2-1}\ . \tag{4.5}$$

By developing this, we have the following result :

$$T_{\sigma\mu\nu} = U_{\sigma\mu\nu} - (1+I_2)^{-1} U_{\sigma kl} C^k_\mu \, C^l_\nu + (I_2^2-1)^{-1}[C^t_\sigma (C^k_\mu U_{tk\nu} + C^l_\nu U_{t\mu l})]$$

$$-(I_2^2-1)^{-1} C^t_{2,\sigma} \, (U_{tk\nu} C^k_{2,\mu} + U_{t\mu l} C^l_{2,\nu}) - 2(I_2^2-1)^{-1} C^t_{2,\sigma} C^k_\mu C^l_\nu U_{tkl} \,. \tag{4.6}$$

The general solution gives us the same result by putting $I_4 = 0$, and using the particular relations (2.11) and (2.12).

We can similarly treat the other cases $I_2 = 0$, $I_4 = 0$.

Post-script

If the coefficients λ of

$$N \equiv \lambda_0 + \lambda_1 C_1 + \lambda_2 C_2 + \lambda_3 C_3 \quad \text{(any number in } [C]_4)$$

are chosen in the algebra $[V]_6$, these numbers form a new algebra $[cV]_{24}$.

To solve

$$U = [e(EE) + e(cc) + 2c(Ec)]\, T \equiv BT$$

is thus equivalent to finding the inverse of B as an algebraic number. The inverse is also a number in [cV] and we can easily find it in the following way.

First, we verify the following results as per the rule of multiplication and the identities for the algebra $[V]_6$.

If

$$L = (C_2 C_2) - I_4(EE), \qquad M = 2(EC_2) + I_2(CC),$$
$$R = 2L + I_2 M, \qquad X = 4(EC)^2 = 2(CC) + 2(EC_2),$$

and $D \equiv I_2^2 - 4I_4$ is the discriminant of the equations

$$x^2 + I_2 x + I_4 = 0,$$

then we have

$$L(EC) = M(EC) = 0; \quad d'\text{où } LX = MX = RX = 0$$
$$RL = DL, \quad RM = DM, \quad RR = DR, \quad M^2 = R$$

and

$$XX + 2I_2 X + D = R.$$

In what follows we use the the following abbreviations : $s = 2 + I_2$, $t_o = 1 + I_2 + I_4$ $x = 1 + 5I_4 - I_2$, and $y = 2 - I_2$.

With B = (EE) + (CC) + 2C(EC), by multiplying it with its conjugate

$$B^* = (EE) + (CC) - 2C(EC),$$

we have

$$BB^* = (EE + CC)^2 - 4C_2(EC)^2$$

or

$$BB^* = \left(1 + \frac{R - sM}{t_0}\right)(t_0 + X - C_2 X)$$

after factorization ; with $\left(1-\frac{\mathrm{R}-s\mathrm{M}}{t_o}\right)\left(1+\frac{\mathrm{R}+s\mathrm{M}}{t_o}\right) \equiv \mathrm{t}$,

$$\frac{1}{BB^*}=\frac{\left[1+\frac{R+sM}{2t_0}\right][t_0+X(1+I_2)+C_2X]}{[t_0+X-C_2X][t_0+X(1+I_2)+C_2X]}$$

after adding the same factor in the numerator and denominator of the right side of the equation.

By multiplying we obtain for the denominator

$$t_0[x+y(X+I_2)+R].$$

We notice that

$$[x+y(X+I_2)+R][x-y(X+I_2)-\nu R] = \Omega = t_+t_- = x^2-4I_4y^2$$

if $\nu = \frac{x-2y}{t_o}$.

We can thus easily express the inverse by the following equation :

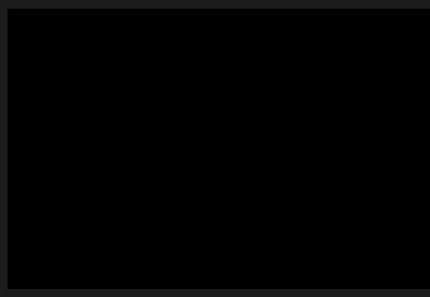
$$B^{-1}=\frac{B^*\left[1+\frac{R+sM}{2t_0}\right][t_0+X(1+I_2)+C_2X][x-y(X+I_2)-\nu R]}{t_0t_+t_-}.$$

By eliminating the brackets, we obtain a result equivalent to the one in the main paper.

English translation of *Bull Soc Math France* **83**, pp. 81-88, 1955 (Gauthier - Villars Publishers, Paris).

The S N Bose National Centre for Basic Sciences was established in June 1986, under the aegis of the Department of Science and Technology, Government of India, to foster, encourage and promote the growth of advanced studies in selected branches of basic sciences; to conduct original research in theoretical and mathematical sciences and other basic sciences in frontier areas, including challenging theoretical studies of future applications; to provide a forum of personal contacts and intellectual interaction among scientists within the country and also between them and scientists abroad; and to train young scientists for research in basic sciences. It has conducted several conferences, workshops, and seminars, and organized S N Bose Memorial Lectures by eminent scientists. Since November 1989 it has been co-ordinating a Theoretical Physics Seminar Circuit, enabling active researchers to travel to the centres on the circuit and exchange ideas with their counterparts. The Centre has its own core group of scientists engaged in research in condensed matter physics, particle physics, foundations of quantum theory, quantum optics and mathematical physics, with opportunities to interact with visiting fellows and scientists.

The S N Bose National Centre for Basic Sciences offers this volume to the nation and the community of scientists as a centenary tribute to S N Bose.



S N Bose : The Man and His Work

Part II : Life, Lectures and Addresses, Miscellaneous Pieces

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S N Bose

The Man and His Work



Life,

Lectures and Addresses,

Miscellaneous Pieces

S N Bose (1894-1974), best known for his seminal contribution to quantum statistics, left behind a substantial body of original work in the form of papers in a wide range of fields including chemistry, spectroscopy, thermoluminescence, statistics, group theory, mathematical physics and Einstein's unified theories, collected in the first part of this volume, with annotative introductions on the papers by distinguished scientists from a later generation.

Bose played a significant role in laying the foundations of science teaching and research at the two major universities of eastern India in the first half of the century, Calcutta and Dhaka (now in Bangladesh). It was in Dhaka in 1924 that he devised the new counting method for light-quanta which reconciled Einstein's concept of light-quanta with Planck's law. Within the next six months Einstein wrote three papers showing that Bose's method, when slightly extended to material atoms, yielded the correct quantum theory of ideal gases. The new counting method has come to be known as the Bose-Einstein statistics. Particles whose behaviour is described by the Bose-Einstein statistics have been named bosons after Bose.

As a teacher, Bose developed a fairly comprehensive model for education in India that focussed on the dissemination of a scientific culture in the country through the regional vernaculars; and was equipped to negotiate with the developmental needs of the nation. His lectures and addresses and miscellaneous pieces, collected in the second part of this volume (several of them translated from Bengali for the first time) provide glimpses into his various emotional and intellectual concerns, passions and obsessions, and friendships, documented in the long biography that is the most exhaustive to date, and brings together a lot of material gathered from various sources including the Einstein papers stored at Jerusalem and the archives of Dhaka University, and rarely seen pieces of correspondence.

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16 Bose delivers the Krishnan Memorial Lecture at the National Physical Laboratory, 12 March 1971.

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Hiralal, Professor Bose's old bearer

Foreword

The Satyendra Nath Bose National Centre for Basic Sciences was established by the Department of Science and Technology, Government of India, in 1986 as a memorial to the Indian scientist Satyendranath Bose. On the occasion of his birth centenary in 1994, the Centre has prepared the present volume incorporating his scientific papers, some selected miscellaneous pieces and addresses, and a biography. Bose wrote and published in four languages : Bengali, English, German and French. Though we have included the original German and French articles, we have provided their English translations also. His writings in Bengali have been collected and brought out by Bangiya Vijnan Parishad, an institution he helped to establish. We have included translations of some of these articles which may be of interest to a general reader.

Several biographical accounts of S N Bose were available. We found that the list of scientific papers was incomplete in all of them, and that not enough attention had been given to his work in Dhaka after his return from Europe. Two published papers — one in German (1927) and another in English (1929) from this period — are included in our collection. Enakshi Chatterjee and Santimay Chatterjee have given a fuller account of his activities in Dhaka — establishing physics laboratories, and helping develop modern research in physics and chemistry.

In some respects S N Bose's career is the story of the triumph and tragedy of Indian science in the first half of the twentieth century. His brief encounter with Europe in the twenties assured his place among the great scientists. He tried to strengthen the scientific base in India but he found little financial and spiritual sustenance. He himself described these two aspects very well, when he said that he was like a comet which came once but never returned, and he seemed to be living on the Moon.

Some comments by later Indian workers about the scientific papers are also included; these hopefully will help an inquisitive young research worker in placing the

scientific papers in the right perspective.

The volume is divided into two parts — the first part aimed at a more specialist readership, containing his scientific papers, with introductory commentaries by later workers in the specific fields; and the second part designed for a more general readership, with the exhaustive biography, his public lectures and addresses, and miscellaneous pieces (several of them translated from Bengali for the first time). While a special effort was made to bring together all his public lectures and addresses, only a selection of the miscellaneous pieces (mostly in Bengali) were translated, since several of them were themselves translations from French or German, or were on themes better covered elsewhere by Bose himself. The biographical resumé at the end gives a chronologically organized account of Bose's life, related to contemporary events and developments that affected his career as man and scientist.

A long list of acknowledgements preceding this foreword covers most of the individuals and organizations who have helped us in several ways. Special thanks are due to the Department of Science and Technology, Government of India, for financial support to the S N Bose National Centre for Basic Sciences for activities in the Centenary year, of which the present publication is one.

C K MAJUMDAR
Director
S N Bose National Centre for Basic Sciences
Calcutta

S N Bose : Life

1

Childhood

Satyendranath Bose was born in 1894 in Calcutta, then capital of British India and the second city of the Empire on which the sun never set. But by the turn of the century there were indications that momentous changes were in the offing. Not unexpectedly, the first rumblings of protest came from Bengal :

> Bengal had a clear half-century of British rule before it spread over wider areas. During the second half of the eighteenth century and the first half of the nineteenth, Bengal therefore played a dominant role in British Indian life. Not only was Bengal the centre and the heart of the British administration, but it also produced the first groups of English-educated Indians who spread out to other parts of India under the shadow of the British power. A number of very remarkable men rose in Bengal in the nineteenth century, who gave the lead to India in cultural and political matters, and out of whose efforts the new nationalist movement ultimately took shape.[1]

Satyendranath's grandfather, Ambikacharan, was one of those 'English-educated Indians who spread out to other parts of India', and his father, Surendranath, though working 'under the shadow of the British power' was so inspired by the nationalist ideal that he started a chemical and pharmaceutical industry on his own. By the time Satyendranath was in school and not yet in his teens there was the outbreak of the 1905 agitation against the partition of Bengal — a political event which changed the lives of many young men of his generation.

So the Bose family followed the general pattern of British-occupied India — the influx of white collar workers to the city, the Kāyastha caste taking more readily to

English education than the Brahmans and getting exposed to western ideas.

Bose's family belonged originally to the village Bara Jagulia in the district of Nadia, not very far from Calcutta. Nadia was the traditional seat of learning known for its Sanskrit *tols*, its scholars, its good manners and chaste diction. However, with the coming of the British and with the emergence of Calcutta, Nadia was fast losing its importance. According to family records the Boses had lived in this village for more than three centuries. The house at Calcutta was built by Durgacharan, great-grandfather of Surendranath. Completed in 1817, the house underwent extensive modification in 1924. Both Durgacharan and his son, Shyamacharan, died in their early fifties. But one of their forefathers, Madhusudan, had lived to be 95.

Ambikacharan spent most of his working life in the north-western regions of India as an accountant in the military commissariat. His eldest son, Surendranath, studied in Meerut Government School after completing primary schooling in the village. He passed his Entrance Examination in 1887 at the age of 17. He was studying in Hooghly College when news came of his father's sudden illness. He rushed to Meerut, but before he arrived, his father was dead.

This was a critical time for the family. The young Surendranath had to take charge of affairs. The family moved from the village to Calcutta, where their house at Iswar Mill Lane had tenants, compelling them to live in a rented house at Jorabagan until the house was vacated.

Surendranath however was a determined young man. He finally qualified himself as an accountant, passing the accounts examination from Sibpur, and worked in various capacities. As an accounts clerk in the Executive Engineering Department of the East India Railways he toured across Assam and North Bengal. Meanwhile, he had married Amodini, the daughter of an established lawyer of the Alipore Court, Motilal Roy Chowdhury. Their first child, Satyendranath, used to accompany his father in all his postings in Assam. He was then old enough to remember later some of the scenes, like logs drifting down the Brahmaputra and the scenic beauty of Assam. He recounted some of his childhood memories in a hand-written magazine he used to bring out with his friends.

Surendranath, by all standards, was a versatile man. He was very much a product of his time, when a new spirit of national identity was emerging :

> Middle-nineteenth century rationalism and Individualism of European culture were still the dominating ideas in the life and evolution of modern Bengal. But the conflicts of political interests between the new generation of English-educated Indians and the British officialdom in the country, and the more fundamental cultural conflict between European modernism and Indian mediaevalism soon provoked a new revolt against this foreign domination in the wake of which rapidly followed a new national self-consciousness which, in the first flush of its new-found pride of race and culture, commenced to repudiate whatever was foreign, irrespective of the intrinsic reason and value of it, and set up a new defence even of those social institutions and religious and spiritual tendencies that had previously been

openly repudiated as false and harmful.[2]

From the seventies of the last century the educated Bengali middle class was becoming conscious of its heritage — it was national and Hindu at the same time. A *Prospectus of a Society for the Promotion of National Feeling among the Educated Natives of Bengal* was published in 1866. The members of this society made a point of not using a single English word in their conversation and they were fined a pice for each English word used. Within a year the Hindu Mela was organized. The purpose of this annual meet was to bring the countrymen together :

> Our meeting is not for religion or pleasure — it is dedicated to our own land — India. Besides it has a greater purpose and that is self-reliance. . . It is a matter of great shame that born as human beings we are for ever dependent on others. So the second motive is to establish the idea of self-reliance in India, so that we can bring about our own improvement through our own efforts and not look up to others.[3]

Songs were written to spread the ideals of self-reliance. A rough English translation of a Bengali poem written by Manomohan Bose for the 1873 Hindu Mela session will give the reader something of the spirit of this new popular literature :

> Weavers and blacksmiths face starvation,
> They can't make a living from the anvil and loom.
> Our own products have lost their market.
> Alas, alas, my country is doomed.
> Needles and threads, even boxes of matches,
> They all come to us by the Sea.
> For lighting a lamp or to eat or to sleep
> For every little thing we are not free.

This was the atmosphere which Surendranath breathed and which to a great extent moulded his mental make-up. It was this spirit which prompted him to start the Indian Chemical and Pharmaceutical Works jointly with Satishchandra B. ahma in 190[illegible].

> Surendranath himself had a keen aptitude for mathematical thinking which because of his father's early death could not find full fruition. He became a qualified accountant and later in life held responsible posts in the Executive Engineering Department of the East India Railways. He was also interested in other branches of science and was the joint founder of the Indian Chemical and Pharmaceutical Works, a modest affair no doubt which still exists, but it came into being earlier than the more famous "Bengal Chemical" of Acharya Prafullachandra. His passion for philosophical studies was no less remarkable. He read through the *Yoga-bashiṣṭha* and the *Gita* many times over and at the same time was quite at home among the dialectical speculations of Hegel and Marx.[4]

The ancestral house was rebuilt in 1924 by Surendranath over the old structure which stands right at the edge of the street, as was the practice those days. Only the front facade of the building can be seen from the road. Divided in three symmetrical

sections, the front facade has two corinthian columns at the two ends. Windows are arranged in vertical lines. Devoid of any other ornamentation, the facade is simple and pleasing. The entrance to the house is from the side, the door leading to the sitting room coming just ahead of the main door. The layout is a typical example of the courtyard style plan — the entry opening to the courtyard and the rooms arranged on the three sides.

Satyendranath was the eldest child and the only son in a family of six daughters. Those days sons were treated differently and given a different kind of education. In fact, the daughters were not sent to school at all. Not having a brother in the family made him a lonely boy, and he made up for that by forming close friendships with boys of his own age, either in the neighbourhood or in school.

Those days daughters were considered more as burdens than blessings. So Satyendranath's mother was often taunted for having given birth to six daughters. But the parents were so proud of their son that the mother is said to have retorted proudly, 'I would rather bear a dozen daughters than another son who is not worthy of being a brother to Satyen Bose'.[5]

Her father was a classmate of Bankimchandra Chatterjee, the famous novelist. Bose's mother was a remarkable woman. Coming from a cultured and affluent background she had to adjust herself to a very different household :

> With only a nominal school education, she had extraordinary ability in managing domestic affairs and was remarkable for her warmth of heart and a high sense of dignity in life. She felt happy beyond measure when she came to know that Rabindranath himself had sought Satyen Bose, out of Dacca, invited him to Santiniketan and dedicated to him his book on Science in Bengali. The parents were proud of their son, and the son is no less grateful to the memory of his parents.[6]

All his life Bose remained devoted to his parents. In a letter to his friend, Maniklal De, he wrote on 3 July 1939 :

> I have come back to Dhaka on 1st July. But mother's state of health is causing me no end of worry. Unfortunately there is nothing I could do. The university is opening — there is pressure of work, so I was forced to come away.

Bose's upbringing can be described as normal and orthodox. His father had a dominating presence until the end. Even when he lived as an old man of ninety with his celebrated son, Satyen had the same alert response every time the old man thundered from upstairs, 'Bodi'. Bodi, incidentally, was Bose's nickname, a short form of Baidyanath, the deity. Over the years Bose's response to his father had mellowed — a kind of friendship had developed which was naturally impossible between a boy growing up in the early years of this century and the head of the family. Two specific orders restricted Bose's activities in adolescence — not to get involved with the revolutionaries (as the eldest child and the only son in the family, he had responsibilities to take over) and not to waste time listening to musical concerts. Satyen Bose

obeyed the first order but not the second. There was a streak of the rebel in him. But he was reasonable. He could understand why his father wanted him to stay away from the *'swadeshi'*[7] boys — but he was a sympathizer all his life. As for music, he did defy his father's orders now and then but would get caught occasionally. Music was not considered a very respectable occupation in orthodox society. Besides, the father was anxious that the son should not neglect his studies. Satyen however could always 'make up', and love for music was something which ran in his veins.

Bose's childhood was remarkably free from any traumatic experience of a personal nature. He had a normal family, a strict father, a loving mother, younger sisters, uncles and aunts as in any ordinary middle class Bengali joint family of the time. The parents laid great stress on education, but that again followed the general pattern. Had it been a childhood like Tagore's, away from close contact with parents, brought up under servants on whose whims depended the child's happiness,[8] then the memories would have persisted. But the very fact that Bose had a happy and normal childhood explains why he seldom talked or wrote much about it.

The family background explains much in S N Bose's character — his urbanity and innate sophistication, his love of art and literature, his love for perfection, his liberal and open mind, his originality, a sense of family loyalty and national pride. Unlike many of his eminent predecessors and contemporaries, Satyendranath was city bred. M N Saha came from an obscure village where facilities for education were practically non-existent. Sir P C Ray too was from a village and never severed his rural ties. Sir Jnanchandra Ghosh came from Bihar. S N Bose was always 'in the mainstream' so to speak, and perhaps this accounts for much of his casualness and his absence of the fighting spirit — for him to be accepted in society was never an uphill task.

Bose's urban background also explains the friendships he formed. Maniklal De, his neighbour and classmate, was particularly close to him. The circle of non-scientific friends was large, and they too were mostly Calcuttans. The tie was very subtle. The reason Bose was never close to Jnanchandra Ghosh as the latter was with Saha again has a lot to do with their not sharing an urban background.

The childhood years were important, but it is a pity we do not know more about his pre-school days except for such details that he would tear the pages of a book after he had gone through it, or that he would be doing sums on the cemented floor of the store room and that too under orders from his father. It seems he was playful and restless like all children of that age. The memoirs of his friend, Girijapati Bhattacharjee, show that he spent a lot of time at his friend's place, playing carrom or chess. He used to make up for that by staying up late. There was no electricity those days — it was either a lantern or an oil lamp. It is possible that his poor eyesight was a result of this.

Satyen's schooling began at the age of five. At first he was sent to Normal School which was close to their Jorabagan home. It was the same Normal School Rabindranath Tagore went to for some time. Later, when the family moved to their own house at Goabagan, he had to be put in the neighbouring New Indian School. It was a good school whose principal, Khudiram Bose, was noted for his radical ideas in the field of education. But Surendranath soon felt the urge to put his son into an even better school

where his talents would be sharpened by keener competition. And so in the final year of school, Satyen was admitted to the famous Hindu School.

During the first decade of the present century, the Hindu School and Hare School were two schools of distinction. When Satyendranath was in Hindu School, the rival Hare School had Ishan Ghose, the famous Pali scholar, as the headmaster, of whom the Hare boys were justly proud. The Hindu School had Rasamaya Mitra, who was an able headmaster and a devoted teacher. His textbook on English composition, grammar and translation was considered to be a model for students. The Hindu School had other equally inspiring teachers. The Bengali teacher, Saratchandra Shastri, was primarily responsible for creating in the students a passion for Bengali language and literature. In the final class in school the boys had to study English, Bengali, history, geography, mathematics and Sanskrit. The prescribed textbooks were Gaurisankar Dey's *Arithmetic and Algebra*, *Geometry* by Hall and Stevens, *World Geography* by Dudley Stamp, Adhar Mukherjee's *History of India* (the first chapter opened with the observation : 'India is the epitome of the world') and Rowe and Webb's *Grammar*. The prescribed books for the languages were university selections. As supplementary books they read Kipling's *Jungle Stories* and *Sītār Banabās* and *Kādambari* in Bengali by Iswarchandra Vidyasagar (1820-91), social reformer and educationist, and Jogin Bose's Life of Michael Madhusudan Dutt (1824-73), eminent Bengali poet.

In spite of weak eyes since childhood, Satyendranath was a voracious reader. His favourite poets were Tennyson and Tagore. According to his school friend, Girijapati Bhattacharjee, he was particularly fond of 'In Memoriam' and could recite the entire poem. He also knew Kalidasa's *Meghadoota* in Sanskrit by heart.

The mathematics teacher of the Hindu School, Upendranath Bakshi, was a legend. He was quick to recognize the signs of genius in the boy. On one occasion he awarded Satyen 110 marks out of 100 in a test for having worked out all the alternatives and some of the sums by more than one method.

The genius which later manifested itself as a wizard with numbers was unfolding during this period.In Girijapati's words,

> Our friendship began in 1908, when we were both in the Hindu School, Calcutta, though he was a year ahead of me. Even at school, Satyen was marked for his extraordinary intellect. Our mathematics teacher predicted that Satyen would one day be a great mathematician like Laplace or Cauchy . . .
>
> I remember, during our school days he came to our home one day and said we ought to be making instruments. What instruments? Why not a telescope, or a replacement battery for a pocket lamp, maybe? I found a large and a small sized magnifying lens and a tin-smith mounted them for us. Our schoolboy telescope gave about X 6 or X 7 magnification but as the lenses were uncorrected, the images were very distorted. Our battery lit the pocket lamp all right, but it was feeble and soon went dead. We made coal gas too by filling a closed earthenware with lumps of coal and placing it on a fire.[9]

Bhattacharjee's account is full of many other details but mostly related to college days because he talks of Bose explaining physics and chemistry to his friends — subjects which were not taught in school then. One can only assume that his childhood was not particularly eventful. The most important thing to have happened was the partition of Bengal in 1905. Bose was then a boy of twelve. It made a very lasting impression because his childhood memories never seemed to go beyond this point. Even in his Convocation address given at the Calcutta University in 1973, when he talked of the most formative period of his life, his earliest memory seems to be that of 1905 — the protest against the partition of Bengal :

> In August 1905 Lord Curzon, then Viceroy and Governor General of India, announced the decision of the government to divide Bengal. To divide and rule is a famous British imperial policy, and it was now being applied in Bengal as a check — so it seemed to the Bengalis — on the growing nationalism of India. The partition became effective on October 16. . .

The protest against the partition was led by Surendranath Banerjea, who organized the movement called *Swadeshi*. The intent of the movement was the support of home industry, and incidental to its advocacy was the boycott of English goods. Banerjea and others also started the *rākhi bandhan* (lit. tying the thread that protects) as a visible protest against partition. It took the form of the mutual tying of a piece of yellow string round the left wrist as a symbol of unity. The day chosen for the ceremony was the day partition was to become effective. Wherever men met on that day, each person was to tie a bit of yellow string round the other's wrist. This had a powerful appeal to the youthful imagination. The students were allotted a role in the day's proceedings. It was for them to check that the unity string was universally worn. No fire was lighted, no meal was cooked that day. This was Bengal's way of signifying to the government its view that partition was an outrage. Some students even took off their shoes and their shirts to show their anger at the humiliation of the people :

> We students found our way to the protest meetings that were held with great frequency and we learned the salutation of the hour which was *Bande-Mātaram* . . . an allusion to India in which affection for the mother-land and defiance of the foreign ruler were inextricably mingled. . . The mere sight of an Englishman in the distance was enough to set off any group of students shouting 'Bande-Mataram'. It was our *Marseillaise* and we made good use of it.[10]

Small wonder then that a boy of Satyen's temperament would be swayed completely. To a boy of eleven it was an emotional feeling — the mother figure of the country, *Bhāratmātā*, in chains was enough to fire his imagination and keep passions boiling. Bose's friend, Niren Ray, has aptly described the atmosphere : 'Bliss it was . . . in that dawn to be alive, to be a young student was the very heaven.'

Another contemporary, Jnanendranath Mukherjee, who was to make a name for himself in chemistry, has written in his autobiography about those heady years :

> I remember vividly that I was placed at the head of a procession with a

> small flag of the Indian National Congress in my hand. Everybody shouted *Bande Mātaram* and spoke of Surendranath as the national leader. I began to feel that nationalism and patriotism were very important. That was perhaps the reason I joined the Anushilan Samity while at school at Burdwan and learnt 'lāthi' play.[11]

Satyendranath was due to sit for his Entrance Examination in 1908. But unfortunately, he was down with an attack of chicken pox just two days before the examination. As a result, he lost one year. He had to appear the next year. That was probably the best thing that could have happened to him. Otherwise, he would not have had the stimulating company of a remarkable group of classmates, the most prominent of whom was Meghnad Saha.

2

College

1909-1915

Satyendranath had done very well in Sanskrit, history and geography. He also had a natural aptitude for languages and was already taking private lessons in French. But he decided to study science. The motivation was part of the nationalistic sentiment shared by most of his contemporaries. It was in a way an urge to do something for the country. No study of language or literature could have equipped them for that — that was what most of the bright students thought. Somehow science to them was a way to take the country forward towards development and prosperity. Knowledge of chemistry also had its uses. In the words of Sailen Ghosh, a friend and contemporary :

> I had got a firm hold on the idea that the things to be learned in the science course might be of great use in the business of freeing India and I was determined to persevere. I found later that a great many others reasoned similarly. Originally chemistry appealed to me most by reason of its connection with the business of making bombs. Afterwards physics claimed my attention because of the need for some device by which bombs could be exploded without injury to the person handling them.[1]

So Satyendranath joined the Intermediate Science class. Some of the others to join Presidency College in 1909 were Maniklal De, Jnanchandra Ghosh (who came from Giridih), Jnanendranath Mukherjee (who came from Bardhaman), Nikhilranjan Sen, Pulinbehari Sarkar and Amaresh Chakrabarty. 1909 was a remarkable year — it was the year of the Alipore Bomb Case. The political atmosphere was highly charged. A year ago the first bomb had been thrown at Muzaffarpur at a British official. Khudiram was caught and hanged. Overnight he became a martyr and a darling of the people. Songs

written about him have been popular ever since. Secret societies were being formed. It was but natural that college students would not remain unaffected.

> The years 1904-1906 saw the great upheaval known as the Swadeshi agitation, which aimed at, among other things, the educational reconstruction of the country. The Presidency College, a Government institution devoted avowedly to the task of Western science, was obviously not a proper nursery for Swadeshi education, but although the college carried on its normal duties, heedless of the storm blowing outside, the agitation deeply stirred the student community, and many pupils of Presidency College played a leading part in propagating Swadeshi and in founding the National Council of Education.[2]

The nationalist passion also turned out to be intellectually stimulating. Many of the 1909 batch of students stand out for their academic brilliance.

> The early decades of this century was a time when the air was more bracing and the wind was more fresh than it ever was or has ever been. The national spirit was high at all levels of human activity; and the development of Science was only a part of that activity.[3]

In a way it was a significant year in the scientific development of the country. P C Ray wrote in his autobiography about this extraordinary batch of students :

> In that memorable year some members of the brilliant group of students, who were afterwards destined to play a conspicuous part in notable research, took their admission in the Presidency College.[4]

Two other equally illustrious students to join in the B Sc class were Meghnad Saha from Dhaka and Sailendranath Ghosh from Cuttack. For students interested in science, Presidency College was the best place. So competition to get in was intense. One can get some idea of the situation prevailing in 1909 from the following excerpts taken from *An Indian Revolutionary*, a short autobiography by Sailendranath Ghosh. Sailen Ghosh was a brilliant student of physics and also a natural leader. He was one of those classmates of Satyendranath who was directly involved in the struggle for freedom. Ghosh was also instrumental in bringing Saha and Bose to physics.

As Ghosh recalls,

> The University of Calcutta at that time served the needs of seventy-two millions of people in Bengal, Bihar, Orissa and Burma. Later on, other universities were opened at Dacca, Patna and Rangoon, so that now the University of Calcutta draws its students from about forty-eight millions. Last year enrolment was thirty-nine thousand distributed amongst the fifty-four or more affiliated colleges. For those who, like myself, were interested in science, Presidency College was admittedly the best. Hence it was the most difficult to enter, especially because the number of students in each class was limited. Preference was given, naturally enough, to its own intermediate graduates, and after those had been provided for, there

were thirty places left for outside students. For those thirty places there were two thousand applicants.[5]

Though the 1909 batch was the most brilliant group in the history of that college, there were others who were also destined for greatness. P C Mahalanobis, Nilratan Dhar and S K Mitra were senior to this group. Subhaschandra Bose joined in 1913 — his brother Satish was in the same class as Satyen Bose.

At that time Presidency College had brilliant teachers like P C Ray, J C Bose, Surendranath Maitra, D N Mallick, Shyamadas Mukherjee and C E Cullis. The English department had P C Ghosh, Manmohan Ghosh and H M Percival. It was Mr Percival who was so impressed by Bose's answer in the Intermediate test that he added an extra ten marks and the remark, 'this boy is a genius'. In fact he called 'this boy' to him before he left for England to wish him well. Meanwhile Satyendranath was also expanding his circle of friends. Maniklal De was a classmate, and as he too came from a family long settled in Calcutta — in fact in Beadon Street, not very far from Iswar Mill Lane — a lifelong friendship developed. They met regularly after class. Maniklal took Bose to such places as the Tagore house and to the Vichitra Club meetings. But that came later, towards the end of their college career. In the Entrance examination both of them had secured the same marks — they shared the fifth position in order of merit. In the Intermediate examination Bose stood first and Maniklal second. In the B Sc class they separated — Bose opted for Mixed Mathematics and Maniklal for Chemistry.

Surprisingly, none of Bose's classmates was as close to him as Maniklal. In one of his letters written from Purulia, Satyendranath expresses his adolescent sentiments at being separated from his friend. Written in chaste Bengali, this is how it reads in English translation :[6]

Dear Manik,

Today, on the day of Vijaya, I am so far from you. You too are so far away. Others are scattered too. How I wish we were together. It would have been so nice to have enjoyed a chat with all of you.

My mind goes out to you, hankering for your company from this distance. Do accept my greetings on this sacred day. How I wish I could send my heartfelt wishes to my dear ones through the wind.

Yours
Satyen

Apart from the noble cause of serving the motherland, science had an added attraction, at least for the more ambitious students who flocked to Presidency College. The stars on the faculty were P C Ray and J C Bose. The thrill of watching these stalwarts from close quarters was something no other college could offer. About the methods of these two great men, Satyendranath has written in his autobiographical sketches :[7]

Dr Ray used to teach chemistry right from the first Intermediate classes. We used to assemble on the first floor gallery in the north-east corner of the

> old building. Very often students from other colleges came to attend Dr Ray's lectures. His English was simple with no attempt at pedantry — in fact his style was colloquial and light-hearted so that the basic points of chemistry would register in the minds of the students. . . .
>
> Those who had the opportunity of attending his chemistry classes can never forget his gestures and mannerisms. To give one example, he would embrace Sitaram, the bearer, standing next to him in order to explain molecular attraction. He seemed to enjoy the ripples of laughter which followed such demonstrations. He used to tell us stories about eminent scientists — like Lavoisier, Dalton, Berzelius, Pasteur. He often talked of Nagarjuna and the alchemists of ancient India. It was not unusual for him to recite a couple of Sanskrit *s'lokas* followed by a long quotation from English literature. Altogether the memory of those two years still remains vivid in my mind.
>
> I never had the reputation of being obedient and docile. Talking back to my elders has always been my problem. Something happened in Dr Ray's class one day, the details of which I do not recall. For some reason he had the impression that I was not attentive and causing distraction to others. I was summoned to the teacher's platform to take my seat on the railing, segregated from the others and next to the table with all the apparatus where the great teacher himself used to stand and deliver his lectures every day. My eyesight was poor and I could now watch every aspect of the experiments very clearly. I also came in contact with senior students engaged in important research work in the professor's chamber at the back. After class I used to listen to their discussions and watch them at work.
>
> But this does not mean I had antagonized my teacher — in fact, I used to receive his affectionate pats and punches all the time. . .
>
> It was only in the first two years of college that I had the opportunity of being one of Dr Ray's students. I was never considered one of his favourite boys, though he had a great fund of affection for me. I used to come and go like a comet in his circle. Perhaps that is why he used to joke about me and say that I was a burning example of his pet theory of 'Misuse of the Bengali brain.'

J C Bose was a little aloof, not easily accessible to the students. Nevertheless, he continued to influence them by his remarkable research and his standing up to the Europeans — he was to the young nationalists an example and a model. Satyen Bose has written about the thrill they used to have watching his wireless experiments — what Marconi had done, we could also do! Later in life Satyendranath recalled glimpses of the great man :[8]

> Bengali periodicals often published accounts of Jagadis Bose's discoveries — he had experimentally proved the existence of life in the non-living. This was well publicized not only in our country but also before the elite in other countries where the professor had presented his case himself and was

listened to with respect.

As one enters Presidency College one notices a glass-enclosed room. Acharya Jagadischandra used to do his research here surrounded by mysterious-looking gadgets. To be able to take lessons in science from this world-famous scientist seemed to my adolescent mind the ultimate thing in life.

On the other side of the ground floor sat Acharya Prafullachandra, engrossed in his chemical research. Like me many students flocked to Presidency College with the hope of getting their first science lessons at the feet of these two teachers. I had the good fortune of learning chemistry from Acharya Prafullachandra in the first year. But it was only after two years that I had the opportunity to study under Jagadischandra. Meanwhile like others who had extra-curricular interests I also went to the library from time to time and flipped through the pages of *Response in Living and Non-Living*. It was the era of the magnetic crescograph in Jagadischandra's research career. Delicate instruments were being made in the Presidency College workshops under Jagadischandra's directions. We used to look forward to his lectures. But the professor was so involved with his research that he could not spare much time. Nevertheless I will always remember the few days when I had the opportunity to study under him. Most memorable were the days when he talked about electrical waves using demonstrations with his own instruments.

The research instruments that Hertz had used when he discovered radio waves were not suited for classrooms. Jagadischandra put his mind to it and developed a new kind of Coherer. His instrument was so fine it could produce minute ether waves with wavelengths of one-sixth of an inch, the tiniest electrical waves in ether. Very soon he demonstrated in the classroom that electrical waves had all the properties of ordinary light waves.

The accounts of his gadgets and demonstrations have been recorded. His skill in invention and experimentation has been recognized everywhere. Perhaps the instruments which he used for his demonstrations are still kept in the Bose Institute.

Since I was more inclined towards theoretical science, I joined the applied (mixed) mathematics class after B Sc. Meanwhile Jagadischandra's laboratory had shifted to the building which houses the Baker Laboratory. Our seniors were allowed to carry on their research there.

By the time Satyen Bose had graduated to his senior classes, Calcutta had ceased to be the capital of India. All Central Government offices had moved to Delhi. A bomb attack on the Governor General, Lord Hardinge, sent a shock wave across the country. The reaction of the students can be gauged from an incident that took place in Presidency College. Sailen Ghosh recalls :

> He (Professor E P Harrison, who taught physics) was lecturing one day and had become nervous and irritable as the best of teachers will at times. After failing to get either the attention or the answers he desired, he quite lost his temper. 'Answer properly', he called out. 'Don't chatter like monkeys'. This remark produced a storm of protest. The students felt that his unguarded utterance had manifested his contempt for them and had been an expression of deep-seated racial antipathy. On either count their resentment was aroused.
>
> The class broke up at once, and the angry students poured out into the corridors. My impression is that it was I who offered the suggestion that they should refuse to go back to class until Mr Harrison had publicly apologized. At any rate that decision was made, and in the afternoon Mr Harrison was without a class, while the whole college seethed with fervent discussion. By the end of a couple of hours the student body had become unmanageable and the authorities were very much concerned. Finally Mr Harrison sent for me, taking that I would be willing to help him out of the trouble.
>
> I told him I knew what they were thinking and that there was only one thing to do. 'If you authorize me to tell them you are willing to make a public apology, I believe they will be satisfied.'
>
> 'But if I do that, will they not be justified in taking I did mean to be offensive?'
>
> 'No. I think that if you make the apology, they will accept your explanation.'
>
> 'All right, let me know what to say.'
>
> By this time the novelty of the strike was gone, the proposal was accepted and, when the apology was made, good feeling was shared. Thereupon Mr Harrison arranged a party at his house to which about a dozen were invited, and the incident closed pleasantly enough.[9]

Satyendranath also took a leading role in organizing the students' strike. Some of his biographers refer to student meetings in the college ground where Satyendranath was vocal about the issue.

But the political atmosphere was quite volatile and most of the students were directly or indirectly involved :

> As time went on, the colleges were drawn within the area of espionage. The state of affairs was particularly distasteful to the Principal of Presidency College, who held stoutly to the idea that no good can come of treating the students as a hostile body. He was humoured to some extent, at least to the point that Indian inspectors of police were not allowed to enter the college until he had been notified. Thus after the first of these alarms, there was a delay of four or five minutes, during which various secret hiding places were filled with the articles the police would have been glad to find on the

> persons of the students. As it was, they found nothing. Somehow it was learned that a certain student was responsible for this raid, either in the quality of spy or through some carelessness of speech. He was found dead in bed not long after and was believed to have committed suicide by taking poison.
>
> The Principal thereupon issued orders that there was to be no visitation of the student rooms until he had been notified and could be present in person. The police came one day, waited for him, inspected every room in his presence and found nothing. At five minutes past three one afternoon a prominent Hindu official was shot in front of the Medical College, near Presidency College. We did not know of the occurrence until school was over for the day and we went out into the still excited crowd. Next the police came to the physics class to arrest a classmate of mine. The professor was indignant. He produced the record to show that the boy was present between three and five. The Principal was in a rage when he heard of the action of the authorities and he told them what he thought of their clumsiness. Some time later I heard that the police were back after the same boy, and I went quickly into the Principal's room and told him. He came out and put a stop to the actions of the police.[10]

When Satyendranath was in Presidency College, the Principal was Henry Rosher James. H R James was a liberal Principal and used to listen to class representatives on general matters. Moreover, his sympathy for the cause of the students made him very popular with them. Unfortunately, he was forced to leave because of his uncompromising stand on the Oaten issue.

In January 1916, a year after Bose and his batch had passed out, there was a student strike in protest against Professor E F Oaten. One day there was loud talk in the corridor when Professor Oaten was taking his class in an adjoining room. Oaten was angry, he came out of the classroom and 'violently pushed back' the crowd. The students insisted that the Professor should apologize, but apparently Oaten was made of sterner stuff than Harrison and much ill feeling was generated. After two days of a students' strike the matter was settled but the strikers had to pay a fine. This caused resentment, and about a month later it flared up again. This time again it was Oaten who was involved — he had manhandled a first year student. This time the student representatives, instead of going to the Principal, decided to take law into their own hands and gave the Professor a good thrashing — he was 'beaten black and blue.' Subhaschandra Bose was among them, and along with other 'troublesome boys' he was expelled. This was a turning point in Subhaschandra's career. In his own words : 'Little did I then realize the inner significance of the tragic events of 1916. My Principal had expelled me, but he had made my future career. I had established a precedent for myself from which I could not easily depart in future.'[11]

Things did not turn out to be good for the Principal either. The government of Bengal had issued a communique ordering closure of the college. The order was issued without the knowledge of the Principal. Infuriated, he met the Member-in-charge of Education,

hot words were exchanged and the Principal was suspended for 'gross personal insult' to the Member. Subsequently the Principal was reinstated but he retired shortly after. Interestingly, Professor Oaten did not nurse any bad feelings. He wrote in his reminiscences, 'Of course we had our troubles, strikes and student rows. What college of young men is for ever peaceful?'

A very remarkable feature of the 1909 batch was the close ties formed between some of them — like Meghnad Saha, Jnan Ghosh and Satyendranath in particular. It persisted long after they had left college. Jnan Ghosh and Satyendranath were colleagues until 1939. When Ghosh left Dhaka for Bangalore to take up the Directorship of the Indian Institute of Science, S N Bose gave a moving farewell speech. Those days he usually avoided giving public speeches and was hardly ever seen attending student functions. One of his students, A K Dasgupta, has given an account of that farewell meeting :

> I still remember what he said in the farewell for Ghosh. As was his habit, he spoke softly. He said : 'My mind goes back to the Presidency College days when Meghnad, myself, Jnan and a few others were in the same class. We used to dream of doing great things for the country. I am happy that Jnan in his new position will be able to achieve what we used to dream.'[12]

Dasgupta has also given his impression of the close bond between Saha and Bose :

> Saha used to come down to Dhaka occasionally, as external examiner or to give talks to the students. On one such occasion he began his speech in Curzon Hall with these words : 'Myself, Satyen and Jnan — we are like brothers. We must meet at least once a year.'[13]

In the I Sc examination, Satyendranath topped the list with his close friend, Maniklal De, coming second. Meghnad Saha, appearing from Dacca College, got the third position. The fourth and fifth positions were taken by Jnanchandra Ghosh and Prankrishna Parija respectively.

Satyendranath, Nikhilranjan and Meghnad then opted for Mixed Mathematics, Jnan Ghosh, Jnan Mukherjee and Maniklal De chose Chemistry. Sailen Ghosh, Amaresh Chakrabarty and Snehamay Dutt studied Physics. Once again Satyendranath maintained his position in the B Sc examination, Saha came second and Nikhilranjan third. This was repeated in 1915 : Satyendranath stood first in M Sc, Saha came second.

Satyen was married at the age of 20, while he was still a student of the M Sc class, to Ushabati, the daughter of a renowned doctor, Jogindranath Ghosh. On her mother's side Ushabati was related to the Dutt family of Rambagan. One branch of this family had embraced Christianity and produced two young talents, Toru and Oru, both of whom wrote poems in French. Satyen obviously had no say in the choice of the bride. He was not too keen to get married so early but he could not go against the wishes of his mother. But if his mother had known that this early marriage would prove to be a stumbling block in the son's career, she would probably have thought twice. But in 1914 the Palit endowment with its strange conditions still lay in the future.

3

Early Career

1916-1921

Equipped with a first class M Sc degree Satyendranath now stepped into the wide world. It was in one sense the best of times and in another sense the worst. The Swadeshi mood was everywhere. For an inspired young generation the sky was the limit — there was nothing they could not do; they could even lay down their lives for the cause of the motherland, if need be. Satyendranath, just out of the university, must have felt the same way. But unfortunately, this elation was a state of mind. In everyday terms, what could he really do — how and where could he use his fine intelligence and superior knowledge? For about a year this brilliant product of the university groped in the dark, not knowing what to do. His friend and classmate Saha was going through the same trauma. There were just no jobs for them.

Then the mist cleared. It seemed the stage was set for them. It was as if the entire effort of building the University College of Science, the magnificent donations of Palit and Ghose, the split in the nationalist camp of education, the final taking over by Sir Asutosh, were all geared to one purpose — of providing a foothold for M N Saha and S N Bose so that, almost literally, they could reach for the stars. Subsequent events were swift and dramatic, but first let us take a look at the background.

One manifestation of the Swadeshi spirit was the Indianization of education. There was a feeling that along with the boycott of British goods the students should turn to their own culture and tradition. English education only resulted in alienation from their roots. The National Education movement gathered momentum, but very soon differences came up. One group wanted to discard everything foreign and have a complete structure of education which was hundred per cent Indian. The other group

believed that extreme nationalism could not take them very far. Led by people like Taraknath Palit (1831-1914), eminent barrister, and Nilratan Sircar (1861-1943), eminent physician and entrepreneur, they laid emphasis on science and technology, and formed the Bengal Technical Institute. But somehow this could not attract enough students. Disappointed, Sir Taraknath Palit handed over all his assets to Sir Asutosh Mukherjee who believed that changes could be made within the framework of the existing universities.

Princely donations from Palit and Rashbehari Ghose (1845-1921), eminent jurist, enabled Asutosh to create two professorships in Physics and Chemistry and one each in Botany and Applied Mathematics. Thus the University College of Science could come into being. The new centre for teaching and research acquired a character of its own right at the beginning from a stipulation laid down for these chairs — all the posts were only for Indians. So, starting off as an ally of the British, Asutosh was able to achieve the objectives of the national school.

While all these developments were going on, Satyendranath and his contemporaries were preparing themselves for the task which Asutosh was going to vest on them but about which they were still in the dark.

The time was 1915, the First World War was raging in Europe and its consequences were being felt in India. But war or no war, the job situation for educated Indians was indeed very bleak. The bright ones aspired for the Imperial Education Service, the IES. In his address as the General President of the Indian Science Congress P C Ray summed up the situation in the following table he had drawn up : [1]

TABLE SHOWING THE COMPOSITION OF THE EXISTING SCIENTIFIC SERVICES

NAME OF THE SERVICE	OFFICES (IMPERIAL GRADE)		AVERAGE PAY OF	
	EUROPEANS	INDIANS	EUROPEANS	INDIANS
Botanical Survey	2	0	1000	0
Geological Survey	16	0	1010	0
Zoological Survey	3	1	970	700
Agricultural Service	38	5	1000	460
Forest Service	9	1	1040	660
Medical and Bacteriological Service on Civil employment	24	5	1220	520
Indian Munitions Board	11	1	780	300
Meteorological Department	10	2	970	770
Veterinary Department (Civil)	2	6	1100	
Educational Service	34	3		
Indian Trigonometrical Service	46	0		

Only three Indians qualified for the IES. J C Bose was one, but he got it after three years of long struggle. Even P C Ray with all his acumen was in the provincial service and was appointed as an acting IES at the fag end of his career. That also was probably due to the service rendered by the Bengal Chemical during the war.

However those who passed their M Sc in Chemistry like Jnan Ghosh, Jnan Mukherjee, Maniklal De and others were slightly better placed. P C Ray who had already accepted the Palit Chair had an established laboratory and had plans for his students.

Satyen Bose and Meghnad Saha had no such patron. Saha had to maintain himself and his brother who was studying in a college, and had to send money home. He gave a number of tuitions covering the length and breadth of the city on a bicycle. He wanted to sit for the Finance examination but was denied permission because of his contact with the revolutionaries. During his Dhaka days he was in touch with the Anushilan Samity activist, Pulin Das. In Calcutta the more famous Bagha Jatin used to come to the rented house at 110 College Street which was shared by Saha, J C Ghosh, N R Dhar, J R Dhar and J N Mukherjee. It was from this house that Bagha Jatin went to his last mission and died fighting with the police at Balasore.[2] Satyen Bose had the advantage of staying with his parents. But he was married — his first child was born in 1916. His father wanted him to try for the railway service but the idea did not appeal to him. For one year he privately coached the prince of the Gouripur State, Pramathesh Barua (who would later became famous as a film actor and director), for the princely sum of two hundred rupees, much more than the stipend Sir Asutosh was going to offer him later.

Chemistry offered better job opportunities. What could a person, even one as bright as Satyen Bose who had specialized in mathematics, do? He tried his hand at research. He went to Dr Ganesh Prasad who had just joined the University. Dr Prasad (1876-1936), a D Sc from Allahabad University, used to teach at Queen's College in Benaras before coming to Calcutta. He later became the Hardinge Professor of Mathematics (1923-36). The students were awed by his scholarship and the difficult question papers he used to set. But again luck was not in Bose's favour, and he failed to establish a rapport with Dr Prasad.

> The students flocked to him for training in research. They were the best science students of Calcutta though several of them had not secured high marks in Ganesh's paper in their M Sc. But the fault lay with the teachers at Presidency College — at least that is what Ganesh Prasad thought. The young students had to stomach adverse comments about their former teachers, too scared to answer back. After my M Sc I too presented myself before Ganesh Prasad who was also my examiner though I had not fared as badly as the others. Dr Prasad was kind to me at first but I was notorious for plain speaking. I found it difficult to bear his tirade against my teachers. I had dared to counter his adverse criticisms. This infuriated him. He said — you may have done well in the examination but that does not mean you are cut out for research. Disappointed I came away. I decided to work on my own.[3]

Around this time he came across an advertisement — it was a teaching job offered by the Bihar Government. With letters of recommendation from Principal James and Dr Mullick, he sent in his application only to be told later that they were on the look-out for someone less qualified — he was too good for their requirement. He had also applied for a job at the Alipore Meteorological Office but was turned down for the same reason.

One wonders why Bose did not approach J C Bose who was about to set up his own research institute next door to the Science College. The obvious answer seems to be that J C Bose had already drifted towards plant physiology. Satyendranath with his background in mathematics could not have been of any use to him. This, of course, is an assumption. From his autobiographical writings it does not appear that S N Bose ever approached the great Sir Jagadis.

When all avenues seemed to reach a dead end, there was a glimmer of light from a very unexpected quarter. Sailen Ghosh who had topped the Physics list that year had a bright idea. Apart from being a brilliant student and a very resourceful person, Sailen Ghosh had the added distinction of being known to Sir Asutosh. While he was still doing his M Sc under Professor Harrison, Ghosh's paper was published in the *Indian Journal of Astrophysics,* and that attracted the favourable notice of Sir Asutosh.

Meanwhile Asutosh had an ambitious plan to start postgraduate teaching and research in the Science College. C V Raman and D M Bose had already been given appointments though they were held up for the time being, D M Bose in Germany and C V Raman in the Government Accounts Department.

A short biographical note on D M Bose will not be out of place here because his orbit crossed that of S N Bose more than once. Debendramohan Bose was born in Calcutta on 26 November 1885. His father was a physician and probably one of the first Indians to have visited the United States. His uncle, Anandamohan Bose, was however more famous as the first Indian Wrangler in the Mathematical Tripos at Cambridge and as President of the Indian National Congress in 1898.

Debendramohan lost his father early but received the care and patronage of his famous uncle. After doing his M A in Physics in 1906 he worked as a research scholar under his uncle. He went to England in 1907 and worked in the Cavendish Laboratory under J J Thomson and then moved to the Royal College of Science in London from where he did his B Sc in Physics with a first class.

Back in Calcutta he was appointed the Ghose Professor after a short spell of teaching at City College. Again he sailed to Europe, this time to Berlin on being awarded the Ghose Travelling Fellowship for two years' advanced study in physics. But his stay turned out to be much longer — the First World War broke out and he could not leave Germany. Neither was he permitted to submit his thesis. But his forced sojourn was a blessing in disguise because he was there when new developments in the quantum theory were taking place. He could attend meetings addressed by stalwarts like Planck, Einstein, Rubens, Warburg, Franck, Hertz, Born and others. At last he got his Ph D from Berlin University and returned to India in July 1919.

When he came back, two enthusiastic lecturers in the department wanted to hear

from him all about the latest developments in physics. In fact, he not only helped Meghnad and Satyendranath with books and information but also helped the latter to improve his German. With his aptitude for languages Satyendranath picked up German quite fast.

C V Raman's (1888-1970) coming to Calcutta was accidental. He had graduated from Presidency College, Madras. His first research paper was published in the *Philosophical Magazine* even before he got his M A. But with all his brilliance in physics he had to enter government service under pressure from the family. Fortunately, after his posting in Calcutta he had the opportunity of doing his research in his spare time in the oldest research institution of the country— the Indian Association for the Cultivation of Science. Even before he left the Finance Department to become a full time professor, he was a star attraction to students who used to flock to the IACS to listen to him.

The postgraduate Physics department was yet to start functioning. Sir Asutosh was in the meantime scouting for equipment. Sailen Ghosh, an able assistant in this venture, recalled :

> Sir Asutosh Mukherjee, Vice-Chancellor of the University, sent me on a tour of the colleges in the neighbourhood of Calcutta to see what instruments could be begged or borrowed with which to set up a department of research in Physics in the newly created College of Science. I reported to him one day that I had found some good material at a college maintained by the Maharaja of Cossimbazar. 'That is good,' he commented. 'I am going up there next week to unveil a portrait. You had better come with me.'
>
> So I accompanied him, and we were the Maharaja's guests for the day. I have a very distinct recollection of the magnificence of the service. A many course dinner was served, and I was presented with an immense plate, made of pure silver, on which were one hundred and eight small silver cups, each containing a separate article of food. Good manners required that the guest should partake of part of all of the contents of every one of these cups. We did not use forks or spoons.
>
> Meantime, on the Vice-Chancellor's instructions, I got myself shown about while he was delivering his address. Later, when the Maharaja was personally conducting him through the building and they came to where I was, he said to me, 'Well, have you found anything here that you must have?' I told him there were many things, some of which were not being used.
>
> By this time the Maharaja was interested, and the Vice-Chancellor told him of our plans. 'Just let me know what you want,' was his comment, 'and if we can spare it you may consider it yours.'[4]

Ghosh's family used to value the relation he had with Asutosh. Mariam, Sailen Ghosh's daughter, has written in her memoir :

> While at Calcutta Dad often visited Calcutta University; he was a student there years ago (1913-1915). At that time he had helped establish the

Physics Laboratory and was a friend of Sir Asutosh Mukherjee.[5]

This clearly shows that Ghosh's family was aware of his early associations with Sir Asutosh. Obviously Ghosh's relations with the great man was so cordial that the children drew the obvious conclusion that he was a friend.

Since Sailen Ghosh had easy access to the great man, it was possible for him to go up to Sir Asutosh with the bold suggestion to start teaching subjects in the University that were not taught at Presidency College. The idea appealed to Asutosh. But the problem was the dearth of good teachers. The rest of the story has been recorded by Satyendranath himself :

> One day we were called up by Sir Asutosh. Meghnad, Sailen and I went up the steep stairs to the library, to the special chamber where Sir Asutosh sat. We were naturally meek and submissive and overawed by his august presence. He had heard that the younger generation wanted more modern subjects to be introduced in the university curriculum. He asked, 'What subjects are you competent to teach, boys?' 'We'll try our best to teach whatever you want us to, sir.'
>
> He smiled. We had only heard of the many new discoveries in physics, most of them made in Germany — new improvements and new discoveries. Planck, Einstein, Bohr — these were just names to us Bengalis. To know more about them one had to read German books or research journals in other languages. During the war most of these journals did not come to India.

Thus a deal was struck. A special allowance of Rs 125 per month was agreed upon. The young hopefuls had promised Asutosh that they would equip themselves with all the latest developments in modern physics. But it was easier said than done. Modern physics was already a novel concept in the West. Here in the colonial backwaters the surging waves of new ideas hardly made a ripple. Books and journals were the only sources of information. But from where would they get those latest publications?

There was another lucky break for them when they got access to the excellent library of the versatile teacher, P J Brühl of Sibpur Engineering College. Brühl was German with a doctorate in Botany. His ill-health had made him change over to physics — an indoor subject. He came to India for its warm climate. Brühl's library had Planck, Boltzmann, Wien — all the masters of modern physics. Meghnad Saha already knew German; Satyen Bose too was taking German lessons. Meanwhile they also had free access to Sir Asutosh's huge library. Bose had the added advantage of knowing French. They also procured a book on Relativity. Thus a partnership started between Saha and Bose which was to result in some joint papers and their combined effort to translate Einstein's relativity papers into English.

In his long article on Education in Bengal and Sir Asutosh, Satyen Bose has given a detailed account of the cross-currents in the education scene those days. That article is more interesting because of Bose's grudging tribute to the great architect of education. Bose had always been a sympathizer of the national school with their stress on

the vernacular. In his later years it became almost a mission with him. But in 1916, Bose had every reason to be grateful to Sir Asutosh because he was unknowingly acting as a catalyst to a major turn in Bose's career.

The young recruits had given an undertaking to Sir Asutosh and they were determined to keep it. The two friends, Saha and Bose, distributed the subjects among themselves for convenience. Saha chose quantum theory, thermodynamics and statistical mechanics while Bose studied the theories of electromagnetism and relativity. Within a few years both were going to make fundamental contributions to the subjects of their choice.

But there was a setback. Sailen Ghosh had already been appointed a lecturer in Physics (after all Sailen Ghosh had stood first in Physics, Saha and Bose were students of Mixed Mathematics) and Asutosh had arranged the Palit Travelling Fellowship for him to go to Harvard. It is not known if Sir Asutosh was aware of his political contacts. Two days before he was due to sail, his passport was impounded, his room ransacked and summons came from the Commissioner of Police. Sailen Ghosh had no other way but to leave the country in a hurry. Later he surfaced in the USA and spent the next twenty years of his life organizing revolutionary activities and mobilizing public opinion against British rule in India.

As far as the two young teachers, Saha and Bose, were concerned, they were one less on the faculty of their newly formed department. From Bose's memoirs we come across another name — that of Jatin Seth who too was interned by the police about the same time when he was going to join the department. Seth was a National University graduate who was sent to America for higher studies. After his return he was also asked by Sir Asutosh to join but he was prevented from doing that.

So they were two short. Apart from Saha and Bose, Asutosh had also collected other bright products of the University — Sushil Acharya, Sisir Mitra, Jogesh Mukherjee and Phanindra Ghosh. In the face of stiff opposition Asutosh gave the green signal to these young men. In the words of Satyendranath :

> But the chemists did not approve of these schemes hatched by the younger generation. They apprehended that the three-storeyed building would soon come to accommodate other sciences too. In their opinion Asutosh was being too hasty. Prompted by some immature youngsters he was going in before the right time. He should wait for Deben Bose and Raman. These youngsters were hardly capable of carrying the heavy burden.[6]

But the youngsters proved that such fears were ill founded. They devised a curriculum, classes were started and soon they were publishing original research papers.

Bose's first paper, written jointly with Saha, came out in 1918 in the *Philosophical Magazine*. His next two papers were based on purely mathematical problems. They were published in the Bulletin of the Calcutta Mathematical Society. His 1920 paper on the 'Deduction of Rydberg's Law from the Quantum Theory of Spectral Emission' in *Phil Mag* attracted the attention of appreciative physicists — one of whom was W A Jenkins. His collaborative work with Saha to translate Einstein for the benefit of students who did not know German had a foreword by P C Mahalanobis — thus

bringing three stalwarts of Indian science together in a single volume. Since the book is no longer available, we give below the contents and excerpts from the introductions:[7]

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7.	*Notes*	*165-185*

The following is an excerpt from the biographical note by Saha :

> Einstein's studies on Relativity were commenced in the year 1905, and has been continued up to the present time. The first paper in the present collection forms Einstein's first great contribution to the Principle of Special Relativity. We have recounted in the introduction how out of the chaos and disorder into which the electrodynamics and optics of moving bodies had fallen previous to 1895, Lorentz, Einstein and Minkowski have succeeded in building up a consistent, and fruitful new theory of time and space.
>
> But Einstein was not satisfied with the study of the special problem of Relativity for uniform motion, but tried, in a series of papers beginning from 1911, to extend it to the case of non-uniform motion. The last paper in the present collection is a translation of a comprehensive article which he contributed to the Annalen der Physik in 1916 on this subject, and gives, in his own words, the principles of Generalized Relativity. The triumphs of this theory are now matters of public knowledge.
>
> Einstein is now only 45, and it is to be hoped that science will continue to be enriched, for a long time to come, with further achievement of his genius.

The following is an excerpt from the introduction by Mahalanobis :

> Einstein's theory connects up the law of gravitation with the laws of motion, and serves to establish a very intimate relationship between matter and physical space-time. Space, time and matter (or energy) were considered to be the three ultimate elements in Physics. The restricted theory fused space-time into one indissoluble whole. The generalized theory has further

> synthesized space-time and matter into one fundamental physical reality. Space, time and matter taken separately are mere abstractions. Physical reality consists of a synthesis of all three.

It is gratifying that the first English translations of these famous papers came from two young men, entirely self taught in physics. The English reviewers were not kind though :

> The translation cannot be called a good one. In a work of this kind we expect a fairly literal translation, but in the present book there are numerous errors in translation, and the choice of English equivalents for German words is frequently unfortunate. In many instances the mathematics is faultily reproduced. The numbering of the pages is not continuous, but recommences at the beginning of section 4, and the omission of the footnotes from the originals is regrettable. Provided it is studied with care, the translation will nevertheless be of service to those who are unfamiliar with German, and wish to grapple with the pioneer works on this subject, some of which are rather inaccessible.[8]

This volume, published by the Calcutta University in 1920, for all its amateur production values (there are printing mistakes and slipshod pagination as pointed out by the reviewer) is of more than historic value now in the light of the recent communication from Professor Max Jammer of Bar-Ilan University, Israel, reproduced below :

> The M N Saha and S N Bose translation of Einstein's papers on relativity, published 1920 in Calcutta, does not contain the error which C B Jeffery and W Perrett committed in their well-known 1922 translation. It concerns Einstein's definition of simultaneity at the very beginning of the kinematical part of Einstein's famous 1905 paper 'On the electrodynamics of moving bodies' (Annalen der Physik, vol.17). The British translators misread Einstein's 'nun' as 'nur' and rendered thereby erroneously a sufficient condition as a necessary condition, thus excluding the possibility of alternative definitions of simultaneity, a possibility contended by the proponents of the so-called conventionality thesis of distant simultaneity (Reichenbach, Grünbaum and others).[9]

It is a pity that it has not been reprinted or revised after the first edition was exhausted.

Any aspiring young scientist at this stage of his career would look for opportunities to go abroad, to get acquainted with fresh ideas at their source. Needless to say, Satyendranath too felt the same way. As he himself has written — it was his dream to set foot on intellectually stimulating Europe, more so in those years 'which shook physics'. Was it not possible for him to get a scholarship or a travel grant? Let us take a brief overview of the scene.

A few Government of India scholarships tenable in the UK were available and these were later transferred to the provincial Government. One of these was awarded to Nilratan Dhar in 1915 and another to Amaresh Chakrabarti in 1918. The Taraknath Palit Scholarships for studies abroad had a peculiar condition attached to them — the

student had to be a bachelor and continue to be so during the tenure of the scholarship. Satyendranath did not know about this stipulation, but when he appealed to Sir Asutosh, he was 'lovingly' told of this limitation. There was the Guruprasanna Ghosh Scholarship, but the amount was just about enough for a stay abroad for three or four months. Meghnad Saha was awarded this scholarship. He borrowed money, added to it his PRS grant and made it to Europe in 1920. Sisir Mitra followed later. The Palit recipients were Snehamay Dutt, Jnan Ghosh and Jnan Mukherjee, all in 1919, and Bidhubhusan Ray in 1922. Only Satyendranath was left out — not a very happy situation for him.

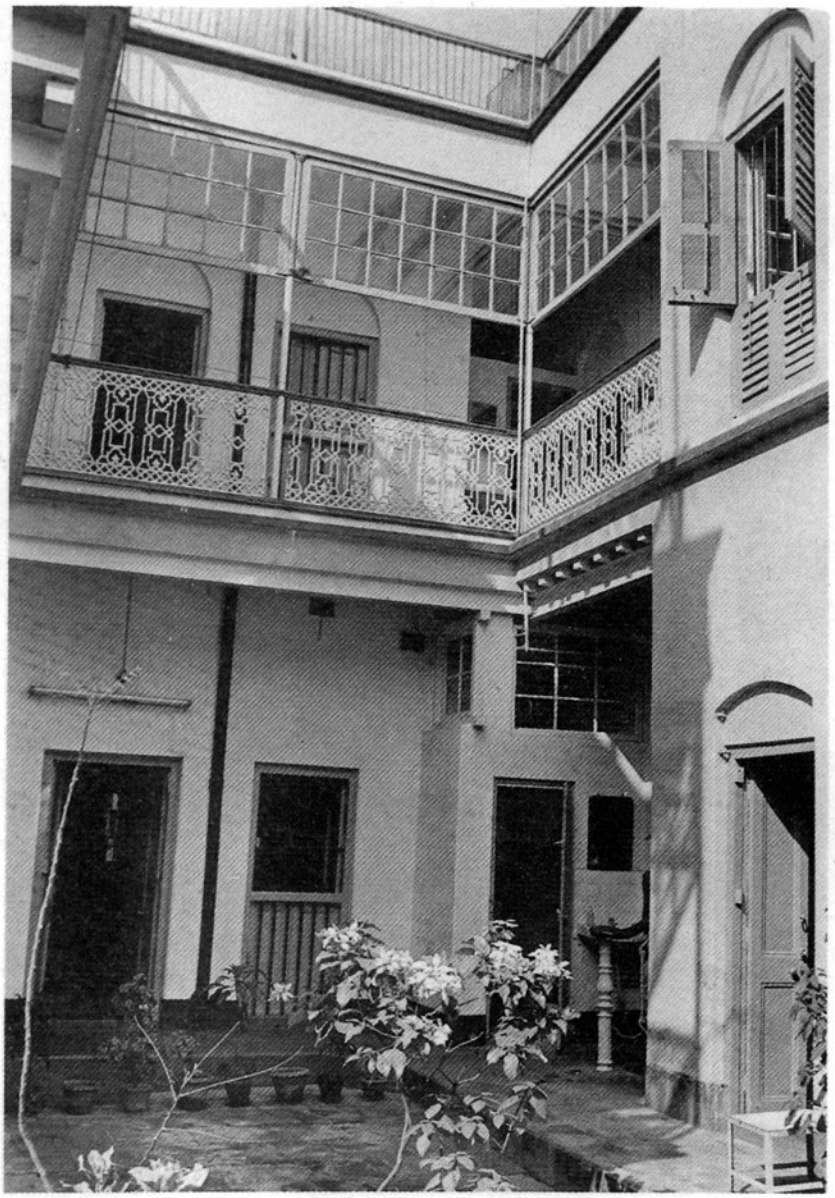
Temperamentally there had always been a streak of the rebel in him as a result of which he could not become one of P C Ray's favourites. This was repeated with Ganesh Prasad. Bose could not get along with him. His outspokenness once again made him fall out with Sir Asutosh. On several occasions he differed with him. In fact, he did what nobody else would have dared to do — point out a mistake in a Mathematics question paper set by Sir Asutosh himself. But this was a minor irritant. The problem of personality clashes with other towering figures was also inevitable to some extent. The situation is best described by Dr D M Bose :

> During the 1920s the situation in the Physics department of the Science College was becoming rather uncomfortable; there were too many able scientists crowding together who were provided with indequate laboratory accommodation, technical resources and apparatus; consequently a certain amount of heat was generated. The situation became somewhat eased by the migration of some of the physicists to other universities, by the gradual expansion of accommodation and resources. The first physicist to migrate was Satyendranath Bose.[10]

Many of Bose's friends like Niren Ray would later wonder why he chose to move to Dhaka :

> A hundred per cent Calcuttan by birth and residence, he had hardly any tangible reason to leave Calcutta for Dhaka. Increased salary and a lift in status? Throughout his life these considerations have never influenced his course of action.[11]

But the reason was apparent. The overriding consideration was the opportunity to visit other centres of learning in the world, an opportunity he was not likely to get in Calcutta. It was not so much overcrowding in the department or his not having the best of relations with Sir Asutosh. Again, it would have been against his nature to take the initiative. The call came from Hartog, the enterprising Vice-Chancellor of the newly founded University at Dhaka — an alien land to a true Calcuttan.

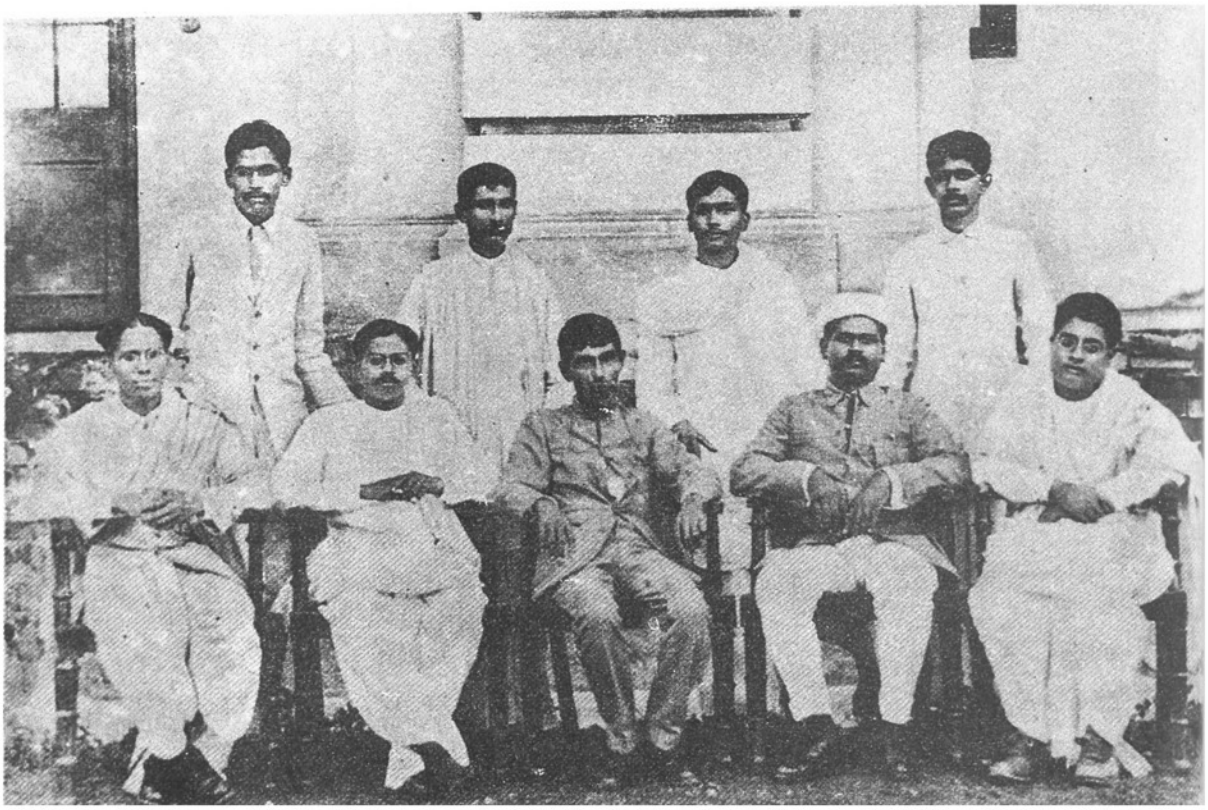


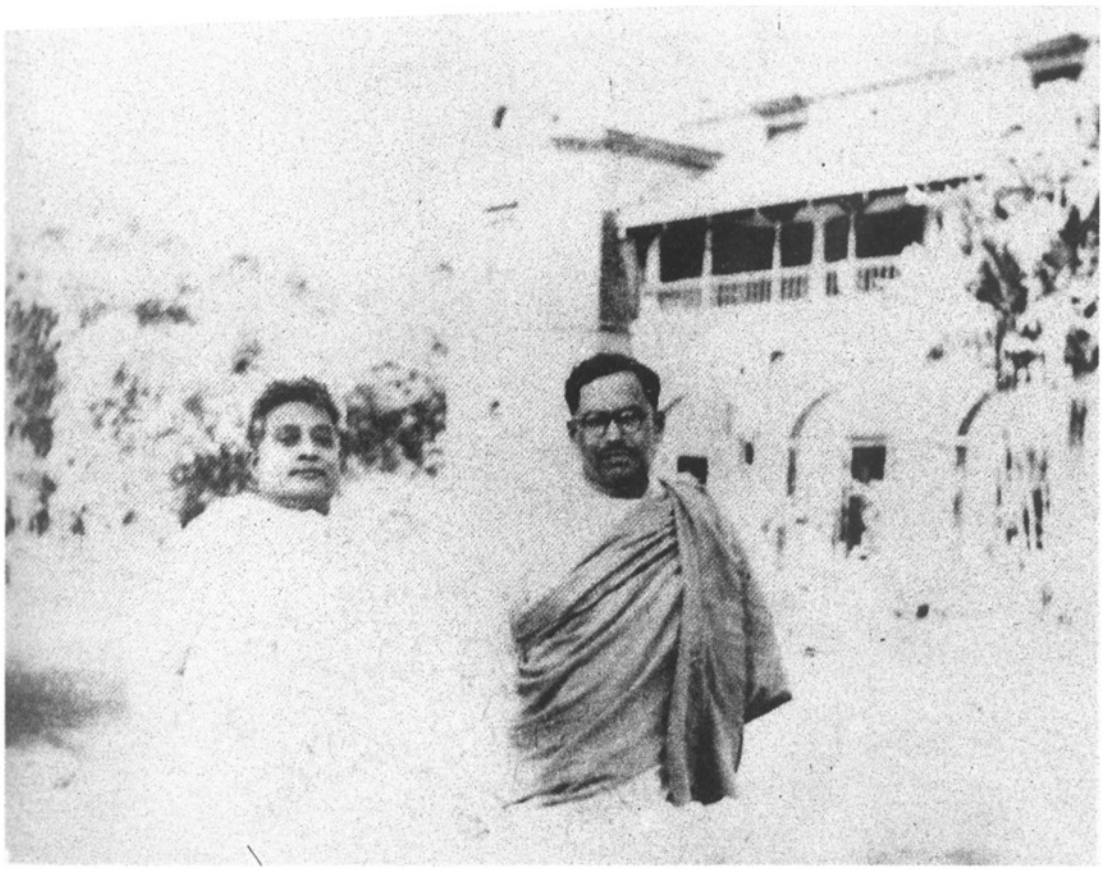




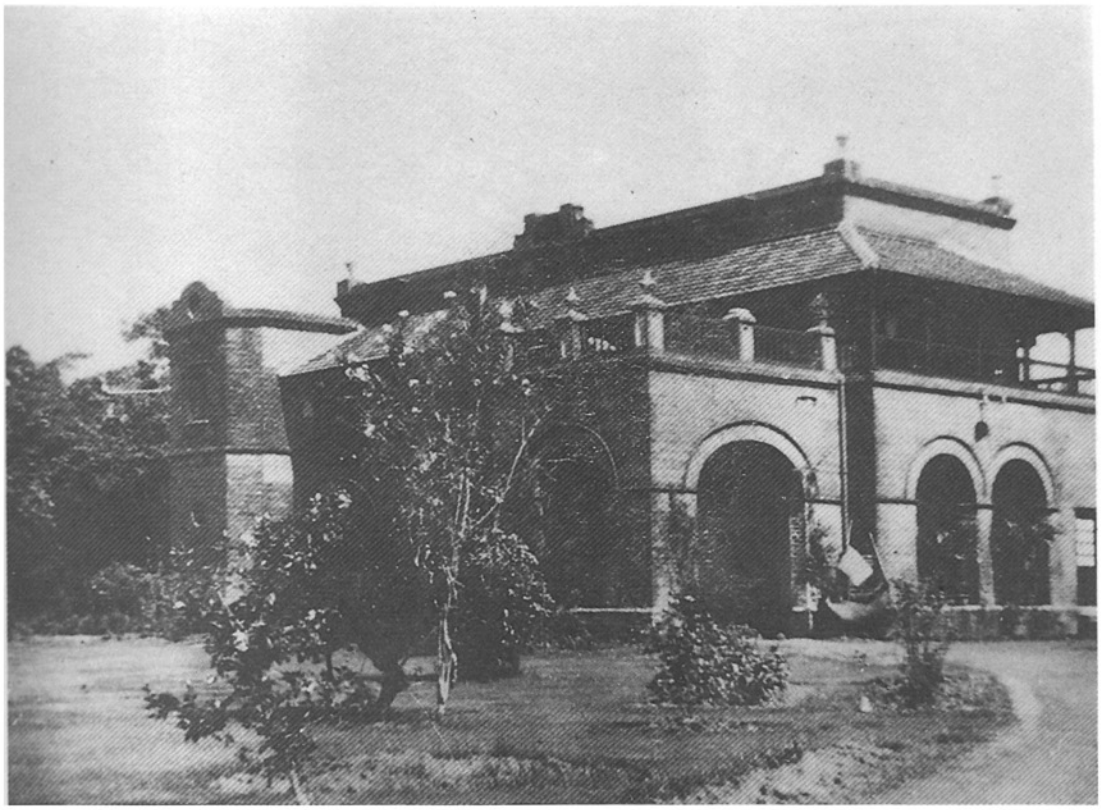


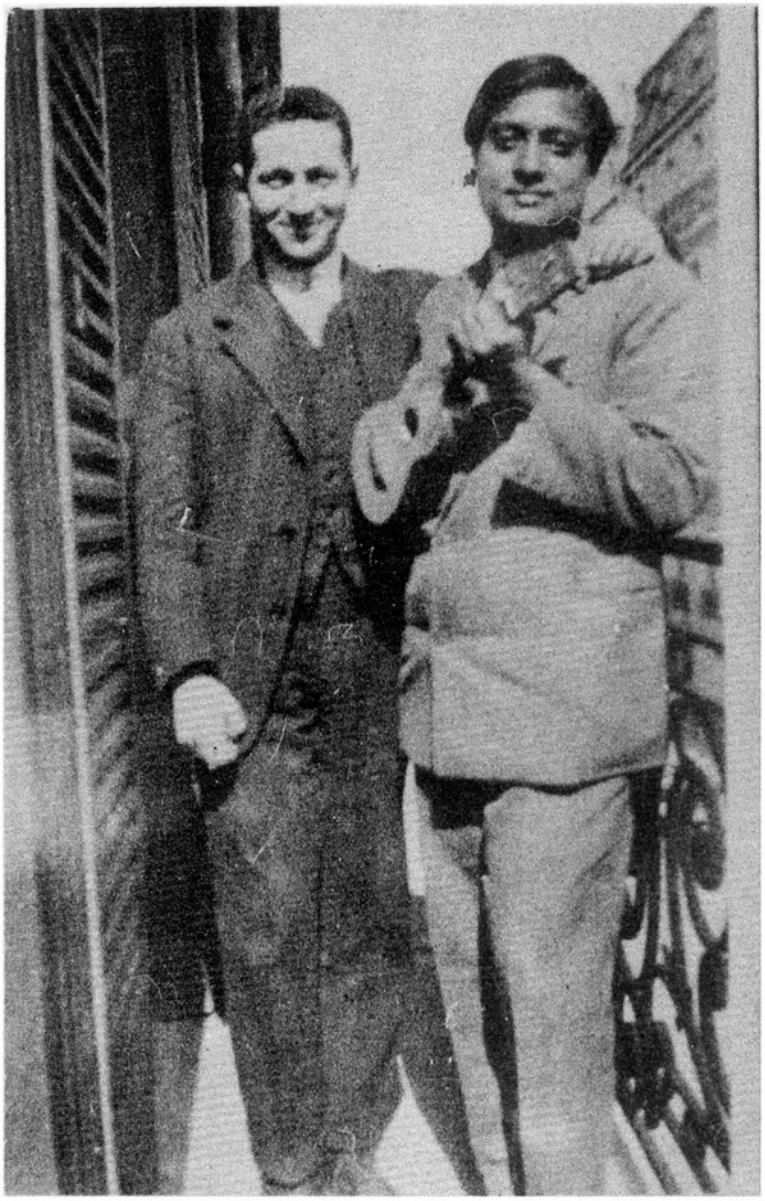






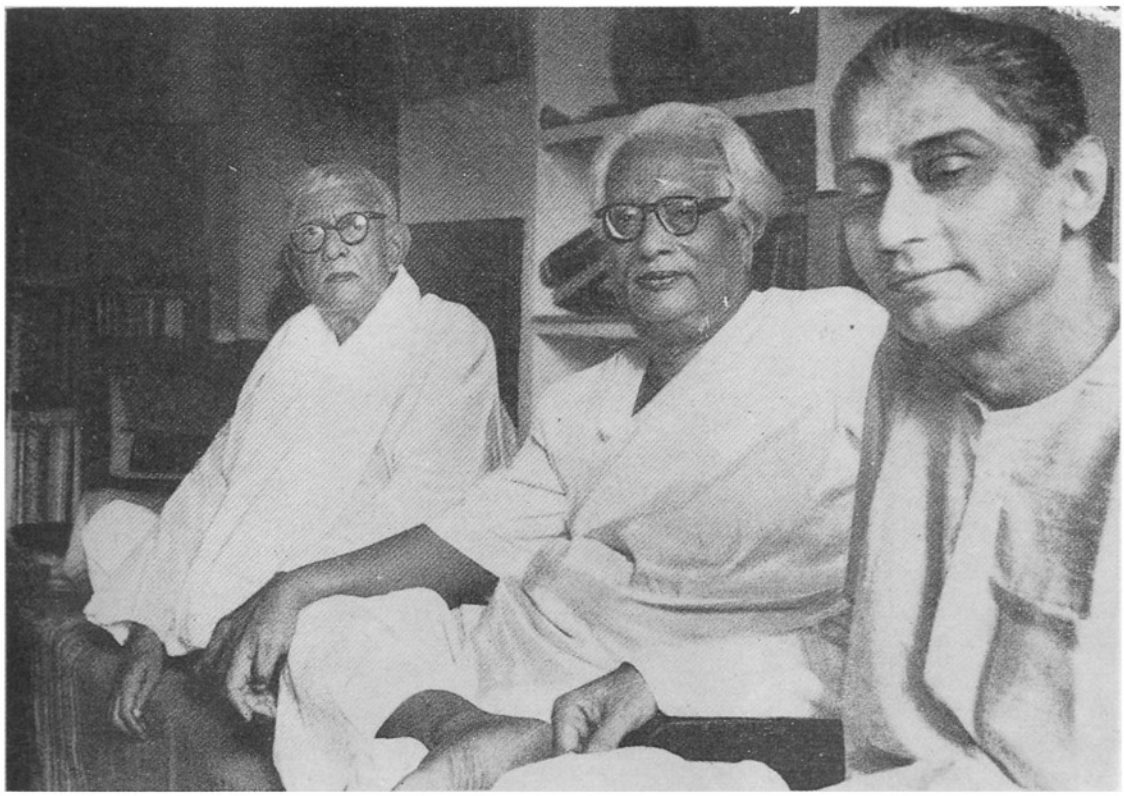




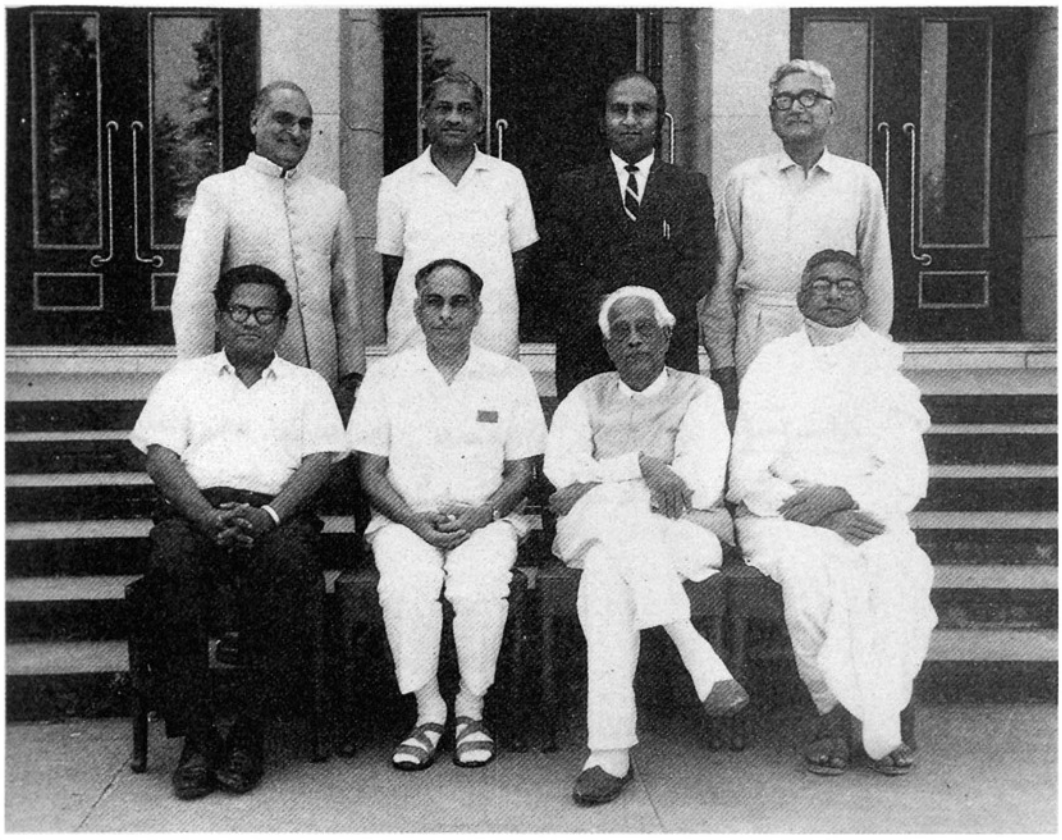














4

Dhaka University

1921-1924

The University of Dhaka owes its origin to several factors, e.g. the desire of the Muslim community of East Bengal to ensure the educational progress of the community; the desire of the Government of India to create a new type of residential and teaching university and relieve the congestion at the University of Calcutta. But the most determining factor was political.

P J Hartog, the first Vice-Chancellor of the University of Dacca, referred to its political origin in the first meeting of its Court on 17 August 1921. The University was said to be a compensation to the Muslims for the annulment of the partition of Bengal. The Muslims of East Bengal had welcomed the partition of Bengal in 1905. They had hoped that the creation of a new province where the Muslims formed the majority would facilitate their progress. In pre-Partition days most of the colleges were located in or near Calcutta. Out of forty-five colleges only fifteen were in East Bengal and Assam.

The formation of the new Province of East Bengal and Assam was in many ways a great boon to its people. In the short span of its existence this new province showed remarkable progress in various fields. The Calcutta University Commission noted that during the period 1905 to 1911 significant educational progress was achieved in the new province. The declaration of the annulment of partition on 12 December 1911 came as a rude shock to the Muslims of East Bengal and Assam. A Muslim deputation consisting of Sir Nawab Salimullah, Nawab Syed, Nawab Ali Choudhury, A K Fazlul Huq and other leaders met the Viceroy on 31 January 1912 and expressed their fear that the modification of the partition might retard the educational progress of their

community. As compensation for the annulment as well as a protest against the general antipathy of the Calcutta University towards the Muslims, the deputation pressed for a University at Dhaka (still, of course, spelt in the colonial manner as Dacca). Consequently, the Government of India recommended the constitution of a University at Dhaka and a communique was published to this effect on 2 February 1912.[1]

Prominent leaders of Calcutta vehemently opposed this plan. On 16 February 1912 a delegation headed by Sir Rashbehari Ghose met the Viceroy and expressed apprehension that the creation of a separate University at Dhaka would be in the nature of 'an internal partition of Bengal'. They also contended that the Muslims of East Bengal were mostly cultivators and were not likely to benefit in any way by the formation of a University. In his reply Lord Hardinge referred to the progress made by the people of East Bengal and Assam during the few years of the partition and said :

> When I visited Dacca I found a widespread apprehension particularly among Muhammadans who formed the majority of the population, lest the attention which the partition of Bengal secured for the Eastern province should be relaxed and that there might be a setback in educational progress. It was to allay this not very unreasonable apprehension that I stated to a deputation of Muhammadan gentlemen that the Government of India were so much impressed with the necessity of promoting education in a province which had made such good progress during the past few years that we have decided to recommend to the Secretary of State the constitution of a University at Dacca and the appointment of a special officer for education in East Bengal.

The Viceroy assured the delegation that no proposals which could lead to the internal partition or division of Bengal would meet the support of the Government of India; and he added that from the fact that he announced the intention of the Government in regard to Dacca to a deputation of Muhammadans it did not follow in any way that the new University would be a Muhammadan University; it would be a University open to all — a teaching and a residential University.[2]

Before the Dacca University came into being, there were two colleges in Dhaka. The oldest was Dacca College, founded in 1841. Classes up to the postgraduate level were held in Dacca College run by the Government. The intermediate and undergraduate courses were taught at Jagannath College which had grown out of the Kishorilal Jubilee School — both were patronized by the zamindars of Baliadi. With the University functioning, only the Intermediate courses continued to be taught in these colleges; the undergraduate and postgraduate classes shifted to the University.

At the outskirts of the city of Dhaka lay the sprawling grounds of Ramna — 'a splendid site of about 450 acres'. In 1905, following the partition of Bengal when Dhaka became the capital of a new province, a new township had come up on the grounds of Ramna. New buildings were constructed, some of which were administrative buildings while others were residential quarters for government employees. The buildings were widely spaced, each having a lawn and garden. When the partition was annulled, Dhaka ceased to be the capital. Later, the newly built secretariat building housed the

office and the Arts Department of Dacca University. Most of the residential quarters were handed over to the University to accommodate teachers. Only a few residential buildings were occupied by British judges and magistrates.

The Court of Dacca University was the parallel body to the Senate in Calcutta and the Executive Council had the same status as the Syndicate. The Court held sessions twice a year. There were as many Hindu members as Muslims in the Court. The University graduates elected lawyers of the Bar Library to the Court.

The most distinguishing feature of Dacca University was its residential character. The 'Hall' was the primary residential unit of the University, and it was designed to play an important part in its social and intellectual life. Each University teacher had to be attached to a Hall and each teacher had responsibility for the general guidance of a number of students and their tutorial care in special subjects. Great importance was attached to such tutorials and the Halls were intended to serve both residential and tutorial functions. Each Hall accommodated more than 400 students under the charge of a Provost. A Hall was subdivided into four or more houses, each providing residence for about seventy-five students under the immediate control of a house tutor. Each Hall had tutorial rooms, common rooms, a library, a lecture theatre or assembly hall and the Provost's office. The Provost and house tutors were responsible for discipline. Each Hall had a society, facilities for games, debates and literary activities. In a residential system the students and teachers were part of one community, working to one purpose and helping each other to a common end.

There was a separate Muslim Hall for the Muslim students of the University. The idea was not to segregrate the Muslim and Hindu students but to enable them to mix as equals and to join in the general life of the University 'in a far more satisfactory manner than has hitherto been possible.' Although there were special facilities for the encouragement of Muslim students and of Muslim studies, the University was open to all.

Two of the Halls were named after the two existing colleges — the Dacca Hall and the Jagannath Hall. Both were meant for Hindu students. The old hostel of Dacca College was converted to Dacca Hall which could accommodate 150 students. Jagannath Hall could have another 200 and the Muslim Hall had a capacity of 300. These latter Halls were newly constructed. Even students who did not stay in these hostels had to be associated with one of the Halls. All students and teaching staff had to attend the cultural programmes in the Halls. The house tutors supervised the meals. There were roll calls at night to check whether the students had returned to the Halls in time. Initially the tutorials were held in the Halls but this was found to be inconvenient. So the tutorial classes shifted to the University. A lecture hall with a capacity of about 700 became the venue for student meetings and cultural programmes. The University also provided facilities for games, sports and physical education in the gymnasium under the guidance of instructors of physical education.

The development of Dacca University as a new model University was an achievement to be credited to its able Vice-Chancellors and teachers. P J Hartog, the first Vice-Chancellor, was a man of wide vision and imagination. During his five year term he

worked very hard to put the University on a solid foundation. Referring to the success of the education system, Hartog said, 'I asked a student who had begun his work under the former system what difference there was between his former training and his present. He said : Sir, we work harder and we see much more of our teachers.'[3]

After he had taught in the University College of Science in Calcutta for four years, Bose had an offer from Dacca University. In his own words :

> In 1921, I was at Calcutta. The Dhaka University had come into being. The first Vice-Chancellor, Mr Hartog, was recruiting new teachers. He called me up in Calcutta one day. Someone in a high position had apparently told him about me. I met him and eventually got the offer of a Readership at Dhaka in 1921.[4]

We do not have precise information about when the meeting took place, but as is evident from the following letter,[5] Hartog had asked Bose to send all his offprints along with his bio-data.

22, Iswar Mill Lane,
Calcutta
Dated the 15th Jan., 1921

Dear Mr Hartog,

In compliance to your wishes, I am sending you the reprints by post. I also enclose herewith a brief statement of my career.

I am quite willing to go as a Reader in Experimental, or Mathematical Physics, on an initial salary of Rs. 400, with the prospect of increment according to any reasonable scale, provided my first appointment be at least for three years, with the assurance that it will be made permanent, if my work there prove satisfactory.

I remain,

Yours sincerely,
Satyendranath Bose

Two days later Walter Jenkins, the newly appointed Professor of Physics at Dhaka sent his recommendations to Mr Hartog :[6]

To
The Vice-Chancellor of the University,
Dacca.

Physics Department
Dacca
Jan 17, 1921.

Dear Mr Hartog,

I have looked over the applications and carefully read through the research work of the people who may possibly be elected to the Readership in Physics and in my opinion the most suitable appointment would be that of S N Bose or Meghnad Saha. Both are able and original workers and while Saha has done more and perhaps better work than Bose, the latter's last paper in the Phil Mag shows him to be extremely capable. He has not so far had the opportunities that Saha has had and I should be inclined personally to

support his claims. From what I know and have heard of Bose I should say that he would work enthusiastically and easily with other people and I think an effort should be made to secure his services.

I remain,

Yours sincerely,
Walter A Jenkins

In the light of future developments this letter of Jenkins acquires great historical significance. Jenkins was making a comparison between the two most suitable candidates (obviously there were other applicants). His assessment was very fair and very English. Saha must have noticed the advertisement in *Nature* or some other scientific journal and sent in his application from England. It does not seem likely that his candidature was being considered without his knowledge. However Jenkins, like the true sportsman that he was, had a natural sympathy for the young man who was denied opportunities. But what he had no way of knowing was the exact situation. It was not as though Saha was given a better deal by the authorities. He had done his D Sc and PRS (Premchand Roychand Scholarship) and, anxious to verify his ionization theory, he had gone abroad borrowing money from friends and well-wishers. The Guruprasanna Ghose travelling allowance was not enough to cover all his expenses.

Anyway, the letter also makes two interesting points. One is the possibility that Jenkins foresaw — 'his last paper in the Phil Mag shows him to be extremely capable'; and the other was Bose's ability to get along with people. These two were very correct assessments. However, the offer was already made, so this assessment was a routine affair.

On 10 February a firm offer went to Bose from the Vice-Chancellor. The following is his letter of acceptance.[7]

22, Iswar Mill Lane,
Calcutta,
The 14th February, 1921.

Dear Mr Hartog,

I am glad to receive your letter of the 10th instant offering me the post of a Reader in Physics in the University of Dacca. (Rs. 400–50–1200). I accept the offer with thanks. I shall see Sir Ashutosh Mukherjee in a day or two and I hope to be released from here by the middle of April. I believe there will be no difficulty in my joining your University in June or even earlier.

In the meantime I shall be glad to render such assistance in setting the courses etc. in Physics as can be done by correspondence.

I am
Yours truly,
S N Bose

To
P J Hartog Esq., M A, C I E
Vice-Chancellor, University of Dacca.

It is one of those curious coincidences of history that Satyendranath Bose whose childhood memories were full of the nationalist agitation against the partition of Bengal in which he too had played a part, was destined to play a different active role in a situation which was a direct fall-out of that 1905 event. The birth of the new University in Dhaka was directly related to the annulment of that partition.

The offer was accepted and Bose proceeded to Dhaka. With him were many others like the historian R C Majumdar, Mohammed Shahidullah, Haridas Bhattacharyya and Nareshchandra Sengupta — all hand-picked by Hartog. Later Jnanchandra Ghosh also joined the University on his return from England.

Hartog had already had several meetings with the newly appointed heads of departments most of whom were erstwhile Calcutta University teachers. Persons like R C Majumdar had made several trips before the University actually started functioning from 15 July 1921. But since the Head of the Physics Department was Jenkins, it is unlikely that Bose was ever invited to such meetings. So this was his first visit and his first assignment in a strange place far away from the city he had known since childhood. The actual distance between Calcutta and Dhaka is only thirty minutes by air today, but then it was a slow journey by train and steamer. From Calcutta one had to take a train to Goalando, then a steamer to Narayanganj and from there to Dhaka by train. Crossing the wide Padma, a river immortalized by Rabindranath Tagore in his writings, was an experience by itself.

Bose reached his new place of work in the newly built area of Ramma on the outskirts of Dhaka. After the brick and mortar jungle of Calcutta, the lush green of East Bengal must have been a welcome change. The 'noble group of buildings, with great space and green playfields around them' (as Sir Mirza Ismail decribed them) were a far cry from the narrow lanes of North Calcutta. Did any teacher of the Calcutta University have acres of land to plan a garden in? In the twenty-five years that followed Bose had ample scope to make unique horticultural experiments, and soon his garden acquired the fame of being one of the best in Dhaka.

For all its visual charm the new set-up must have seemed like wilderness to Bose in 1921. One can try to imagine the mixed feelings. Here was a young Calcuttan in 'an alien land', as any typical Calcuttan would say, but fortunately, and in a manner very uncharacteristic of a typical Calcuttan, Bose did not display any such condescending emotions. His letters are full of zest and hope and eagerness to try out something new and occasionally perhaps a little disappointment, as is evident from the following letter written to Saha in July the same year :

> *It has been well over a month since I moved to your part of the country. Work has not yet started. Your Dacca College had quite a few things but due to utter neglect they are in a bad way. Perhaps I need not elaborate. On the table of the Sahibs are scattered lots of Nicol Prisms, lens and eye-pieces. It would require a lot of research to determine which one belongs to which apparatus. We do suffer from lack of journals here, but the authorities of the new university have promised to place orders for some of them along with their back numbers. They are talking about having a separate science library. . .*[8]

The situation however changed soon. By 1924 the library was filling up. In fact, the

authorities were criticized for their expenditure on books and periodicals, to which Hartog's reply was, 'I would far rather spend money on books . . . than on bricks and mortar'.

Was Bose then a Samson in exile? Well, not quite. Dhaka may have been physically distant, but Bose came here ready for a new beginning. If he felt himself to be in exile, it was not because of leaving Calcutta but a profounder sense of intellectual isolation. He was aware of things happening in Europe, the new Mecca of modern physics where all his classmates and colleagues had gone. Saha was in Germany when Bose moved to Dhaka. One can imagine his desperation. In one of his memoiral pieces he has written how he 'sighed for Albion's distant shore', but he was twice unlucky. In a letter written to Saha the underlying sentiment of feeling left out can be easily perceived.

Dhaka 19. 7. 21.

My dear Meghnad,

I have not heard from you for a long time. I used to hear about you often from Abinash and others while I was in Calcutta. Have you decided to boycott me for my irregular correspondence? Jnan and I are in the same place now. He tells me that you have been to Germany and met many stalwarts. I also heard that you were supposed to visit Munich. Can we expect a graphic account of your experience?. . .[9]

In his later years Bose used to tell his students, 'We used to live on the moon — do you understand, on the moon!' This best describes the intellectual isolation he felt those days.

When Bose joined the Physics Department, it had the following faculty : W A Jenkins, M Sc, IES, Professor (ex-officio) under statute 7(1)(ii); S N Bose, M Sc, Reader; R N Ghosh, M A, Reader; and Quazi Mutahar Husain. A whole new department had to be started to teach B Sc honours and M Sc courses. Bose taught thermodynamics and Maxwell's electromagnetic theory. The first batch of students graduated in 1924. But soon life became very uncomfortable for Bose when on 12 April 1923 he was informed by the Registrar that due to stringent financial conditions the University was forced to revise the grades and offer him reappointment for one year on a fixed salary of Rs 500. What was the reason for this sudden economic crisis? To quote R C Majumdar :

> It went wrong right at the beginning. The new Act of 1919 required the minister of education to be an Indian. The first minister for education was Mr Provash Chandra Mitra. On assuming office he declared the inability of the Government of Bengal to pay such high salaries to the teachers of Dacca University. He proposed a reduction in the salaries. The Government of India had kept aside a fund for Dacca University from the time it was proposed. When Dacca University came into being, it had a deposit of fifty-five lakh rupees. Now the University demanded the sum. The Government of Bengal argued that the buildings had been provided to the University against the sum.
>
> Hartog informed the Government of India. The Government of India had earlier approved the pay scale of the teachers. At this juncture they ex-

pressed their inability to intervene. The new law prevented the Government of India from issuing any directive in such matters to the Government of Bengal. The latter released a sum of Rupees five lakhs per year to the Dacca University. The pay scales of the entire teaching staff had to be lowered.[10]

Bose was asked to intimate within three days whether he accepted the offer. In reply he wrote a long and hard-hitting letter to Hartog accusing him of going back on his word :

. . . Thus the letter from the office has been an unpleasant surprise to me. The short term within which I am asked to make up my mind is distinctly unfair. I shall like to think on it and I want to find out how far my original impressions about my appointment letter were correct; whether my interpretation was a reasonable one, the only interpretation which will suggest itself to an honest man.

Of course I hasten to add that I do not want to cling to my post here simply on the strength of my original letter of appointment, if I feel that the university does not think my services worth having in the original terms.

I would like to point out however that I have not been fairly treated by your University. I have been drawn away from an honourable post by offers which you are now so eager to forget. If after two years of honest and conscientious work this is the only recognition by your University, I think it will not be very difficult for any one to see how the University is ushering forth a new regime in the educational history of Bengal.

May I hope to hear from you, soon, so that I can at any rate make up my mind before I go away to Calcutta. . .[11]

This letter was undated. Acknowledging the receipt of this letter, Hartog answered, trying to make his position clear :

12th April, 1923

Dear Mr Bose,

I have received your letter (undated) today. In view of the change in the financial condition in Bengal, the University gave notice to all teachers who were engaged for an indefinite term or whose appointments terminated during the present year that such appointments should come to an end on June the 30th, 1923 and you duly received such a notice. When you were originally engaged, I had no reason whatever to think the original plans in regard to the University would not be carried out and you were offered your original engagement in perfect good faith. The Executive Council have decided, as I informed the Court, to offer engagements for one year only to a large number of teachers this year including yourself but they hope to be in a position in the first term of the next session to offer permanent engagements in accordance with the scheme approved by the Academic and Executive Councils. Had you attended the meetings of the Academic Council at which this matter was discussed, you would have been fully aware of what was done. It is my desire, and I feel sure, it is the desire of the Executive Council to retain your services on the new terms offered. I think that it is entirely unfair to suggest that the University is eager to forget the terms originally offered to you. These terms were fixed not by the Executive Council but by the Chancellor on the advice of the officers of the Education Department and it is not the fault of the University that it is at present not placed in a

position to carry out the original permanent scheme.

You have during the past two years received a higher salary than that which you received in the University of Calcutta. The terms offered are not equal to those which were originally contemplated but they are certainly not inferior to those offered to the Post-Graduate staff of the University of Calcutta.

I should be very glad to see you at the earliest possible moment and if you do not wish to accept these terms it will be clearly necessary to take immediate steps specially in view of the fact that Professor Jenkins will not be returning to the University until after the Puja Holidays.

I think I ought to remind you that you have been given every facility possible to carry out your own research work and the University has not been unappreciative of the work which you have done.[12]

I am,
Yours sincerely,
Vice-Chancellor

To
S N Bose, Esq. M Sc
Reader in Physics

A comparison of the salaries was not very fair because had the prospects not been better, the question of Bose's leaving a secure job in Calcutta would not have arisen. Another hit below the belt came in the last paragraph — the authorities allowing Bose to carry on his own research! Within a year Hartog was rightly paid back when Bose's 'own research' won for Dhaka a place on the scientific map of the world.

Letters went back and forth. It was evident that Hartog was not really willing to let Bose leave, and Bose, too, stuck to his stand. In a letter to Hartog on 1 May 1923 from Calcutta where he had gone during the summer holidays Bose wrote :

I have received your letter of April 27. You have asked me to state the terms on which I shall be willing to stay on in Dacca. May I reiterate once more that the decision of the Ex. Council has been quite unjust in my case. I am not asking for fresh terms. I shall be satisfied if the Executive Council recognizes that my original term of appointment is of a permanent nature and as such cannot be said to have terminated after two years. I don't know whether the other Readers are all in positions similar to mine. But as for myself I think I am asking only what has been originally offered to me and nothing more.

I shall be in Dacca after the vacation and shall stay on pending the decision of the Executive Council. But I am afraid I can't give any undertaking to serve till 8th October. If the University authorities persist in their unjust decision, I shall have to seek for job elsewhere . . . [13]

They kept exchanging letters. Both parties were hurt and offended; Bose because he was left in the lurch, Hartog because aspersions had been made on his sense of fair play. Actually, the third party in this battle was the Bengal Government which had caused all this trouble and heartbreak. Bose was justified in offering arguments in self-defence. For Hartog the situation was made more delicate by the fact that Bose was his chosen man. Bose after all had not applied on his own for the job.

From the long correspondence that followed one aspect of Bose's character becomes

very clear. For all his apparent easy-going nature he was 'a man of steel' when it came to fighting for justice. Years later his close friend Bishnu Dey, the poet, wrote :

> . . . that he has astonishingly determined and directive power against meagre resources and massive opposition, has been seen in the small-scale but persistent zeal over the Vijnana-Parishad and Jnan-O-Vijnan as also throughout the tragic episode of reorganization at Santiniketan.[14]

But in Dhaka the adversaries were stronger and Bose was just a young man at the threshold of his career — the issue was his survival. As we shall see, he did not give in and finally had his way. His insistence that his case be treated separately resulted in the formation of a committee, later known as the Bose Committee, to interview him and to consider his terms for staying on. Bose had taken a very uncompromising stand, but Hartog and Jenkins were not ready to let him go.

The committee consisted of P J Hartog, Jnanchandra Ghosh and P K Ghosh, Bar-at-Law. Later Jenkins was also co-opted into the Committee. Bose meanwhile was getting impatient. On 18 December 1923 he wrote to the Registrar :

> *. . . May I also add that I joined the University after the vacation on the understanding that my case will be separately considered by the Executive Council. I have been told that the decision will be arrived at as quickly as possible : and though about three months have elapsed I have had no news about the decision. . .*[15]

It appears that he had asked the committee for a two years' study leave and insisted that his pay should continue in the same scale as originally proposed. The Bose Committee finally took a decision on 2 March 1924. It was as follows :

1. Mr S N Bose to continue his work as Reader in Physics in the University till the end of the present session on his present salary;
2. Mr S N Bose to be granted two years' study leave as from the beginning of September 1924;
3. Mr S N Bose to receive a total advance of Rs. 13,800/-. . .[16]

While all these tussles were going on between Bose and the authorities, there were other developments taking place on a different plane. Bose was teaching thermodynamics and Maxwell's electromagnetic theory to the M Sc classes. He was also studying the theory of relativity and the quantum theory. These new discoveries in modern physics were creating new waves in Europe. When D M Bose returned from Germany in 1919 after a long internment during World War I, he had presented to Satyendranath a copy of Max Planck's *Thermodynamik und Wärmestrahlung*. Max Planck, the German physicist, was the main architect of modern physics. He founded the quantum theory in 1900 by proposing that energy was absorbed and released in the form of quanta, that is, in indivisible packets rather than in a continuous manner as required by Maxwell's electrodynamics. He introduced his famous formula $E = h\nu$. Bose read Planck's papers on the distribution of energy from a black-body based on this new theory. Being a perfectionist, he would not accept any ad-hoc assumption while working out a theory. So he was not happy with Planck's derivation which had ad-hoc assumptions.

It was around March-April 1924 that Bose wrote his famous paper, 'Planck's Law

and the Light-Quantum Hypothesis' and, in continuation, another paper entitled 'Thermal Equilibrium in the Radiation Field in the Presence of Matter'. He communicated the first paper to *Philosophical Magazine*. He was so confident that it would be published that the second paper has a footnote referring to the first paper — 'to appear in Phil Mag'. But *Phil Mag* kept silent. Not hearing from them, Bose now took the bold step of sending the first paper to the highest authority in the subject — Albert Einstein — with this covering letter : [17]

Physics Department,
Dacca University,
Dated, the 4th June, 1924.

Respected Sir,

I have ventured to send you the accompanying article for your perusal and opinion. I am anxious to know what you think of it. You will see that I have tried to deduce the coefficient $(8\pi\frac{\nu^2}{c^3})$ *in Planck's law independent of the classical electrodynamics, only assuming that the ultimate elementary regions in the Phase space have the content* h^3. *I do not know sufficient German to translate the paper. If you think the paper worth publication, I shall be grateful if you arrange for its publication in Zeitschrift für Physik. Though a complete stranger to you, I do not feel any hesitation in making such a request. Because we are all your pupils though profiting only by your teachings through your writings. I do not know whether you still remember that somebody from Calcutta asked your permission to translate your papers on Relativity in English. You acceded to the request. The book has since been published. I was the one who translated your paper on Generalised Relativity.*

Yours faithfully,
S N Bose

Again on 15 June 1924 he sent the second paper to Einstein with the following covering letter : [18]

Physics Laboratory
Dacca University
Dacca, India
15th June, 1924

Respected Master,

I send herewith another paper of mine for your kind perusal and opinion. I hope my first paper has reached your hands. The result to which I have arrived seems rather important (to me at any rate). You will see that I have dealt with the problem of thermal equilibrium between Radiation and Matter in a different way, and have arrived at a different law for the probability for elementary processes, which seems to have simplicity in its favour. I have ventured to send you the type-written paper in English. It being beyond me to express myself in German (which will be intelligible to you), I shall be glad if its publication in Zeitschrift für Physik or any other German journal can be managed. I myself know not how to manage it. In any case, I shall be grateful if you express your opinion on the papers and send it to me at the above address.

Yours truly,
S N Bose

In his first letter to Einstein, Bose addresses him as 'Respected Sir', in the second letter as 'Respected Master' and in the third it was to be 'Dear Master'. These forms of salutations may seem archaic, even quaint to western minds, perhaps also to the modern generation of Indians. But traditionally in India the seniors demand a special form of address. When writing to elders, even today the form in Bengali is 'Sreecharanakamaleshu' meaning 'to the graceful lotus feet'. A simple 'Dear Sir' or 'Dear Mr Einstein' would not do — it was simply inconceivable to a man of Bose's culture and upbringing. Since there was no exact English substitute for the conventional Bengali way of addressing a senior, the best polite expression he could think of was 'Master' which has the weight of the entire teacher-student relationship going back to history. Einstein was a master or 'guru' in the Ekalavya tradition — the master not being aware of the devotion of a student with whom he has had no direct contact. The first paper was translated into German by Einstein himself and published in *Zeitschrift für Physik* with the following note :

> In my opinion Bose's derivation signifies an important advance. The method used here gives the quantum theory of an ideal gas as I will work out elsewhere. —*A Einstein*

In the latter half of the nineteenth century the spectral distribution of radiation emerging from a small aperture in the wall of an enclosure maintained in thermal equilibrium was measured, but the nature of the distribution could not be explained by any existing theory, till Planck was able to provide one. Einstein further extended Planck's hypothesis. He suggested that the radiant energy travels in packets or 'light-quanta'. Even then the theoretical derivation of Planck's formula was not entirely satisfactory because he had replaced classical electrodynamics with ad-hoc hypotheses. Attempts were made by others to improve upon this. It was the unsatisfactory nature of the derivations of Planck's formula that prompted Bose to work on it on his own, and he finally succeeded in providing an entirely self-contained derivation. But what Bose actually did was more than derive a formula — he introduced a new concept into physics, later named Bose Statistics. Einstein understood the deeper significance of it and immediately applied it to the case of ideal gases and found a new result, viz. the Bose-Einstein condensation.

Purnangshukumar Roy, a student and close associate of Bose, offers this account of how that paper came to be written :

> When Bose left for Dhaka his acquaintance with statistical mechanics and gas theories was rather superficial compared to his familiarity with the theories of electromagnetism and relativity. It was some time after March 1924 that Bose had a meeting with Saha when during the course of the discussion Saha referred to the papers by Pauli (1923) and Einstein and Ehrenfest (1923) published in the recent issues of *Zeitschrift für Physik*. (In fact Saha is supposed to have left the papers with Bose.) He complained about some strange relation in the Pauli paper and asked Bose to examine it. What is the 'strange' relation that Saha had talked about?

> In the first half of 1923 the works of Compton and Debye on the X-ray scattering off an electron considerably agitated the minds of the then leading physicists. It was, however, young Pauli who took up the problem of finding a quantum theoretical mechanism for the interaction of radiation with free electrons. He subjected the interaction to the requirement that electrons with the Maxwellian distribution of velocities were in equilibrium with radiation; it was, of course, assumed that the spectral distribution of radiation obeyed Planck's law. Pauli thus obtained an expression for the probability of a Compton interaction between a photon and an electron. The expression, however, consisted of two parts. One part depended on the radiation density of the primary frequency alone, while the other depended also on the radiation density of the frequency which arose through the Compton process. It was this second term which was intriguing and puzzling from the philosophical point of view : the existence of this term sought to imply that the probability of something happening depended on something that had yet to happen. The paper by Einstein and Ehrenfest was a generalization of Pauli's work.
>
> It is to this 'crazy idea' — this is how Bose often referred to the work of Pauli — that Saha drew the attention of Bose. It is thus young Bose was inducted into the brilliant papers of Debye (1916) and Einstein (1917). It is thus he was led into the evergreen wonderland of radiation and statistical physics.[19]

In his derivation of Planck's radiation formula from quantum statistical mechanics, Bose treated the photon as a particle and obtained an expression for the number of cells in phase space occupied by the radiation which was one-half the factor that appears in Planck's formula. He introduced a factor 2 to take care of the polarization of the photon. In the paper one finds the statement that the factor 2 arises from the two states of polarization. Polarization was usually attributed to the transverse wave nature of radiation and not to particles. However, a few years later in 1931, C V Raman and S Bhagavantam in their paper 'Experimental Proof of the Spin of the Photon' wrote,

> . . . we understand from a personal communication by Prof. Bose that he envisaged the possibility of the quantum possessing besides energy $h\nu$ and linear momentum $\frac{h\nu}{c}$ also an intrinsic spin or angular momentum $\pm \frac{h}{2\pi}$ round an axis parallel to the direction of motion. The weight factor 2 thus arises from the possibility of the spin of the quantum being either right-handed or left-handed corresponding to the two alternative signs of the angular momentum.[20]

We find several versions of how those papers came to be written. Shyamadas Chatterjee had asked Bose :

> Sir, how did the idea of Bose statistics come to you? A child-like smile lit up his face. Then he recounted the background— It was in early 1924, Meghnad Saha had come to Dhaka in connection with M Sc practical examinations.

He brought with him the just published paper of Pauli on photon. Saha was not happy with the theoretical explanation given. He asked his friend to think over it.[21]

W A Blanpied who had interviewed Bose writes :

Here (at Dhaka) he lectured, read, thought, 'spent many sleepless nights' thinking about the Planck law and late in 1923 submitted a paper on the subject to the *Philosophical Magazine*. Six months later the editors of that journal informed him that (regrettably) the referee's report on his paper was negative. Undeterred, he sent the rejected manuscript to Einstein, and thus took the first step in the formulation of quantum statistics.[22]

J Mehra, who had also interviewed Bose, writes :

It was some time around March 1924 that Bose had a meeting with Saha. 'Saha was a guest visiting Dhaka; it was his native land. He came out to see me and stayed with me for a time. I told him about the derivation of Planck's law which we had to teach our boys, with all its contradictions, and how I felt the necessity of getting a derivation without inner difficulties.' In the course of their conversations, 'Saha drew my attention to certain aspects of equilibrium between radiation and the electron-gas, and what Einstein and Ehrenfest had published in a recent issue of the Zeitschrift für Physik. Saha told me that Pauli had utilized Einstein's 1917 idea. What seemed to be happening in Pauli's work was that in order to apply the quantum condition you had to know exactly what was going to happen afterwards. So there were certain difficulties and Saha pointed them out to me.'[23]

M N Saha wrote :

The Berlin period also saw the phenomenal rise of the quantum theory, and of wave-mechanics which, taken together, have thrown a flood of light on radiation and atomic phenomena. Though in the midst of these great discoveries, Einstein appears to have been little attracted to them, except on two occasions.

The first occasion was in 1917 when, poring over the implications of the Bohr-atom, he wrote a paper on Einstein A and B-coefficients, the A-coefficients denoting the spontaneous transition probability of excited states to lower ones, the B-coefficients denoting the transitions to higher and lower states under the effect of radiation. These contributions are rightly regarded as fundamental to atom-physics.

It was these considerations which led to a collaboration between Einstein and Ehrenfest under the title *'Quanten theorie des Strahlungsgleichgewichts'* in the Zeitschrift f. Physik, vol. 19, pp. 301-306. While the author of this note was studying this paper together with S N Bose in 1924, the latter was led to a very ingenious deduction of the Planck law of radiation. He showed that the number of cells A in a phase space can be determined

by the expression

$$A = \frac{1}{h^3} \int \int \cdots dp_r \, dq_r$$

the integration extending over the whole phase-space covered by the particles.

One can then proceed to fill up the phase-space by the number of quanta available, by using combination with repetition (a device already taken by Debye), and thus get the Planck law in a very simple manner. This is in fact what was done by S N Bose.[24]

All these reports except the one by Saha were supposed to have been orally communicated by Bose. Saha's account came out in a journal not normally read by physicists and was probably overlooked. We consider Saha's version to be the most authentic account of the genesis of the paper.

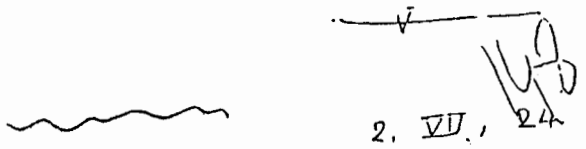
Einstein's comments on his paper provided him the final passport to go to Europe for a period of two years. It was a handwritten letter, a postcard in which Einstein had expressed his admiration for the paper, adding that he considered it to be a major contribution. So, if the authorities were reluctant to grant study leave earlier, they could not refuse it now. The postcard also helped him in securing a visa promptly from the German Consulate in Calcutta.

2.7.24

Dear Colleague,

I have translated your work and communicated it to Zeitschrift für Physik for publication. It signifies an important step forward and I liked it very much. In fact I find your objections against my work not correct. For Wien's displacement law does not assume the wave (undulation) theory and Bohr's correspondence principle is not at all applicable. However, this does not matter. You are the first to derive the factor quantum theoretically, even though because of the polarization factor 2 not wholly rigorously. It is a beautiful step forward.

With friendly greeting,
Yours
A Einstein



2. VII. 24

Lieber Herr Kollege!

Ich habe ihre Arbeit über-
-setzt und der Zeitschrift für
Physik zum Druck übergeben.
Sie bedeutet einen wichtigen
Fortschritt und hat mir
sehr gut gefallen. Ihre
Einwände gegen meine Arbeit
finde ich zwar nicht richtig.
Denn das Wien'sche Ver-
schiebungsgesetz setzt die
undulations Theorie nicht voraus
und das Bohr-che korrespondenz-
prinzip ist überhaupt nicht
Verwendet. Doch dies thut
nichts. Sie haben also richtig
den Faktor quantentheoretisch
abgeleitet wenn auch wegen
des Polarisations-Faktors 2
nicht ganz streng Es ist
ein schöner Fortschritt.

Mit freundlichen Gruss

Ihr A Einstein.

(Sd)

The postcard from Einstein in facsimile

5

Europe

1924-1926

Bose sailed from Bombay and reached Paris on 18 October 1924. There has been a lot of speculation about his long stay in Paris which was not his original intention. For one thing, he had to collect his mail from R L De who had been sent from the University of Dacca on deputation to Mme Curie's laboratory. Paris had its attractions too, and Bose had friends who were already in the city. There was a third factor — it had to do with his revolutionary contacts. This will be discussed in detail in the last chapter.

In Paris Bose put up at 17 Rue du Sommerard, a building which housed the Indian Students Association. This association had brarches in various European cities. It sheltered students who were involved in the nationalist movement. Bose's old friend, Girijapati Bhattacharjee, was already in Paris . He had accompanied Rabindranath Tagore there, a few weeks before Bose's arrival. The meeting was a pleasant surprise for both. This is how Girijapati has described their meeting in Paris :

> I had preceded him but had left Paris for a short visit to London. When I returned to Paris, I found Satyen lodged in a 'pension' at 17 Rue de Sommerard where my lodging was also arranged by Dr Prabodh Bagchi (later Vice-Chancellor, Visva-Bharati.)
>
> Several Indian students were also staying at the Pension including Dr Bagchi, Dr Subodh Mukherji (Auditor General, E I Railway), Dr Thawakali (student, Philosophy), Arya Chowdhury (artist) and D G Tendulkar. Sahed Suhrawardy (poet and later Bageswari Professor of Art, Calcutta University) and Dr Niranjan Chakravarti (later Director General, Archaeological

Survey of India) joined later. The house was a rendezvous for Indian students residing in Paris. Bose's paper was then well known in the academic circle in Germany, France and elsewhere, but was unknown to the Indian students in Paris. From an innate modesty, he never spoke about it to anyone.[1]

Bose's main interest seems to have been to visit the modern laboratories of Europe, particularly in radioactivity and X-ray crystallography. Initially, his idea might have been to spend one year in England and another in Germany. Hartog's letters to Lord Rutherford and William Bragg introducing the promising young man corroborate this. For Germany Bose needed no introduction.

However, his plans were flexible, as Hartog mentions in both the letters. Bose reached Paris on 18 October 1924, but five days later, Rutherford replied to Hartog expressing his inability to accommodate Bose. This letter could not have reached Dhaka before the second half of November, because we have Hartog's letter to Rutherford, dated 20 November 1924, saying he was sending his (Rutherford's) letter to Bose in Paris. Meanwhile, Bose had been writing to Hartog. He could not have known about Rutherford's refusal before the third week of December. But he was looking around in Paris, and from his correspondence with Hartog we come to know that he was working at the X-ray laboratory of Maurice de Broglie and from January the following year would be given facilities for work in the Radium Institute of Mme Curie. This is Hartog's letter to Lord Rutherford :

Office of the Vice-Chancellor,
Dacca University.
Ramna (Dacca), September 12, 1924.

My dear Rutherford,

This is to introduce to you Mr Satyendra Nath Bose, who is a Reader in Physics in this University and who has been given study-leave to work in Europe for 2 years. He is a distinguished young physicist and mathematician, who has been working on Relativity and cognate subjects. He translated into English some of Einstein's memoirs with Meghnad Saha. I enclose a copy of a postcard recently received by him from Einstein with regard to a paper of his on Light quanta, which Einstein has translated and published in the Zeitschrift für Physik. Mr Bose's plans are not settled, but I think it is possible that he may wish to stay in Cambridge for a time and work in the Cavendish Laboratory if you would give him permission to do so. I should add that Mr Bose is not only my colleague but also my friend and I am sure you will be glad to know him and help him in any way that you can.

We have a really good Physics Laboratory here, but it is very difficult to get new apparatus made on the spot, though we have a workshop and a glass blower.

To
Sir Ernest Rutherford,
Cambridge.

Yours sincerely,
Sd.[2]

He wrote a similar letter to Sir William Bragg :

Office of the Vice-Chancellor,
Dacca University.
Ramna (Dacca), 30th Sept., 1924.

My dear Bragg,

This is to introduce to you my friend and colleague Mr Satyendra Nath Bose, Reader in Physics in this University, to whom we have just given two years' study leave to be spent in Europe. Mr Bose is a brilliant young man. I enclose a copy of a postcard he has just received from Einstein who translated and published his last paper in the Zeitschrift für Physik. I am not quite sure where Mr Bose will spend the greater part of his time, but I should be very grateful if you would give him any advice in your power when he goes to London.

We have spent a considerable amount of money on fitting our Physical and Chemical Laboratories which are I think probably as good as any in India. I will send you a copy of our Pamphlet of General Information and of our last Annual Report which will give you some idea of what we are doing. I expect to be home on leave early in the summer of next year and I then return to finish up my five years' term of office on November 30th, 1925.

With very kind remembrances,

To
Sir William Bragg, F R S,
Davy (Faraday) Laboratories,
Albemarle Street,
London. W.

Yours sincerely,
sd.
Vice-Chancellor [3]

On his arrival in Paris Satyendranath found a letter from Hartog along with these letters of recommendation waiting for him. He wrote to him on 26 October : [4]

17 Rue du Sommerard,
*Paris V^e^*
26th Oct. (1924).

Dear Mr Vice-Chancellor,

I have arrived on the 18th instant and found your letter, with the letters of recommendations waiting for me at R L De's place. Many thanks for kindness. I shall be seeing the people at Paris now, and shall write to you again very soon.

Hoping this will find you in best of health.
I remain,

Yours sincerely,
Sd.
S N Bose

Meanwhile, Rutherford had replied expressing his inability to accept Bose. Hartog sent Rutherford's letter to Bose in Paris. Unfortunately that letter is lost. We have however Hartog's letter to Rutherford :

Sir E Rutherford, F R S, *Nov. 20, 1924.*
Cavendish Laboratory, Cambridge.

My dear Rutherford,

Many thanks for your letter of October 23, which I will send on to Bose who is now in Paris. I can quite understand the pressure on your space.

I herewith enclose a copy of the postcard from Einstein.
With kind regards,

Yours sincerely,
P J Hartog

Sir E Rutherford, F R S,
Cavendish Laboratory, Cambridge

There is no record of any reply from Bragg in the file. Meanwhile Bose had kept in touch with Hartog, sending him reprints, reporting the progress of his work :

17 Rue du Sommerard
*Paris V^e^*

Dear Mr Hartog,

Received your kind letter of the last mail. I am at present working at the X-ray Laboratory of M de Broglie. Madame Curie also has given me hopes of allowing me facilities for work in the Radium Institute from the beginning of the new year.

I am also sending by this mail the reprint of the papers that have appeared in Z. für Physik.

Hope this will find you in perfect health, With best wishes for happy new year,
I remain,

Yours sincerely,
Sd.
S N Bose.

22nd January 25
17 Rue du Sommerard
*Paris V^e^*

Dear Mr Vice-Chancellor,

I am sending herewith the reprints which I forgot to send you along with my previous letter. You shall be glad to know, that I have been granted facilities to work in the Curie Laboratory from the beginning of this year. I am also attending the course of lectures by Prof. Langevin, in the College de France. He has been also very kind to me, and has suggested me a theoretical problem on the Relativity Theory to work on.

I hope you are in the best of health and spirits,

Yours sincerely,
Satyendranath Bose [5]

Bose was already well known for his paper, and 'doors opened' wherever he went. However, his first encounter with Mme Curie was interesting. In his own words :

> Langevin, a student of Pierre Curie, was then the Director of the Municipal School where radium was discovered. He had read my paper and welcomed me warmly. He asked me what I wanted to learn, how long I wished to stay etc. I went to Madame Curie with a letter of introduction from him. I wanted to join her Institute and learn to work with radioactivity. I was allowed entry to her small chamber. The great elderly lady sat there in black clothes. I could recognize her from pictures of her that I had seen. I handed her the letter of introduction. She greeted me affectionately and said that there was no way she could disregard a recommendation from such a person. You will certainly get an opportunity to work with me, she said. But not right now, after three or four months. Get to know the language. Otherwise you will find it difficult to carry on in the laboratory. You are in no hurry, I presume.
>
> She spoke at ease in chaste English for about ten minutes. I had no opportunity to tell her that I knew a French of sorts already. I had been at it for the last ten years at home. I came away resigned to carrying out her instructions. I was in Paris for the next five or six months and had the opportunity to work at the Radium Institute for some time.[6]

His friends at 17 Rue du Sommerard had no idea of the waves he was making. His friend, Girijapati, has written a detailed account of the reaction of his friends :

> I came to know of it by chance. It was February 1925. A cold wind had swept Paris the previous night with drizzles. When I woke up in the morning, the city was under a blanket of snow. Something impelled me to come out of my room and knock on Bose's door. 'Entré vous' came the quick response. Entering I found him awake, reading Dante's La Divina Commedia in the original. He asked me to sit and rang for two cups of chocolate and croissant. Turning towards me, he said he had a small present for me. He gave me a copy of the reprint of his paper 'Plancks Gesetz und Lichtquanta'. I knew very little German and Satyen explained the contents of the paper to me step by step ending with the note Einstein had added : 'Bose's method of derivation of Planck's Law, in my opinion, signifies a forward step. The method applied here yields the Quantum theory of ideal gases as I will show elsewhere'. Satyen had received the reprints the previous evening and I learnt from him that it was Einstein himself who had translated his English paper into German for the publication. 'Why didn't you tell any of us about your paper and Einstein's acceptance?' I asked. He said, he had shown his manuscript only to D M Bose who thought it was so novel and outstanding that it deserved to be sent to European physicists for their opinion. The snow continued to fall and Satyen reverted to Dante, without saying a word about the laudatory opinion of the great Einstein. I sat amazed at his composure and unconcern till he finished Dante.[7]

Bose had written to Einstein, in fact a week after his arrival in Paris, expressing his desire to work with him. For all his flexibility of plans, about one thing he was very definite — that was to work with Einstein : [8]

17 Rue du Sommerard
*Paris V^e^*
26-10-1924

Dear Master,

My heartfelt gratitude for taking the trouble of translating the paper yourself and publishing it. I just saw it in print before I left India. I have also sent you about the middle of June a second paper entitled 'Thermal Equilibrium in the Radiation Field in the Presence of Matter'.

I am rather anxious to know your opinion about it, as I think it to be rather important. I don't know whether it will be possible also to have this paper published in Zeitschrift für Physik.

I have been granted leave by my university for 2 years. I have arrived just a week ago in Paris. I don't know whether it will be possible for me to work under you in Germany. I shall be glad, however, if you will grant me the permission to work under you, for it will mean for me the realization of a long-cherished hope.

I shall wait for your decision as well as your opinion of my second paper here in Paris.

If the second paper has not reached you by any chance, please let me know. I shall send you the copy I have with me.

With respects,

Yours sincerely,
S N Bose

The second paper was already published but Bose was not aware of it. For some reason Einstein had not informed him, as he had done earlier. However, now a prompt reply came : [9]

Dr S Bose
17, Rue du Sommerard
Paris.

Berlin W. 30
3 November 1924

Dear Colleague :

Thank you sincerely for your letter of 26 October. I am glad that I shall have the opportunity soon of making your personal acquaintance. Your papers have already appeared sometime ago. Unfortunately the reprints have been sent to me instead of you. You may have them at any time. I am not in agreement with your basic principle concerning the probability of interaction between radiation and matter and have given the reason in a remark which has appeared together with your paper. Your principle is not compatible with the following two conditions :

1) The absorption coefficient is independent of the radiation density.

2) The behaviour of a resonator in a radiation field should follow from the statistical laws as a limiting case.

We may discuss this together in detail when you come here.
With kind regards.

Yours
A Einstein

This letter written on 3 November must have reached Bose in a couple of days. Presumably, he was upset by Einstein's objections because Bose himself thought this to be his best work. As many of his students recall, he used to tell them to pay more attention to his second paper. He did not reply immediately, worked on the problem for a while and wrote a third paper before he replied to Einstein. The following letter of 27 January 1925 is extremely important as it holds clues to the third paper 'sent under a separate cover' : [10]

17 Rue du Sommerard
*Paris V^e^*
27th January 25

Respected Master,

I received your kind note of 3rd November in which you mentioned your objections against the elementary law of probability. I have been thinking about your objections all along and so did not answer immediately. It seems to me there is a way out of this difficulty, and I have written down my ideas in the form of a paper which I send under a separate cover. It seems that the hypothesis of negative Einstrahlung stands, which, as you have yourself expressed, reflects the classical behaviour of a resonator in a fluctuating field. But the additional hypothesis of a spontaneous change, independent of the state of the field, seems to me not necessary. I have tried to look at the radiation field from a new standpoint and have sought to separate the propagation of Quantum of energy from the propagation of electro-magnetic influence. I seem to feel vaguely that some such separation is necessary if Quantum theory is to be brought in line with Generalised Relativity Theory.

The views about the radiation-field, which I have ventured to put forward, seem to be very much like what Bohr has recently expressed in May Phil. Mag. 1924. But it is only a guess, as I cannot say honestly to have exactly understood all he means to say, about virtual fields and virtual oscillators.

I am rather anxious to know your opinion about it. I have shown it to Professor Langevin here and he seems to think it interesting and worth publishing.

I cannot exactly express how grateful I feel for your encouragement and the interest you have taken in my papers. Your first post card came at a critical moment and it has more than any other made this sojourn to Europe possible for me. I am thinking of going to Berlin at the end of this winter, where I hope to have your inestimable help and guidance.

Yours sincerely,
S N Bose

Langevin had gone through this paper and thought it was worth publishing, as mentioned in this letter. Curiously enough, this paper was neither published nor was it subsequently found in the Einstein archives. It was Bose's strange loyalty to Einstein which prompted him to make a great mistake in his career. Langevin could very well have communicated the paper to one of the French journals and a preprint could have been sent to Einstein. The paper does not exist, but the letter does. The new ideas introduced in this paper have been corroborated in course of time. In fact, Einstein himself agreed to one idea twenty-one years later, though Bose's contribution was not

acknowledged. It is a matter of great regret that few have worked on Bose's second paper though he used to tell his students that he considered it to have been wrongly assessed. Included in the first part of this volume is the first and so far the only in-depth analysis of Bose's second and the 'lost' third paper done by his last Ph D student, Partha Ghose.

Bose had no way of knowing that by 1921 Einstein's creative period was nearing an end. It was Bose's 1924 paper that provided a spark after which his mind strayed to other fields and activities. Bose did not know that Einstein never had Ph D students working under him, nor that he had only mathematical assistants who too did not stay for long. Besides, from the early twenties there was a politically motivated anti-relativity camp in Germany which continued to trouble Einstein. He declared himself a Jew and was busy raising money for a Jewish University in Palestine :

> I am a national Jew in the sense that I demand the preservation of the Jewish nationality as of every other. I look upon Jewish nationality as a fact and I think that every Jew ought to come to definite conclusions on Jewish questions on the basis of this fact. . . The Jewish nation is a living thing and the sentiment of Jewish nationalism must be developed both in Palestine and everywhere else.[11]

In 1921 he was asked to go on a fund-raising tour of the USA on behalf of the Hebrew University. He agreed, missing the Solvay Congress for the first time since the end of the War!

From the following record compiled by Abraham Pais,[12] one can have an idea of Einstein's involvement in the cause during the period 1924-25 :

1924 As an act of solidarity, E. joins the Berlin Jewish community as a dues-paying member.
E. edits the first collection of scientific papers of the Physics Department of the Hebrew University.
The 'Einstein-Institute' in Potsdam, housed in the 'Einstein-Tower' starts its activities. Its main instrument is the 'Einstein-Telescope'.
Ilse E. marries Rudolf Kayser.
June. E. reconsiders and rejoins the CIC.
June. 7. E. states that he does not object to the opinion of the German Ministry of Culture that his appointment to the Prussian Academy implies that he has acquired Prussian citizenship. (He retains his Swiss citizenship.)
December. E.'s last major discovery : from the analysis of statistical fluctuations he arrives at an independent argument for the association of waves with matter. Bose-E. condensation is also discovered by him at that time.

1925 May-June. Journey to South America. Visits to Buenos Aires, Rio de Janeiro, and Montevideo.

E. signs (with Gandhi and others) a manifesto against obligatory military service.
E. receives the Copley medal.
E. serves on the Board of Governors of the Hebrew University (until June 1928).

The first paper published by Einstein after reading S N Bose's paper on Planck's law and the light-quantum hypothesis was communicated on 20 September 1924 in which he continuously refers to S N Bose as D Bose. The second paper by Einstein was communicated in December 1924 in which he 'arrives at an independent argument for the association of waves with matter'. It is indeed curious that in the Einstein archives in Jerusalem there are two pages in S N Bose's handwriting in which the equation on which this revolutionary conclusion is based is derived by Bose in his typical fashion. The marking on this note is : [13]

Undated. Probably 1925, Berlin
A. E. E., dft. 2 pp
remarks on "Fluctuations in density"

S N Bose, Dacca, India (6)
Scient. Corr. File
Folder "B-Misc. II"

Writing undated letters was one of S N Bose's annoying traits. So there is no way of knowing whether this calculation was done before or after Einstein wrote his second paper on Bose gases.

None of the papers sent by Bose to Einstein could be obtained in the original from the Einstein Archives in Jerusalem. So one does not know what changes or modifications were made by Einstein if any to the first paper. Blanpied writes, 'He [Bose] also claims that his first Zeitschrift paper had anticipated the idea of treating left-and right-circularly polarized photons as distinguishable states, but that Einstein had recommended he delete that section.'[14] This has been corroborated by Partha Ghose to whom S N Bose had remarked that 'the old man' had struck off his idea of photon spin. Why did he not point out to Einstein that it was originally his idea? Bose answered with a smile — what matters is that the idea had turned out to be right, not who proposed it first.[15]

Perhaps all this will provide a clue to the riddle (which has worried many of his biographers) as to why he did not produce a single paper while in Europe with the best of opportunities.

It was characteristic of Bose that his interests dispersed over a wide variety of subjects. For instance, he was interested in the application of X-ray crystal structure analysis. In France, Bose was introduced to Maurice de Broglie who was doing original researches in X-ray crystallography. He was also invited to stay at the de Broglie estate in the country. Bose made friends easily and during his first visit to Europe established lasting friendships with many European scientists. Bose's experience with X-ray crystallography was later put to good use when he went back to Dhaka in 1926. He developed a well equipped X-ray crystallographic laboratory at the Dacca University.

After spending one year in France in the company of the luminaries of modern physics, Bose left for Berlin. On 8 October 1925 he sent a message to Einstein seeking an appointment. They did not meet immediately for Einstein was not in Berlin at the time. As soon as he returned, Bose met him.

He spent his time in Germany studying, attending seminars, colloquia and meeting people. During his stay in Berlin he came in contact with such towering figures of the new science as Fritz Haber, Otto Hahn, Lise Meitner, Walter Bothe, Hans Geiger, Peter Debye, Max von Laue, Wolfgang Pauli, Werner Heisenberg and others. They were the people who gave a new meaning to the German tradition of scientific scholarship. Germany, or more correctly, Berlin was the centre of learning that attracted scientists from all over the world. In a letter written in 1921 to Saha and J C Ghosh who were then in Europe, P C Ray advised them to spend as much time in Berlin as they could manage:[16]

> . . . *Never in your life will you get such a chance of meeting great men of science. England has only mediocre people, barring a few. Moreover they are incapable of appreciating our work because we belong to a subject race. . .*

While in Germany Bose visited Einstein frequently and talked about many subjects besides physics. The revolutionaries were very active in India at the time — anti-British sentiments ran high.

> One day he confided to me. 'I think Englishmen are better than other Western colonial nations, and I feel that they are far better than the French and Dutch. You should not be surprised at a German like me praising the English (after the World War). Now tell me, do you really want that the British should quit your country?' I said, 'Of course — we all want to determine our own destinies ourselves.' He was not quite convinced. He raised a hypothetical question. He said, 'Suppose there was a button near you and all the Englishmen would quit India if you were to press it. So, would you press that button?' I smiled and said, 'If God were to grant me such an opportunity, I will not hesitate even for a moment to press it.' He said, 'Really?' and kept quiet for a while. I asked him, 'Well, why do you Jews then want to establish a new Israeli State? Even you seem to be fairly inclined in its favour.' He said, 'Of course, I can now understand what you are saying — it is an emotional matter and cannot be understood rationally.'[17]

Bose's name has always been associated with Einstein's, more so in non-scientific, non-academic circles in India to the extent that in an Indian television quiz show the two competing teams were named after Bose and Einstein. This popular image has persisted, but it hardly does justice to the originality of Bose. It is true that Einstein was responsible for recognizing his originality and convincing others of its importance for which Bose was ever grateful, true to the Ekalavya tradition, but to think that Bose shines in the reflected glory of 'the Master' is neither right nor fair.

Bose continued to visit 5 Heberland Strasse, the small flat where the Einstein family lived. There is hardly any record available about what transpired between them. In his memoirs and autobiographical sketches Bose has carefully avoided mentioning the academic discussions they had. However, we have this account from a younger scientist whose family was very close to Bose :

> One day he was reminiscing about his Berlin days. . . . Einstein had given him a problem to solve. Satyen-Babu told me, 'I thought about it for three days and then I went to see him with my solution! Einstein went through it and said, "Well, this is not wrong, but not the way I want it — this clue does not open up any new possibility. Please don't waste my time with such trifles." You know that is what the old man said.' Satyen-Babu's mind seemed to go back to the past and for some time he did not speak.[18]

Clearly he was disappointed as he must have been about Einstein's rejection of his 'third paper.' But Bose's reaction was very true to his character. It was not easy to contradict a man like Einstein. In early life he was known for his outspokenness and had antagonized Ganesh Prasad and annoyed P C Ray and Sir Asutosh. Why then was he afraid to speak up to Einstein? Was he not very sure of the solution himself ? Or was it because he held Einstein in much greater esteem than either P C Ray or Asutosh? Perhaps he found a true 'guru' in him — a 'guru' he was in search of. Once again we are back to the Ekalavya syndrome.

In the story of the loyal Ekalavya, when the guru demanded his pound of flesh, Ekalavya was only too happy to oblige. After all, the 'guru' is entitled to his 'dakshina', the fee. Like Ekalavya, Bose had a kind of unquestioning devotion to Einstein. But it went much beyond the master-disciple relationship. No matter how Bose might have regarded Einstein, the 'master' certainly considered him to be his equal, a 'colleague'.

A clue to Bose's attitude can be found in the articles he wrote on Einstein — they appear in this volume. In them he pays tribute to the man, the champion of the oppressed Jewish race who was forced to leave his own country. In him and in the Jewish situation Bose found a parallel with the Indians under the British :

> Afterwards, both India and Israel became independent states almost at the same time. Unfortunately, the Englishmen's divisive policy has sown the seeds of racial conflict in the two young States. Nobody can say what the final outcome will be.
>
> I left Germany in the middle of 1926. Then, anti-Semitism took a serious turn in Germany. Einstein had to leave his own land and take shelter in Princeton before the start of the Second World War. It was from there that he propagated his latest theory. He stood for world peace and he unambiguously prayed for it. His immortal words about Gandhi will remain indelible in our history.[19]

So it was to the other Einstein too that Bose paid his tribute — a kind soul hounded out of his country, a man who held aloft the ideals of peace and friendship. The situation

in Europe was deteriorating. For Bose the continued campaign against the Jews was a cause of worry because many of his German friends were Jews :

> Many scientists I knew had to leave Germany because of their Semitic blood. I was very friendly with Franck, Einstein, Born, Ewald, Szilard and Mark. A frenzy of Aryanism seized Germany. The Black Shirt volunteers took hold of the administration all over Germany. The wave crossed German frontiers — in 1938 Austria came under Nazi occupation. Letters stopped coming. All news of Europe was blacked out. Then the Second World War broke out.[20]

6

Back to Dhaka

1927-45

BOSE IN THE UNIVERSITY

While Bose was in Europe a vacancy was created in the Physics Department back home, following the departure of Professor Jenkins. The post was advertised. Some of Bose's friends advised him to apply and since he did not have a doctorate degree to obtain a testimonial from Einstein, which he obtained. Recommendations also came from Paul Langevin and Hermann Mark.[1]

16.3.26

The recent works of Mr S N Bose, especially his theory of radiation equilibrium, signify in my opinion an important and enduring progress of the physical theory. Also in personal discussion with Mr Bose, I have got the impression that he is a man of unusual gift and depth, from whom science has much to expect. He has also at his command an extensive knowledge and certain ability in our science. As university teacher he will certainly develop a successful and prosperous activity.

Sd. A. Einstein

Paris, the 26th April 1926
10 Rue Vauquelin

...que Francaise
re' Egalite' Fraternite'
Ecole Municipale de
Physique et de Chimie
Industrielles

Office of Director

I the undersigned, Paul Langevin, Professor at College de France, Director of Ecole de Physique et Chimie de Paris, certify that Mr Satyendranath Bose, Reader of Physics at the University of Dacca, has spent a year in Paris in 1924-25 and worked under my direction.

I have the highest esteem for the personal merit and the works of Mr Bose whose own researches have been pursued here and whose presence has contributed to increasing the scientific activity of our university. I am particularly happy that the initiative of the University of Dacca has permitted this useful stay of Mr Bose in Paris.

Sd. P. Langevin

Berlin, 9 May 1926

Mr Satyendranath Bose is at present conducting a physical experimental investigation about the refractive index of Roentgen rays in the Kaiser Wilhelm Institute in Berlin-Dahlem. His large and profound knowledge which stretches over the whole of physics as well as the wider territory of chemistry marks Mr Bose out prominently. His most valuable quality which makes him of inestimable value to a collaborator is his deep and clear insight in the fundamentals of our science, his wealth of fruitful ideas and his capacity to combine theoretically important questions with experimentally feasible tests in an exceedingly happy manner. From the presence of Mr Bose, our institute derives the greatest benefit all the more as he understands, in a masterly way, how to make the most difficult questions clear through discussion — a quality which seems to make Mr Bose eminently suitable specially to the profession of teaching.

Dr. Hermann Mark
Kaiser Wilhelm Institute
for Chemistry of Fibrous Materials
16 Fambay Road, Berlin-Dahlem

In spite of all these strong recommendations the selection committee offered the post to Professor D M Bose. In case of his refusal their second choice was S N Bose. It was only to be expected that the mantle of his great uncle would eventually fall on D M Bose. J C Bose had in fact groomed him to that end. They were close neighbours and in close contact as long as J C Bose was alive. And D M Bose was happily settled in his Ghose chair since 1914, and there seemed no apparent reason why he should suddenly make a clean break with the past and venture into a new set-up. Hence it seemed quite unlikely that he would wish to be considered for the job. The decision might have been clinched by the telegram sent by Arnold Sommerfeld, the external expert, who wrote:

> Both the candidates are capable. S N Bose is a famous theoretical physicist and D M Bose is an established experimentalist and theorist; perhaps the latter should be chosen.[2]

It is also quite possible that D M Bose's name was intentionally recommended, with the full knowledge that he would not accept in which case the offer would automatically come to the second choice, S N Bose.

On 27 September 1926 the post was offered to D M Bose. The Minutes of the Executive Council meeting held on 27 September 1926 read as follows:

> RESOLVED : That Dr. D.M. Bose , M.A., Ph.D., be appointed Professor and Head of the Department of Physics on a fixed salary of Rs. 1000 (Rupees One thousand) only per mensem for a period of two years in the first instance from the date on which he joins the appointment, subject to the terms of a contract to be approved by the Executive Council: and that failing Dr. D.M. Bose, Mr. S.N. Bose, M.Sc. be appointed to above post on the same terms and conditions.[3]

This was another setback for S N Bose, who, on rejoining the University on his return, refused to take over as acting head of the department. Dr R N Ghosh, who was officiating since Jenkins left, continued as the acting head.

Finally, when D M Bose declined the offer, S N Bose was appointed Professor and head of the department. The same year he also became the Dean of Science. There was an ironic sequel to the episode. When C V Raman left Calcutta, S N Bose was a member of the selection committee for the Palit Chair that chose D M Bose for the post.

Two significant events had taken place in Dhaka while Bose was away.

In January 1926 Rabindranath Tagore visited Dhaka, where he delivered two public lectures at the Curzon Hall, on 'The Meaning of Art' and 'The Big and the Complex'. He also spoke to the students in their Halls. Tagore was accompanied by two professors from the University of Rome — Carlo Formichi and Giuseppe Tucci, who also gave talks; Formichi on 'Meditative and Active India' and Tucci on 'The Idealistic School in Buddhism'.

Hartog had retired in January 1926; G H Langley succeeding him as Vice-Chancellor. In recognition of his service Hartog was knighted before his departure. The absence of Professor Jenkins was also felt because besides being the Head and Dean he had been President of the University Athletic Club, the first President of the University Students' Union and also the Provost of the Dacca Hall.

Bose's two-year stay in Europe had been in many ways a turning point in his career as a teacher. His long visits to some of the most modern universities and laboratories had opened his eyes to the increasingly important role of science. Shaped in the formative period of his life by a spirit of nationalism, he had chosen science as a vocation in the hope that it would give him an opportunity of doing some good. Initially, he had only a vague idea of what that good could be. Now his ideas had crystallized; it was his mission now to introduce the latest developments in science — both in physics and in chemistry — to the university back home. That is what engaged

all his attention. During his student days J C Bose had been an inspiring model. Following on his instance S N Bose now tried making his own apparatus. In the words of S D Chatterjee :

> Just as he insisted upon deriving mathematical equations himself, rather than relying upon the literature (indeed he disdained to 'look things up'), he likewise insisted upon designing his own experimental equipment. Returning to Dacca, he launched a drive into experimental physics.[4]

After coming back Bose began to reorganize the Physics Department, trying to make use of his experience in the European laboratories he had just visited. His first communication however was in chemistry in the German journal *Zeitschrift für Physikalische Chemic* in 1927 on 'Messungen der Zersetzungsspannung in nicht Wässerigen Lösungsmitteln' jointly with Sushilchandra Biswas. Two years later his paper on the beryllium spectrum jointly with S K Mukherjee was published in the *Philosophical Magazine* in 1929. These two papers are enough to confute the common misconception that there was a lull in Bose's creative output from 1924 to 1936.

In 1928 Arnold Sommerfeld came to India. His first visit was to Bangalore from where he was to visit Dhaka on Bose's invitation. But unfortunately he fell ill and could not come to Dhaka. In a letter to the Vice-Chancellor he wrote :[5]

> *Dear Sir,*
>
> *I thank you sincerely for your kind invitation to the University of Dacca. It would have been a great pleasure for me to deliver lectures at your University and to meet your staff. Unfortunately I lost 14 days in the Hospital at Bangalore with fever. So I am obliged to shorten my engagements and to renounce the visit of Dacca as well as on the visit of Lahore, Jaipur, Bombay. On the other hand it would be a great pleasure for me indeed if Professor S N Bose could decide to see [me] at Calcutta where I shall stay from the 4th to October 13th. . .*

Bose went down to Calcutta to meet Sommerfeld. During the visit of Sommerfeld C V Raman arranged a grand function at the Indian Association for the Cultivation of Science where he gave his talk on the discovery of the new radiation. On this occasion S N Bose met Raman and saw his spectra and is supposed to have commented: 'Professor Raman, you have made a great discovery. It will be called the Raman Effect, and you will get the Nobel Prize.'[6]

In 1929 Bose presided over the Physics and Mathematics Section of the Indian Science Congress held at Madras. C V Raman, who was yet to get the Nobel Prize, was the General President. Young Chandrasekhar who was then only nineteen read a paper. It was a session that could be described as a conjunction of present and future stars.

It was in the the idyllic atmosphere of Dhaka that Bose's versatile genius began to blossom. He did not confine himself to the subject of his own specialization, mathematical physics, but encouraged his colleagues and students to undertake experimental work also and helped them with new ideas both in theory and in experiment. During his stay at Dhaka the Physics Department developed special facilities for research work in X-ray spectroscopy, X-ray diffraction, magnetic properties of matter, optical spectroscopy including Raman spectra, wireless etc. The scientists who came to Dacca

University to work as his colleagues were to make very important contributions later.

His method of working too was quite unconventional. Pratul Rakshit who was carrying out research under Dr Jnanchandra Ghosh in the early thirties tells us in his memoirs how any academic would bring his problems to Bose who would solve them effortlessly. He recalls an FRS and scholar of botany discussing his research topics with him. Bose was also a widely recognized resource person in organic chemistry.

To quote Rakshit :[7]

> Let me recall a personal experience. It was 1935-36, when I was working on the nature of the molecules of ascorbic acid (Vitamin C). Dr Jnan Ghosh was my supervisor. The results of several of my experiments could not be explained. Dr Jnan Ghosh was anxiously advising further experiments. One evening, Acharya Satyendranath was walking past our laboratory. Suddenly he asked me, 'What are you working on?'
>
> I described my experiments to him and their strange results. He heard me attentively and left. On reaching the laboratory the next morning around seven, I found him pacing down the corridor. The laboratory was locked. 'Why are you so late?' he complained. I unlocked the room. He went in and took his seat beside a small table. I was quite embarrassed. 'Show me your laboratory notebook,' he said. He went through the experiments, questioned me on the finer details and read the notebook as well. He was interested in my work. I felt honoured.
>
> Then onwards, he would come to my room every morning and leave for his home only in the evening. He used to go through every detail of each of my experiments. Often in the afternoon he would tell me, 'Why don't you go home and have lunch, I will keep an eye on your experiment'– as if he was a classmate, He would leave my room only to attend a class. His constant presence encouraged me to put in more effort. Occasionally he would suggest new experiments. My Professor, Dr Jnan Ghosh would also come once a day. These four or five months I really had the chance to learn so much. Then one morning Satyendranath analysed my results in a sheet of paper, and advised a few more experiments, indicating at once the results I should expect. 'Show this plan to your supervisor,' he said and left my room. That was the last time he spoke on this matter. He never came to my room again, once he had solved the problem.

In this connection Dr Rakshit quotes K S Krishnan (who was a Reader at Dhaka then),[8] in personal conversation, 'Dr Bose finds his pleasure in complex problems. His enthusiasm dies once he solves them. He throws the proofs in the waste-paper basket, never bothering to send them to any journal.'

Krishnan was one of the outstanding scientists to have joined the Physics Department. He had started his research career at the Indian Association for the Cultivation of Science, Calcutta, under Sir C V Raman , collaborating with Raman in his celebrated work on the new scattering of light, later named 'Raman Scattering'. Later Krishnan

joined Dacca University in 1929 as a Reader in Physics and stayed there till 1933. While he was at the Indian Association, Raman and Krishnan's theory of magnetic birefringence led to a method of determining molecular magnetic anisotropies, and Krishnan's interest in the subject continued and grew. At Dhaka Krishnan got the full benefit of Satyendranath's encyclopaedic knowledge, and found a congenial atmosphere for developing his own line of research.

Among the other physicists at Dhaka was Kedareswar Banerjee who joined the University as a Reader in Physics in 1933 and stayed on till 1943. Banerjee, a product of the Calcutta University, had his early research training under C V Raman. His original work on the diffraction of X-rays by liquids was useful in the analysis of crystal structures. He visited several renowned European laboratories in 1931. His stay at Dhaka was beneficial both to him and to Bose. The X-ray laboratory of Dhaka soon became one of the finest of its kind in India. There were many research scholars in the Department who made important contributions like R K Sen, Abdul Matin Chaudhuri (who later became Vice-Chancellor of the Dacca University), S Sen, S B Bhattacharyya and C R Bose.

Then there was Satisranjan Khastgir who joined Dacca in 1931, and succeeded Bose as Professor when Bose left.

In his memoirs Khastgir recalls the incident which excited Satyendranath to publish a paper in 1938 'On the Total Reflection of Electromagnetic Waves in the Ionosphere'. In the words of Khastgir' :[9]

> Professor Saha had once come to Dhaka from Allahabad. He gave a lecture in the Physics Department. He addressed a huge gathering at the Curzon Hall. Saha spoke on the problems relating to the reflection of radio-waves from the ionosphere on which he was then working. He asked his friend Bose to work out a solution for an intricate problem like this. Appleton had given three conditions for the reflection of radio waves, Saha introduced a fourth one, based on the hypothesis that there is no absorption of radio waves in the ionosphere. But Saha knew himself that the assumption was arbitrary. So he requested Professor Bose in the open meeting to give a general solution to the reflection problem. After this lecture Satyendranath concentrated on the problem and ultimately succeeded in finding a general solution.

Bose used to invite eminent scientists like C V Raman, D M Bose, Meghnad Saha, Sisirkumar Mitra and B B Roy as external examiners for the M Sc practical and viva-voce examinations, invariably followed by long tea sessions and discussions in Bose's room. Later in the evening there would be lectures by these distinguished scholars. All these helped to create an atmosphere of learning and scientific attitude in the campus.

The Physics Department was however not quite big. Jnan Ghosh had expanded the Chemistry Department in many directions. Ghosh incidentally was the President of the Chemistry Section of the Science Congress session at Benaras. Ghosh was Provost of the Dacca Hall in 1925, and held the position till he left Dhaka. S N Bose succeeded

him as Provost when Ghosh left Dhaka in 1939. Vice-Chancellor R C Majumdar paid glowing tributes to Ghosh at his farewell.[10]

The academic atmosphere at Dhaka was informal and there were no boundaries between different subjects. Particularly the close liaison between the Physics and Chemistry departments stemmed largely from the two professors who were not only good friends but shared the same liberal outlook, openness, devotion to students and love of knowledge for its own sake. It was an atmosphere that suited Satyendranath. Though chemistry was not his first love, he was genuinely interested in the subject. Perhaps the seed had been sown very early by his father (the founder of the Indian Chemical and Pharmaceutical works), nurtured by P C Ray's class experiments, and enriched by his experience in Paris and Berlin. Right from his return to Dhaka he had started guiding students in chemistry and published papers in chemistry. He used to get involved in chemistry experiments, sometimes to help out the students, sometimes to satisfy his own curiosity. One such incident has been beautifully described by Pratul Rakshit,[11] then a student of J C Ghosh:

> Let me relate a personal experience to illustrate how close we could come to him. It was a May afternoon in 1935. The sun scorched the surroundings. On the way to Ramna, the gulmohar trees were aflame with the flowers. The University classes were suspended for the summer holidays. But work continued full-steam in the laboratories. This was the best time for research, with our teachers unencumbered enough to spare us most of their time. I was working at the room to the west of the staircase leading to the first floor of the laboratory. At the other table, my friend Bipin Kar was measuring hydrogen peroxide. In the dark cubicle close by my classmate Tirumalli Ramachar was busy working with a mercury lamp.
>
> — 'What are you doing?'
>
> I turned around on hearing the familiar voice. Satyendranath must have been standing there for quite some time. He was dressed in his vest and simple sandals. Like always he wore a smile. He was holding a flask, with a colourless liquid which looked like water. 'I am told that you are quite an expert in bromination,' he said, 'could you brominate this fluid for me?' I took the flask promptly from him and said, 'I'll do it right now.' 'No, not now,' he stopped me. 'Keep it with you. I will come in the evening. Then we can have the quiet. Not many will be using the laboratory then. We will use the laboratory at the other end. I will be with you,' and he left.
>
> It was getting dark. The laboratories were being locked up for the night one by one. Dr Ghosh had left for home. Here and there, a couple of researchers were working. Professor Bose arrived. Taking the flask, I accompanied him to the big laboratory on the eastern end, used for the postgraduate classes. I arranged the necessary equipment. The required amount of bromine was measured and poured in the funnel with great care. The bromine had to be mixed drop by drop with that fluid.

Satyendranath stood beside me. As I began the bromination, I told him, 'Sir, it will take two hours. Please go home.'

There was a hint of gentle amusement in his smile as he said, 'No, I will stay too.' Those who have known him would know that this smile of his had a touch of mystery about it.

He sat with a few chemistry periodicals in Dr Chowdhury's cubicle, at the corner of the laboratory.

The clock went ticking away. I went on mixing the bromine drop by drop. The flask was getting hot. I was cooling it under the tap and again mixing the bromine. The laboratory was deserted. The lights flashed above. Outside, one could see through the large windows, it was getting dark between the jackfruit and lichi trees. Soon after I had begun the experiment, I was having an increasingly burning sensation in my eyes. When an hour had passed, tears were pouring down. 'Sir' came up to me. Without looking at me, he asked, 'It's hurting, isn't it? Your eyes must be burning.' 'Seems as if something like tear gas has hit me,' I confessed.

He smiled and asked, 'Are you sure you will be able to complete it?'

He returned to the cubicle. I continued till I had ten per cent of the bromination left; by then my eyes were in a terrible state. I could hardly see anything through the tears. 'Sir' had meanwhile come to observe me a couple of times. This time he came and told me, 'It's almost done. Let me do the rest. Why don't you go out and have some fresh air?'

He almost forced me to give up and held the funnel. I objected but by then I was almost blinded, and had no way but to let him take over.

At the very end of the laboratory there was a big wash-basin. I opened the tap and splashed water on my face. The spray felt like a sharp stab. 'Oh', I cried out in pain and collapsed on the floor. For a few seconds I must have been senseless.

Haren, who needs no introduction to the chemistry students of Dacca University, was in charge of the keys of the laboratory. Such an experienced and skilful laboratory assistant as Haren is rare. He must have been hovering around waiting for us to finish. As I collapsed on the floor, he rushed to me. On a table nearby, there stood large bottles of distilled water. Promptly, he poured a whole bottle of water on my head. Professor Bose too had by then rushed to my side. With his long experience , Haren had guessed immediately what had happened to me. A volatile gas produced by the bromination had entered me through the pores of my skin, and the splash of water had only plunged it deeper.As Haren poured the water, the chemicals were washed away and I was relieved. I was thoroughly drenched by then. 'It's almost over. I will need another couple of minutes. I will be coming in a moment,' said Professor Bose and returned to the experiment. In a little

while the bromination was completed. As he took off his heavy spectacles to splash water on his eyes, I tried to stop him.

'You need not worry. I have been at it only for five to seven minutes,' he assured me and splashed water on his eyes. He too felt the stab of the pain though it was not as intense as mine. A cry of pain escaped him. Naturally Haren poured water on his face too. Professor Bose got quite soaked in the process. We both came out and sat on the green lawn. It was a dark night. The sky above had twinkling stars. We were soaking wet. We looked quite a funny sight.

Myself, an insignificant student, and Satyendranath, one of Einstein's peers, sat side by side. He was simple, saintly, affectionate, open and generous. When I recall the incident, it seems so strange. How fortunate I was to be in close touch with such great personalities.

It was getting late. I told him, 'Sir, let me accompany you home on my way back.' He objected, 'No, you come to my place. How can you go home in this drenched state?' We went to his place. I took a warm bath and a sumptuous dinner at his home. I reached home much later in the night.

During his twenty-five years in Dhaka S N Bose kept away from meetings or student functions in the university. But when he was Provost of the Dacca Hall he had to preside over all cultural and other events of the Hall. One of his students, Arunkumar Dasgupta, remembers an occasion when R C Majumdar, then the Vice-Chancellor dragged Bose with him to the Dacca Hall aquatic sports. 'He said, "Look who is here." We all stared at the dhoti and kurta clad Satyendranath. He was smiling. Instantly he became the centre of attraction. But he did not stay for long — left quietly.'[12]

This is in marked contrast to his activities in Calcutta. In fact, he was much sought after in Calcutta. He obliged whoever came to request him.

The years between 1926 and 1945 saw a lot of changes. The more peaceful Dhaka of the early twenties came to witness political disturbances, communal riots, famine, and the War. But to get to the background of the events one has to take a look back to the social situation of the early days, and the rôle of the University as an agent of change.

THE CHANGING SCENARIO

When the foundations of the university were laid, Dhaka was a 'sleepy little town' in the provincial backwaters of Bengal and part of the intention of starting a centre of learning here was to develop the area which was predominantly agricultural.

But unlike Calcutta, Dhaka did not owe its origin to the coming of the British. It had a rich and glorious past. In his Convocation speech Sir Jadunath Sarkar gave an excellent summary of the past history of Dhaka, its racial diversity, its spirit of adventure, its liberal ideas : [13]

In the far-off Hindu period, East Bengal was a centre of the highest Sanskrit learning; teachers and writers from these districts attained to supreme

eminence among the Hindus and Buddhists alike.

Due to its geographical position, this part of our province witnessed in the Muslim period a great mingling of races and cultures, probably unequalled by any other part of India. Those enormous arteries of inland navigation, the Ganges and the Brahmaputra, as well as the ocean highway, have met together here, and poured into this land the Mongoloids of the north and the east, Arabs, Turks, Afghans, Persians and Abyssinians from the Islamic west, Punjabi Khatris, Hindustani writers, Rajput warriors and Portuguese traders and pirates, many of whom have taken root in this soil. In consequence of this, Dacca, like the ports of Athens and Alexandria, has enjoyed a richly diversified life, which has developed a remarkable openness of its people's mind to light. In the present age the sons of East Bengal have set an example to the other people of Bengal by their readiness to receive new ideas, their forward-looking spirit which breaks through age-old social conventions and blind traditions, and their power of readily adapting themselves to new environments.

However, the first economic survey done by Dr J C Sinha and others in the department of Economics in 1926-27 revealed a rather grim picture; 401 students were interviewed of whom 284 were Hindus and 117 Muslims. Dr Sinha took an expenditure of rupees nine per month for each member of the family as the margin of subsistence. The survey showed that 62.6 per cent of the Muslim students came from below that margin, 6.3 per cent from families on the margin and only 31.1 per cent from families above the margin. In contrast 69.2 per cent of the Hindu students came from families above this margin. What then must be inferred from this inquiry? In the words of the Vice-Chancellor, G H Langley:

It shows that a large percentage of the students who enter the Dacca University are faced with acute financial anxiety, and that the percentage of such students is exceedingly high in the case of Muhammadans. For many of these the anxiety must be so great as to make it impossible for them to take full advantage of the period they spend at the University. This period should be one of mental freedom. The student should be able to concentrate upon his studies and to throw himself joyfully into the corporate life of the institution. This is not possible for one who is conscious of being a burden upon other members of his family and who knows that he cannot raise his fees without incurring debts. From the facts above mentioned it is clear that in the Muslim Hall the University is faced with the problem of providing higher education for a community which is appallingly poor. Since the University was founded the Executive Council has done what it could from the resources at its disposal to supply this need. Although the number of students belonging to the Muslim Hall has always been less than one-third of the total number of University students, the amounts allotted annually in grants for stipends and scholarships to the Muslim Hall have

> been at least equal to the sum of the amounts allotted to the other two Halls for Hindu students.[14]

And he makes an appeal to the Muslim community to come forward to help, to raise funds for scholarships and stipends.

In a way education and progress were inter-related, particularly in the Muslim community. It is gratifying to see that already in the late twenties and thirties Muslim boys were coming forward to take advantage of the educational facilities. Vice-Chancellor Langley in his 1926 Convocation speech expresses satisfaction at the increase in the number of Muslim students : [15]

> One most encouraging feature of the progress of the University during the past session has been the continued increase in the number of resident Muhammadan students. As the result of this increase it was necessary to expand the accommodation of the Muslim Hall by the addition of the Ramna House. It is not possible to overestimate the importance to the province of the growing enthusiasm of the Muhammadan community in Bengal for higher education, which has been so strikingly demonstrated by the expansion of the Muslim Hall. In the session 1921-22, there were 87 resident students in the Hall and 83 attached students, while in the session 1925-26, there were no less than 170 resident students and 182 attached. This continued expansion has been maintained in the present session. Since the University was inaugurated in 1921, the excellent work that has been carried on among the Muslim students of the University under the guidance of Mr A F Rahman has been hampered by the lack of the separate building for a Muslim Hall. It is, therefore, with the greatest satisfaction that I am able to announce that the University has, with the approval of Government, decided to build a Muslim Hall to accommodate 300 students. This Hall is likely to meet the needs of the Muhammadan community for some years to come. Here Muhammadan students enjoying the privilege of higher education will be able to develop their own characteristic corporate life, in close proximity to the two Hindu Halls of the University, with the possibility of cultivating fellowship with Hindu students who are coming under similar influences, and of uniting with them in the classroom, on the athletic field, and in the activities of the University Students' Union of which they are all members.

The number of Muslim students increased from about 160 to over 400 between the years 1921 and 1930. This number rose to 595 in 1939. Of these 595, 364 were resident students. It seemed the authorities had underestimated the extent to which the Muslim population would take advantage of the opportunities offered by the University of Dacca.

As the Vice-Chancellor R C Majumdar rightly said in his 1939 Convocation Speech,

> The provision of university education for the Muslim students is an object of national importance and must not be looked at from a sectional point of

> view. Politicians of every shade of opinion have stressed the need for a proper understanding and growth of harmony and goodwill between the two great communities of Bengal. It is my firm conviction that nothing would facilitate this task better than the spread of higher education among the Muslims. For so long as the intellectual attainments and educational standard of the two communities are not brought, more or less, to the same level, there cannot be a solid foundation for that communal amity which we all have so much at heart.[16]

The numbers went on increasing and in 1941 the number of resident Muslim students rose to nearly 600.

Meanwhile the Hall spirit continued to flourish. One interesting example is the forming of the Old Boys' Association of Dacca Hall. To quote Langley, again in his Convocation Speech of 1928 : [17]

> Further, I wish to refer to the impressive gatherings of the Dacca College and Dacca Hall Old Boys' Association which were held in April last. This Association was brought into being — largely through the initiative of Professor J C Ghosh, Provost of the Dacca Hall — in 1927, and it held its first annual meetings during the Easter of that year under the Presidency of Mr Basanta Kumar Bose, Chairman of the Calcutta High Court Vakils Association, an old boy of the Dacca College of 77 years of age. The second annual meetings were held in April of this year and Rai Shama Charan Roy Bahadur, of Mymensingh, who has reached his eighty-fifth year, presided. He delivered a most interesting address dwelling principally on his reminiscences of student life in the Dacca College of 60 years ago. The address was followed by musical and dramatic entertainments, and the proceedings terminated with a dinner in which nearly 600 old boys and others participated.

The Hindu Halls were originally run on caste lines. There were separate dining halls for Brahmans and non-Brahmans. But gradually the caste barriers disappeared. R C Majumdar refers to an occasion in his autobiography when M N Saha had come to Dhaka and was amazed to find Hindu boys of all castes sitting at the same dining table. When Saha was a student of Dacca College he was not allowed to drink from a glass because it would be contaminated by his touch. Saha was very impressed by the progress achieved in so short a time.

In his 1929 speech Langley refers to this improvement : [18]

> Another sign of the growing sense of a common life is the breaking down of caste in the Halls. In the Muslim Hall there has always been a common table where the students dine together, and, since the foundation of the University they have arranged an annual dinner to which they have generously invited many Hindu and European guests. Until recently such a gathering was not possible in either of the Hindu Halls. Now, however, caste-distinction in such matters as the taking of food and the performing of worship in

> common has practically disappeared in the Hindu Hall, despite the fact that students of the depressed classes — the so-called untouchables — are admitted. During the past session dinners were arranged in both the Dacca and Jagannath Halls at which I had the privilege of being present. These were attended by practically all the students and members of the staff of the respective Halls and many Muhammadan guests were present.

It was a vibrant life, with scholars and writers gracing the University with their stimulating presence. Tagore visited in 1926, Sarojini Naidu was invited as the chief guest at the Convocation of 1939, Sarvepalli Radhakrishnan in 1941. Honorary degrees were conferred on Rabindranath Tagore, Sir Abdur Rahim, Sir Jagadischandra Bose, Sir Prafullachandra Ray, Sir Jadunath Sarkar, Sir Muhammad Iqbal and Saratchandra Chatterjee. However, it could not remain an island of peace for long. The revolutionaries had always been active in East Bengal though their arena of operation was not confined to this province.

As Dacca University was completely financed by the government, and before Fazlur Rahman all its Vice-Chancellors had been British, the undercurrent of nationalist waves did not always reach the surface. But it could not remain cut off from the movements taking place in the rest of the country. In 1931 the first official reference was made to the political situation, when Jenkins said in his Convocation report : [19]

> During the past year, in common with all other academic institutions in India, the University has suffered somewhat from the disturbed political atmosphere. In the early part of the session vigorous picketing, although not necessitating the closing of the university, caused a fall in attendance of approximately 50 per cent. A little later an unfortunate incident led to the non-attendance of students at the University for seven days . . .

In the same speech he spoke of girl students 'obtaining a foothold.' In 1930 ten girls graduated, and 19 were resident in the Women's Hostel . In March 1936 there were 46 girls in a total student population of 1021.

In 1935 Anderson, the Chancellor, made a scathing attack on the revolutionaries, calling them perverted : [20]

> Love of the motherland is deep-seated and urgent in the Bengali race and so insistent is it that it has taken for many the perverted form of terrorism and for many more the almost equally perverted form of anarchy in the shape of non-cooperation and civil disobedience. Both these forms of imagined service to the country are merely destructive and worse than negative — a fact now realized by the immense majority of the patriotic sons and daughters of Bengal. Thank God the realization has not come too late and though great damage has been done to Bengal that damage is not irreparable.

Ironically, in the same year homage was being paid to the King Emperor and Queen Empress on the completion of twenty-five years of reign. 'The empire-wide celebrations and the spontaneous manifestations of enthusiasm have revealed the greatness of their

Majesties,' said A F Rahman, 'and also that the throne is firmly rooted in the hearts of their subjects.'[21]

Hints of coming political changes were given by K Nazimuddin in 1934: 'We in Bengal are on the threshold of great political changes; the administration of the country will be placed in our hands. The students of today will be the administrators and legislators of tomorrow.'[22]

But the great non-cooperation movement of Gandhi like the earlier revolutionary upsurges left the Muslim community completely cold. As National Professor Abdur Rajjak commented in conversation with Sardar Fazlul Karim, 'It was *their* struggle. It had nothing do with us.'[23]

Thus the rift continued to grow. The two communities did not see eye to eye. Communal riots broke out in 1930, the first spell in January, a second spell in May and June. But inside the campus no tension was felt. Langley's Convocation speech of 1930 is a clear indication of the amity that prevailed then : [24]

> Unfortunately, during the period of the past session Dacca has been disturbed with political and communal strife, but there is cause for genuine gratification that the life and work of the University were at no time seriously interrupted. Within the University, the relations between Hindus and Muhammadans — both among the members of the staff and students — have always been friendly, and it was possible to maintain this friendly relationship when communal tension was felt outside.
>
> When the communal disturbances took place in January last, the University was in session and about 80 students volunteered as special constables. They assembled at the Central Police Office at about 8 o'clock, and for three nights under the direction of police officers patrolled the town in groups containing an equal number of Hindus and Muhammadans. The work of these students was reported by the local authorities to Government, and in March I received a letter from the Chief Secretary conveying the thanks of His Excellency the Governor in Council to them for the assistance they had rendered. In May and June, when Dacca was again disturbed with serious communal riots, the University was in vacation and very few students were in residence in Dacca. On this occasion the Dacca Hall became a place of refuge for Hindu families from Kayettuli and other quarters. For two or three days from the 24th of May there were 700 refugees in the Hall and many of them remained for almost a fortnight. The students in residence placed all available accommodation in the Hall at the disposal of the temporary inhabitants, gave them food until such time as they were able to make their own arrangements, and assisted them in other ways.

But as riots became more frequent in the city campus life could no longer remain unaffected. The students were congratulated for being 'less distracted by conditions outside the university,' but it could not be so for very long. There were unfortunate incidents and gradually campus life was vitiated by interested parties who wanted to

use the students for political gain. In 1941 it was no longer an oasis of peace:

> Unfortunately, the normal academic life of the University was rudely disturbed by the communal disturbances which broke out in the city towards the middle of March.
>
> The continued disturbances in the city and the utter dislocation of its normal life and activities also compelled the University to cancel its invitation to the Indian Science Congress to hold its next session at Dacca. The decision, arrived at with great reluctance and regret, has unfortunately been fully justified by subsequent events.
>
> When the University reopened after an interval of two months signs were reassuring and we resumed our normal work with high hopes. Unfortunately the orgies of riot and murder broke out again in full fury before two weeks were over, and once more the work of the University had to be suspended for more than two weeks. This recrudescence of disturbances was specially unfortunate for the University, as it was the time for the new admissions and the students were naturally scared away from Dacca. There can be hardly any doubt that the prolonged and repeated communal clashes have dealt a severe blow to the future growth of this University, and it is difficult to say how long it will take the University to recover from this terrible setback.[25]

In 1943 there were free fights between two sections of students resulting in injuries, one proving to be fatal. They seemed to be 'drifting towards hate.'

In 1938, when Bose had an offer from Calcutta, his reply to Syamaprasad Mookerjee shows him not quite ready to accept it. Though he puts forth a number of conditions, the hesitation to accept the offer is only too apparent: [26]

Physics Laboratory
Dacca University
Dated, Ramna,
the 28th January 1938

My dear Mr. Mookerjee,

Thank you very much for your kind enquiry. I hope you will understand how difficult it is for me to make up my mind finally in the matter, especially as I do not as yet know the conditions under which the post may be offered. I hold a permanent appointment as the head of the Department of Physics here, and I feel it will not be wise for me to go in for change at this age, and accept any offer which does not mean the same security of tenure, and at least the same facilities for work. At the same time the prospect of being able to be of some service to my own Alma Mater has a great attraction for me. If she thinks my presence in Calcutta will be of some use to her, I feel it will be difficult for me to resist her demands. If it be possible for the Governing Body of the Palit Trust to offer me the same salary as I am drawing now, with an additional house allowance of say about Rs 150/- I may accept the offer. I mention the additional house allowance, as I fear Calcutta will be a very much more expensive place to live in than Dacca.

I hope this will help you in arriving at a definite decision. With kindest regards,
I remain
Yours sincerely
S.N.Bose

Then the war came — and things went from bad to worse. The Chancellor's speech in 1940 referred to 'the grave international events which overshadow your lives and more. . . The present war which started in Poland and spread through Scandinavia and the low countries to France and Britain, may spread even further. Even if hostilities do not move further East, the outcome of this struggle will profoundly affect the progress of mankind for decades to come.'[27]

There are references to the 'unusual' world situation in all the subsequent speeches.The local scene however was made murkier by communal conflicts in Dhaka and even within the University. But the first direct reference to the effect of the war on campus life was made by the Vice-Chancellor M Hasan in his speech of 1944:[28]

> The normal work and life of the University of Dacca were greatly hampered during this period under review by the disturbed conditions of Bengal in general and the conditions in Dacca in particular. The University occupies the unenviable position of being situated in an area which is nearest to the scene of gigantic military operations on the eastern front and, therefore, it has to face great difficulties of a peculiar nature. Gradually its buildings have been taken away for other urgent non-academic uses so that it finds it extremely difficult to carry on its normal activities at the present time. This (along with other factors) has greatly affected the number of students which this season is the lowest on record. This has meant a substantial decrease in the income of the University, while its expenditure has increased by leaps and bounds in other directions. . . Many of our young and promising teachers are forced to leave the University to take up employment elsewhere.

In the war years, the Americans had established an air base near Dhaka which also had by then a number of hospitals for the treatment of British, American and Indian soldiers. Scientists from different foreign countries were also coming in. In one of his memoiral pieces, Bose recalls his encounter with a Chinese scientist who was studying processes of making vitamin tablets in different countries in the hope of finding a way to remedy the acute vitamin shortage that endangered the lives of children in war-battered China.[29]

The effects of the War lingered long after the actual hostilities had come to an end. In his 1948 Convocation Address Vice-Chancellor M Hasan spelt out some of the continuing difficulties :

> The last war brought thus a new type of difficulty; East Bengal was practically in the war zone, and the University had to part with the major portion of its accommodation in order to meet the sudden and pressing demands created by the war. It was hoped that with the end of the war the

> University would not only get back its own buildings but that new buildings would be constructed to satisfy its growing needs. Plans were made and sanctioned and even bricks were collected for some of the buildings when the partition of Bengal created new problems.[30]

Meanwhile riots had become a common occurrence in Dhaka, and a source of great mental agony for Bose. On one occasion trouble broke out at Raipura near Dhaka and took quite a bad turn. The situation could be brought under control only by the intervention of both the police and the army. Some of the students formed a relief team and worked in the troubled areas. Satyendranath as Provost of Dacca Hall was there to supervise the social service programme undertaken by the boys. He would move around with the students, going from village to village, eating whatever meagre meal they were having. Naturally his presence was quite inspiring.

Earlier he had declined the offer of a position at Calcutta University, made by Syamaprasad Mookerjee, then Vice-Chancellor, and son of Sir Asutosh to whom Bose owed a special debt of gratitude. But now, when conditions were getting more and more disturbed in Dhaka, the death of B B Roy in Calcutta left the prestigious Khaira Chair vacant. And when this post was offered to Bose, he was quite willing to accept it. The Dacca University was however reluctant to lose him. The Court of the University at a special meeting decided to offer to extend his year of retirement and a special salary of Rupees fifteen hundred a month. But Bose had already made up his mind.

7

Back in Calcutta

1945-1956

The 1905 partition had unleashed a mass of forces that slowly moved towards an inexorable climax. The partition led to widespread protests, compelling the Raj to backtrack for a while, till there was a forward move with the much publicized Imperial concession, viz. the foundation of a new University. Then events took their natural course.

The Dacca University, in the twenty-five years between S N Bose's entry and exit, had contributed in its way to the development of a Muslim middle class which did not exist before.

> The Muslim students of the University of Dacca who were either residents of the Muslim Hall or were attached to it, belonged to the corporate life of a compact body of students. They were imbued with a spirit of Muslim cultural tradition. The educated Muslim youths formed the middle class of the Muslim society. They were conscious of their cultural tradition and political rights, and aspired to have a rightful place in the political life of the country.[1]

More and more they identified themselves with the objective of the Muslim League and the Lahore resolution of 1940, demanding the creation of an independent Muslim state.

Bose's departure from Dhaka was thus inevitable, part of the larger process through which the sub-continent was heading towards a sharp polarization. But the University authorities were genuinely sorry to see him go. In his 1946 convocation speech the Vice-Chancellor, M Hasan, referred to this loss in the following words:

> I must, however, refer to the great loss this University has suffered owing to the departure of Professor S N Bose who was Professor and Head of the Department of Physics, Provost of Dacca Hall and a former Dean of the Faculty of Science. He is one of the most distinguished physicists in the world, and his departure from the university is an irreparable loss. The university owes a great debt to him for his manifold and distinguished services for which it cannot thank him adequately.[2]

Dacca University soon came to lose several luminaries. Some like M Hasan himself and Dr Mahmud Hussain, Professor of International Relations, left to serve the Government of Pakistan. Dr P Maheswari of the Department of Botany, 'a scientist of international repute, free from all political and religious bias, and truly loved by all,' also left. In 1949 the Vice-Chancellor S M Hossain had to admit ruefully that 'the departure of these senior members preceded by a large exodus of Professors, Readers and Senior Lecturers from almost every department created a critical situation for the University.'[3]

Meanwhile in Calcutta, a quarter of a century had made a lot of difference in the research scene. When Bose had left Calcutta the Physics department was yet in its infancy, struggling under acute financial pressure. Saha had to leave because he could not obtain the money needed for research. But in twenty-five years the research tradition had consolidated in specific fields. The Raman school flourished in the Indian Association for the Cultivation of Science whose facilities he was entitled to use as the Secretary of the Association. Professor Raman's researches in acoustics were carried out by his students at the University Science College campus. But the important area of the scattering of light which led to his discovery of the Raman effect and the award of the Nobel Prize were all done at the IACS. His able student K S Krishnan, who had co-authored all his papers on the subject, had left for Dhaka in 1928, where he had been a colleague of Bose. At Dhaka, Professor Krishnan had switched his interest to crystal magnetism. When he returned to IACS as Mahendralal Sircar Professor, he continued his pursuit in the subject. Many of his students from Dhaka followed him to Calcutta. His work was rewarded with a Fellowship of the Royal Society. However, he had moved further west to Allahabad in 1938; so that when S N Bose was back in Calcutta Krishnan was not there. But his place at IACS was taken by another Dhaka colleague of S N Bose, Dr Kedareswar Banerjee.

After C V Raman's departure Dr D M Bose had succeeded him to the Palit Chair. Professor S K Mitra had become Ghose Professor, and Dr B B Roy, who had been a lecturer, was appointed to the Khaira Chair. After working in the laboratories of Manne Siegbahn at Uppsala and Niels Bohr in Copenhagen, Professor Roy had developed a fine X-ray laboratory at Calcutta. However, it was his untimely death in 1944 that created the vacancy that Syamaprasad Mookerjee took the initiative to fill by bringing Professor Bose down from Dhaka. All the chairs — Palit, Ghose, and Khaira — now carried the same salary status.

Work on the Raman spectra was continued in the Palit Laboratory through the efforts of Dr S C Sirkar. Professor Saha who had come back in 1938 had in the meantime

changed his line of research and concentrated on nuclear physics. When Bose returned to Calcutta B D Nag Chaudhuri was setting up the cyclotron and Dr N N Dasgupta was laying the foundation of bio-physical studies.

There were three professors in the department then — Bose himself, Meghnad Saha, and S K Mitra; and four lecturers — J C Mukherjee, B N Chuckerbutti, D Banerjee and S C Sirkar. S K Acharya, another very old colleague of Professor Bose, was the Special Officer for the Council of Post-Graduate Teaching in Science and Technology. The annual research grant was a meagre Rs 2500 for each Professor and Rs 1000 for lecturers. Any additional support for research had to be channelled through the different schemes financed by the Board of Scientific and Industrial Research and other bodies.

In 1945 the faculty of the University College of Science and Technology could boast of celebrities in almost all departments. The Pure Chemistry department had Professor Prafullachandra Mitter, a younger and close associate of Acharya P C Ray, as Palit Professor; Jnanendranath Mukherjee as Ghose Professor, Priyadaranjan Ray as Khaira Professor and other distinguished chemists like Dr P B Sarkar, Dr J C Bardhan and Dr B N Ghosh.

Dr Bireshchandra Guha was Ghose Professor and Head of Applied Chemistry; the Applied Mathematics department was headed by Dr Nikhilranjan Sen, classmate of Satyendranath. Dr P N Ghose was Head of the Applied Physics department.

It was a happy homecoming and the faculty and the students of the University College of Science and Technology were delighted to have Bose back to the fold. The Khaira Laboratory at that time had two senior research students, Harshanarayan Bose and Kamalakshya Das Gupta. H N Bose later joined the Physics department of the IIT, Kharagpur, and established his school of research in solid state physics; Kamalakshya Das Gupta, known for his experiments on X-ray scattering, later moved to the University of Austin at Texas. Sivabrata Bhattacharyya who had been a student of Professor Bose at Dhaka also joined the Khaira Laboratory in 1945. Bright young students who also flocked to the laboratory included Jadugopal Dutta, Aparesh Chatterjee, Jagadish Sharma, Purnima Sengupta, Biren Dutta, Amal Ghose, Leela Roy and many others. Besides those who joined formally as research students after they had completed their Masters, there were others too, like Asoke Bose of the Chemistry department, Nandadulal Sengupta from Saha's laboratory, Parimalkanti Ghosh, Mahadev Dutta, Ganbehari Banerjee, Purnangshu Roy, Tapen Roy, originally students of the Applied Mathematics Department, who joined the group.

S N Bose brought new life and an atmosphere of informality to the campus. None of the stalwarts like D M Bose, S K Mitra, C V Raman or even M N Saha were so easily accessible to the students. N D Sengupta refers to an occasion when 'a group of young students including myself belonging to different disciplines. . . approached him and sought his guidance to form a study circle. . . we decided to meet on every Saturday after 5 o'clock so that many of us who were not working in the Science College but in other laboratories could attend the study circle. . . none of us including myself, who attended regularly, were officially attached to him. Many of our friends at that time

were lecturers in undergraduate colleges. His doors were open to everybody.'[4]

Earlier, after Saha's return, there used to be a weekly Physics-Mathematical colloquium in the Physics department. It was open to all students though most of them could not follow the discussions between R C Majumdar and M N Saha.[5] But the style of the study circle of S N Bose was certainly less intimidating and the atmosphere quite informal. The Physics-Mathematical colloquium naturally did not continue for long.

Hardly had Bose settled down in his place of work when trouble struck. The entire country was engulfed by communal violence. The great Calcutta killing of August 1946 is now part of history. The University College of Science and Technology stood right in the midst of a very sensitive area which made it impossible for the student community to remain uninvolved. Though army personnel were housed within the campus, research activities did not come to a complete halt — but the situation continued to be abnormal for some time.

This was followed by the partition and transfer of power. Bengal and Punjab were divided. There was a large-scale exodus both ways. The continued influx of refugees upset the entire social fabric.

For an idealist like Satyendranath it amounted to a betrayal. He extended his help to anybody who appealed to him. Some people also tried to exploit his kindness. The long sojourn in Dhaka had made it his second home. The very idea that the part of Bengal where he had spent the best years of his life was now foreign territory was a source of agony. This is the point at which he came — to use a much abused cliché — 'out of the ivory tower' and began to get more involved in public issues. In marked contrast to his way of life in Dhaka, which was sheltered and academic in the purest sense, he now emerged in a different role. His frustration led him to a stronger awareness of his immediate social responsibilities. They could not be shirked any longer.

Independence opened new doors and opportunities.An infrastructure for scientific research and development was set up on a large scale for the whole country. The Council of Scientific and Industrial Research came into being, with S S Bhatnagar at its head.

All the important scientists of the country were invited to participate and act as advisers. The Calcutta School participated in a big way. M N Saha, S K Mitra, S N Bose, B C Guha, J N Mukherjee, D M Bose and many others contributed their share in shaping the future of science in India.

With time came other changes — the Physics family broke up, when Meghnad Saha's dream of a separate Institute of Nuclear Physics at last took shape. The foundation stone of the Institute was laid by Syamaprasad Mookerjee in 1948. Meghnad Saha became its Honorary Life Director while the Palit Laboratory of the Pure Physics department with its research grant was separated from the Physics department and given to the Institute of Nuclear Physics which was run by an autonomous Governing Body.

The Ghose Laboratory of the Pure Physics department and the Electrical Communication Laboratory of the Applied Physics department were integrated to form the Institute of Radiophysics and Electronics, which became one of the departments of the

Technology Faculty of the Calcutta University. Professor S K Mitra became the head of the Institute of Radiophysics and Electronics. Thus the Pure Physics department was left only with the Khaira Professorship. This situation continued for quite some time. After Professor Saha relinquished the Palit Professorship to become the Director of the newly re-constituted Indian Association for the Cultivation of Science, Dr B D Nag Chaudhuri succeeded to the Palit chair. After Saha's death in 1956, Dr B D Nag Chaudhuri assumed the directorship, and the post of Palit Professorship was returned to the Physics department in 1958.

Meanwhile S N Bose continued his experimental investigation in crystallography and radiation physics with a group of very young workers. New fields of research soon opened up in many directions including many applied aspects like studies in minerals and clays. If any problems aroused Bose's interest he would not rest till it was solved. His versatile mind now went forth to explore fresh areas. His methods were his very own, and often proved trying for his students.

Asoke Bose, one of his students, has made interesting observations about his unusual guide.

In his words:[6]

> My feeling is he was interested in experiments because of the challenge involved. He had hardly any interest in the results. 'How can one measure temperature?' he asked once. He did not believe in getting the standard thermocouple from abroad. He told us: 'Make your own nickel chromium alloy. Boil sulphur, fix the temperature.' The sulphur vessel blew up, and exploded and started burning. 'Well, change it then,' he said, 'use something else. Find out for yourself.' After about six or seven months I wrote to a friend in the United States, and brought the standard thermocouple, I made a furnace following a model. He asked me: 'Why is it done in this way?' I said, 'I don't know.' He said, 'You should have known.' I felt rebuked. But he never spoke a sharp word. It would have been a good thing for the country if there were more people like him. But one has to write papers, work for a living. If it was a country like China the method would be adopted as a national programme : 'Close all doors. Let the students learn and stay in this country.' Then it could have worked. But it would be futile if there is only one real teacher and the rest non-teachers. He felt that everything can be done. The countrymen should do everything for themselves, be it mathematics, or machines, or X-rays. If it is adopted by the state it is a sane plan — otherwise it is insanity.

Asoke Bose recalls another incident, from when he and Professor Bose had been working on a problem continually for two months from early morning till night.

> I happened to see a Dutch paper which gave me the idea that our work was worth publishing. I suggested it to him. He would not listen. Paper? What do you want a paper for! Have you understood it? Fine. Now you can throw it away. It was like a game — he was intense, but as soon as he saw the

> solution he had no use for it — he threw it away. It required intense thinking. I could sense what a genius he was — I do not know the definition of a genius. Right from the beginning it was a game to him. But I was proud of it — he wasn't. He never published anything of which he was not proud. But he had a child's curiosity. If he was interested in something he would go five hundred miles, knowing there would be no use for it. He did not mind wasting time. He was doing organic chemistry intensely but he knew that that was not his way of life.

Asoke Bose did his Ph D research under Heisenberg. This is how he compares his two outstanding teachers:

> You can't categorize Professor Bose, nor Heisenberg. The assistants of Heisenberg were very famous people, some of them future Nobel laureates, but Heisenberg was always the quickest to comprehend. Though I was his student I did not have the close contact which I used to have with Professor Bose. We did not meet so frequently and our relations were not on a personal level.Once I had some doubt about some work of Hamilton. Professor Bose immediately took me to the Presidency College library, brought out the original paper and explained it. Heisenberg obviously was more productive — but there can be no comparison. Professor Bose could see through the game of mathematics but Heisenberg was essentially a physicist. Their works were not made to order. Both were geniuses but Heisenberg's German school gave him a lot of cooperation. I think Professor Bose never worked with anyone. He never asked any of us to do the calculation; he used to do it himself. The American strategy is like this. Suppose there are five problems. I give a student exactly work for eighteen months — only I know what I want to publish. Professor Bose's way was not impersonal like this. On the other hand he could not build up students — the gap was infinite. Even Dirac did not have too many students — about Einstein the question did not arise. One day Professor Bose summoned me to his place. He asked me: 'Can you verify these calculations?' I did not know them, so I had to express my inability. He was dissatisfied. 'Oh, so you haven't learnt them.' Actually he wanted someone to verify them — he was very excited. But he did not get the cooperation he wanted. But I was a chemistry student, learning mathematics. I wish there was a good centre. I don't know if he ever got somebody later.
>
> One should not expect people like him to be doing routine work in one particular line. That would be insulting him. They are after the final truth, not caring for what can be achieved on the way. Everything is eternal. He was not interested in small matters — wanted to go to the depths. This only rich people can afford to do. He was an aristocrat.

From 1950 to 1956 there was a group of about ten to twelve young students working in the Khaira Laboratory under Bose. They were mostly working in experimental

research. But Bose was known for his interest in other subjects. His laboratory was also open to professors and students of applied mathematics — they came to discuss problems, sometimes to share ideas,or just to have an intellectually stimulating conversation. One did not need any introduction to walk in. Mrs Mahalanobis used to joke about this easy accessibility, comparing his living room to the five point crossing of Shyambazar.

Since the X-ray laboratory was not equipped for crystallography work, Bose with able assistance from his students constructed a number of precision cameras like a Weissenberg camera, a Differential Thermal Analyser, a micro-focus X-ray tube, and a special type powder camera for clay analysis. Soon the Khaira Laboratory developed into a well-equipped centre for crystal structure analysis. Clay minerals from different parts of the country were studied by X-ray diffraction methods to understand the molecular structure of common clay minerals. Thus Bose demonstrated how the application of X-ray analysis could promote chemical research — as a pioneer in the field.

His habit of taking nothing for granted, of working out everything and carrying on the pursuit of knowledge for its own sake often proved difficult for his students.

One of Bose's Ph D students, Jagadish Sharma, who was with him during this period (1947-54) has conveyed to Purnima Sinha (a student and biographer of Bose) the following experience:[7]

> He was tremendously interested in experiment. Anybody could work on theory, he used to say — perhaps because he himself was good at it. Sitting at his table he could clearly identify the problems — the possible areas of mistake — and would analyse the necessary steps for correction. He used to give a number of suggestions every day. He had a prolific mind. It would have been useful to follow his suggestions but it was not possible in the limited conditions of the laboratory to do all. One would have needed an entire institute for that. I used to select the ones which were possible in that situation. He never reprimanded me for not following many of his suggestions. That solution was important for me, otherwise I could not finish. It was not that I did not learn, but often too much material merely increased my confusion. He was not in favour of setting up a time limit for completing a thesis. His remark after my dissertation had been bound for final submission was: 'How are you going to incorporate changes?' Only a few days ago he had suggested some detailed work. My excuse for not doing it was the rainy season when the apparatus do not work properly. I had said that I would wait till the rains were over. So when he objected to the bound thesis I replied, 'Today is the last date.' 'Last date? What do you mean by the last date?' he was amused. A colleague nudged me and said, 'Beat it.' He did sign my thesis though. The idea of how to measure thermoluminescence, the plan and design of the apparatus — everything was shown by him step by step. And he did it so quickly. Nobody else had done a complete scanning device earlier. This had opened up avenues for research in rapid continuous

> scanning. He reported this work in 1954, at the crystallographic conference in Paris. Biren Dutta and Amal Ghose of our laboratory made names for themselves by their work using this device.
>
> It was a great pleasure working with him. He was like the flowing Ganga, you take whatever you can. In my career I have worked with many people — but I have yet to find someone like him. Those years were the best years of my life. Even insignificant problems were treated with care. Nothing was unimportant for him. Something useful might emerge — that was his attitude.
>
> The way he used to argue was: Start right from the fundamentals. Suppose Niels Bohr is dead, all his papers are lost. Don't refer to books — you try to figure out the entire argument, try to reason yourself.

Even in Dhaka Bose had an 'open house'. The tradition continued in Calcutta, perhaps in a more expansive fashion because the people who interacted with him here represented various disciplines. The canvas was also larger. A glimpse of this aspect of Bose's character can be had from the memoirs of one of his students, Gaganbehari Banerjee :[8]

> While Professor Bose was in Dacca Dr Pratul Rakshit and Dr Pulin Sarkar amongst many others often came to discuss chemistry with him. The discussions could have ranged from ascorbic acid to jute. Mrs Ashima Chatterjee, though always in Calcutta, was in touch with Professor Bose even when he was in Dacca. Once Professor Bose sent her from Dacca a compound she was finding impossible to get in Calcutta. In Calcutta Sahairam Bose was seen in Professor Bose's room discussing for hours. He used to say that he did not feel satisfied unless he could discuss his findings and problems with Professor Bose. Experimental physicists say that they received plenty of ideas from Professor Bose. He published a few papers on experimental physics. The instrument he constructed for the study of thermoluminescence is well known.

These were busy years for Bose. He became President of the Indian Physical Society (1945-48), President of the National Institute of Sciences (1948-50), was nominated a member of the Rajya Sabha and honoured with the title of Padma Bibhusan. A number of foreign scientists visited the Physics department. During the period 1953-1955 he published five papers in quick succession related to the Unified Field Theory. He corresponded with Einstein again and there was some disagreement.[9]

Bose was nominated a member of the Rajya Sabha, the upper chamber of the national parliament, by the President of India, along with some others eminent in their respective fields, like the actor Prithviraj Kapoor, the poet Maithilisharan Gupta, the dancer Rukmini Devi Arundale, and the scholars Kalidas Nag and Radhakumud Mookerjee. Bose was sworn in on 3 May 1954, and his resignation was formally announced on 10 August 1959.

Parliamentary politics however was not his forte and his august presence was hardly

felt in the debates and exchanges. The fact that he was chosen by the establishment may have cramped his style for he was by nature uncompromising and outspoken. His earlier association with the revolutionaries and his open frustration with a truncated independence did not make this new role a comfortable one. Significantly, two of his rare sustained speeches, both of them practically outbursts (on the refugee problem and the Preventive Detention Act 1950), came after Saha's death. It may be recalled that Saha, Bose's classmate and one time collaborator who had won a seat in the Lok Sabha, the lower chamber of parliament, for the same term, was particularly critical of the way the government was handling the East Bengal refugee problem. Saha died on 16 February 1956, and Bose's severest attack on the government came exactly a week later, on 24 February the same year. His third major speech was on the University Grants Commission Bill on 21 March 1955. All the three speeches are included in this volume.

He went on several tours, visiting the famous laboratories of Europe where he met many of his old friends and colleagues. The war had brought many changes. Bose has written a moving account of some of these meetings in his autobiographical writings, particularly in 'Otto Hahn in Memoriam' in Bengali.

In 1953 Bose was invited to attend the World Congress for General Disarmament and Peace at Budapest. Invitations came from Soviet Russia, Denmark and Czechoslovakia. On his way he also visited Geneva, Paris, Copenhagen, Zurich and Prague. He met Professor Pauli in Zurich and Niels Bohr in Copenhagen.

For some reason , however, he was never invited to the United States which was now home to many of his colleagues.

The following year Bose attended the International Crystallography Conference in Paris as a representative from India. Apart from scientific interests he had a special liking for France and the French way of life. The year 1955 saw him in Paris again, this time on an invitation from the Council of National Scientific Research of France. It was the fiftieth year of Einstein's formulation of the Theory of Relativity. It was in Berne that Einstein had promulgated the theory of Brownian movement, the photon hypothesis and the Special Theory of Relativity. Naturally Berne was the most suitable place to celebrate the golden jubilee of a discovery which changed the basic concepts of physics. The conference was scheduled to be held in July, and Einstein himself was expected to attend. Bose was looking forward to meeting him but unfortunately Einstein's death supervened, and the meeting never took place.

He went to England in 1956 to attend the Annual Meeting of the British Association for the Advancement of Science. Two years later he went again to attend the Royal Society meeting where he was elected a Fellow.

In 1962 he went to Sweden and from there to Moscow to attend the Peace Conference. In August 1962 he was invited to Japan to attend a seminar on science and philosophy, organized to commemorate the atomic explosion over Hiroshima and Nagasaki. This visit left a strong impression on his mind. He was particularly impressed by the Japanese method of teaching everything through the vernacular and subsequently he tried to follow the same model.

8

Bose, Tagore and Visva-Bharati

When Satyendranath was a young boy in junior school, Tagore began his famous experiments with education at Santiniketan on a piece of land bought by his father in the rough terrain of Birbhum. Gradually it grew to be the present Visva-Bharati; but already in Tagore's lifetime it had acquired a character which was not exactly what the founder had in mind.

Towards the end of his life Tagore was growing disillusioned. What shape would his institution take after him? Who would be the guiding force? These thoughts continued to trouble him. He even went to the length of saying that whatever he said was not gospel truth, thereby implying the freedom to differ.[1]

The Visva-Bharati was built on many pious ideals — it was not to be a degree peddling factory, it was to open its arms to the whole world, look upon all as its kinsmen. The whole world was of course a symbolic phrase — it did not mean a couple of sympathetic westerners, but free flow of knowledge — the ability to absorb new ideas. By the time Tagore died a kind of self-generating insularity was fast making Visva-Bharati all that the poet never wanted it to be — sunk in narrow rituals, empty paraphernalia without meaning. Little wonder then that when Satyendranath went there, full of new ideas and the urge to do something (as he was to tell his good friend, Annadasankar Ray, later) he was pursuing a mirage. Tagore's ideal university had ceased to exist long ago; what was left was only a shadow of its earlier self.

Had it been otherwise, perhaps the induction of a man like Satyendranath could have given it the much needed new lease of life.In one of his later day speeches Tagore spoke appreciatively of the spirit of Europe: 'it has found its true self-expression

through the light of science — because Science is Truth and Truth leads to Immortality.'[2] Perhaps he would have been the happiest to see a man who had contributed so much to the new science emerging from Europe to have come to this centre. Satyendranath was ideally equipped to take charge. He was well versed in the latest developments in science; he was versatile with his thorough knowledge of music, his passion for languages, his national spirit, his openness.

Historically one can date back Bose's acquaintance with Tagore to the time when as a young man Bose used to visit the house of the Tagores to attend the cultural sessions of the Vichitra Club, which owed its origin to two Tagore brothers, both painters.

> In 1915 Abanindranath retired from the Vice-Principalship after serving the Government School of Arts for ten years (1905-15). He threw himself heart and soul into the work of the Indian Society of Oriental Art, helped by his talented brother Gaganendranath. They began to add to their collection of books and prints, Eastern and Western art, and Rabindranath also cooperated with them. Thus the Vichitra School came to be organized (1914-15) with the principal aim of imparting direct education in art and literature to the ladies of the Tagore family and some of their friends. From the School gradually developed the Vichitra Club which functioned admirably for nearly five years (1914-19).[3]

Pratima Devi (1893-1969), painter and poet, and a niece of Abanindranath's was the soul of the Vichitra School and was the principal supporter of her husband Rathindranath when he was developing that club. Tagore also took active part. Kalidas Nag (1891-1966), scholar and thinker, recalls having sung songs as part of the choir in the Vichitra Club.

Bose used to attend these sessions as the friend of a family friend. Maniklal De was known to the Tagores and postcard invitations used to come to his Beadon Street address. They were all signed by Rathindranath, son of Rabindranath. The text, translated, usually was something like this: [4]

Vichitra
6, Dwarka Nath Tagore Lane
Calcutta

Dear Sir,

You are hereby invited to the Vichitra evening meet tomorrow, Wednesday, at 6 p.m.

Yours
[Signed] Rathindranath Tagore

There was another literary group meeting fairly regularly with Pramatha Chaudhury as its central figure. Chaudhury (1868-1946) was an important writer in his own right, and was married to Indira Devi Chaudhurani (1873-1960), a niece of Tagore's. The group published a periodical in Bengali — *Sabujpatra* — notable for its urbane wit and avowal of the common speech in preference to literary diction. Satyendranath was closely involved with this group, and his friendship with the Chaudhuris lasted till their deaths.

On the young and receptive mind of Bose, the Vichitra Club had left a very strong impression. Many years later, paying tribute to Soumyendranath Tagore (1901-74), Marxist ideologue, poet and musicologist, on the occasion of his seventieth birthday, he said :

> The history of the Tagore family remains an eternal wonder to the country. Abanindranath revived the Indian style of painting. Debendranath turned away from the ostentatious rituals of idolatry to devote himself to monotheism. It was he who rallied the dedicated younger members of the family to set flowing a surge of renaissance in literature and painting — the old myths of the country were recreated in Bengali — Rabindranath bridged the gap between the past and the present. We moved about in the periphery and came to know only later that Soumyendranath, sharing in all this, felt the urge to plunge into the freedom struggle. . . For some time I have had the opportunity of listening to his lectures at several cultural gatherings. I have often felt that this remarkable man has carried on the Tagore heritage. His words draw forth memories from the past, as I seem to hear Rabindranath's words in the Vichitra Club meetings, surfacing from the rich recesses of my mind.[5]

This is the only direct reference we have to the fact that Bose used to be present in the Vichitra meetings. Apparently this shy, handsome young man did not catch the eye of Rabindranath because Bose did not belong to the inner core, and he was never assertive. In his own words, he 'moved in the periphery'. It seems unlikely that he was ever introduced to Tagore, or even if Maniklal had done so it could not have left any impression on the poet, for whom he was just another young admirer, a bright student of the Calcutta University, with not much else to his credit. It is common knowledge that Tagore did not attach any importance to formal education, or even formal academic achievements.

Their next encounter could very well have been in Europe in 1924. When Bose was in Paris Tagore was touring Europe with his entourage. Bose was not unaware of it, but made no attempt to meet Rabindranath, though this time he was no longer 'an unknown Indian'. Bose generally shunned publicity, and was not interested in celebrities as such.

Their first meeting was actually at Dhaka. The circumstances had changed considerably. S N Bose was a name to reckon with. In Dhaka, Tagore was a guest of R C Majumdar (1888-1980), the eminent historian, then Head of the Department of History at the University, but expressed his desire to stay on a boat. A well-fitted barge was provided by the Nawab of Dhaka. Naturally Tagore had streams of visitors. Dr Majumdar in his Memoirs has given an amusing account of how the entire operation was conducted. The students used to run errands from the campus to the river front, while Dr Majumdar and his wife took their position near the landing to supervise. Majumdar's household was naturally left uncared for. The two good friends who came forward to help were S N Bose and J C Ghosh. During the time of Tagore's stay the Majumdar children were under the care of S N Bose and J C Ghosh. It is characteristic

of Bose that he preferred to keep in the background.

During the thirties Bose and Tagore came closer, when Bose visited Santiniketan a couple of times, and requested Tagore to contribute to the science journal *Vijnan Parichay*. and in his turn was asked by Tagore to attend the opening ceremony of the Cheena Bhavan, the Centre of Chinese Studies at Visva-Bharati. In March 1940 Tagore invited him to the spring festival at Santiniketan. Bose was only too happy to accept, but circumstances were not favourable. He could not go. In a letter to Tagore he wrote:[6]

9 March

I could not make it to Santiniketan in this vacation of 'dol'. I was looking forward to getting out of Dhaka this time but my plans were upset because both my son and daughter fell sick. Moreover the practical examinations will start from 17 March. I am supposed to look after the arrangements — that makes my leaving Dhaka the previous week all the more difficult. Perhaps it is not in my fate to watch Vasantotsava. I am feeling sorry for myself.

Do you have any plans for leaving Santiniketan in the near future? If you are there I may go down towards the end of April after my responsibilities are over. Perhaps I might even stay for three or four days. Since I have never had the opportunity of going round all the institutions I wish to do so now. I shall certainly visit if you are there.

I had promised you to come. I'm so embarrassed that I could not keep my word. Please excuse me. With respectful pranaam.

Tagore replied promptly :[7]

March 1940

You would have liked it this time — the celebrations were just about perfect. But it is no use crying over spilt milk.

Of course I'll be here in April. The main reason is that I am stationary, tied to my seat. I'll indeed be happy to have you. Something artificial may be done about the heat, but it may not be satisfactory.

The trees here, stricken by drought, are lifting their fingers to the cruel sky in vain. I am looking forward to the time when you people will bring down the Hitler-like stubbornness of the sky.

Dolpurnima *[signed] Rabindranath Tagore*
1347 [Bengali era]

Earlier in 1937 Tagore, in a rare gesture of affection, had dedicated his book on cosmology — his only work on a scientific subject — to Satyendranath. This book may have been written as an answer to the challenge of another scientist, Meghnad Saha. In his tribute to Tagore Saha had written :

> In him the art of speech has reached its perfection. But we, scientific men, beg to bring one complaint against him. His great powers have not yet been used in the exposition of science — one of the synoptic world pictures of today, of the beauties of the world explored by the physical, the biological and the anthropological sciences.
>
> May we not hope that, like his illustrious predecessor Goethe, he will turn for a while to modern science and give expression in his inimitable poetry

> to the hope behind the invading despair and the harmony behind the modern Babel of jarring voices?[8]

In his introduction Tagore acknowledges the work done by a student of Bose in preparing the text. In his own words:

> Sreeman Pramathanath Sengupta, M Sc, is an old student of yours. He teaches science in the Santiniketan school. Initially I had entrusted him with the job of writing this book. Gradually the responsibility shifted till it was totally on me. But I could not have finished unless he had started; besides as one uninitiated, I would not have dared to take to the unaccustomed course. I have received help as well as encouragement from him.[9]

It is evident that Sengupta had an important role at least in writing a major part of the first draft. So in all fairness Tagore decided to dedicate the book to Satyendranath. It is a common misconception that Tagore's dedication was a form of support of Bose's crusade for the mother tongue. Actually the crusade had not yet started. It was to gather momentum only after Bose's return to Calcutta in 1945. But Tagore could not have been unaware of Bose's beautifully written Bengali article on modern science in the journal *Parichay.*

But this dedication meant a great deal to Bose, still in the prime of life. Like Einstein's postcard it acted as a passport, it gave credibility to his mission for propagating science through the mother tongue. However, neither of them knew at this stage that Bose would be so closely involved in the affairs of Visva-Bharati.

In 1951 Visva-Bharati became a Central University with the Prime Minister as Chancellor. In January 1956 when the Upacharya Dr Prabodh Chandra Bagchi died, Bose was offered the post. Bagchi was an old friend of his Paris days. Bose meanwhile had retired from the Calcutta University. He joined Visva-Bharati in July 1956. In the interim Indira Devi Chaudhurani had acted as the Upacharya. She was an old acquaintance of Bose from the *Sabujpatra* days.

The news that Satyendranath was taking charge as the new Upacharya of Visva-Bharati appeared in all newspapers on 3 July 1956, and 'was welcomed by all sections of the public.' After Rabindranath this university had never had someone of such stature at the helm of affairs. Welcoming the new Upacharya the *Visva-Bharati News* wrote, 'In him we shall find the most wanted leadership for the growth of this institute.'

Bose lost no time in getting down to work. Before the end of the week he had met the Heads of Departments and other officials. He met the Karmi Mandali [the Staff Association] and the senior students. 'It has been very gratifying to hear him reiterate every time that the ideals and the special features of this Institution should continue to be integrated into a cooperative and harmonious fold for the fulfilment of the object of this Institution as a meeting ground of the East and West,' wrote the *Visva-Bharati News.*[10]

It seemed to be an ideal set-up. Satyendranath came with hopes of doing something here — as he was to tell Annadasankar Ray later. But Tagore's ideal proved to be elusive. After two years of vain struggle he was forced to confess in his farewell speech:

'I am reminded of the story of a man standing before a map of France, a lamp in hand, saying, "I am looking for France". I am exactly in the same position now.'

He had many new ideas but what he did not realize was that no matter how noble the purpose, they might go against the set and rigid way of life which over the years had grown almost into a ritual in Santiniketan. It seemed to him a waste that the glass-enclosed *mandir* (temple) should be used for prayer only one day of the week. He wanted to convert it into a reading room. But nothing could be changed; it had to be the way Gurudev had planned it. Bose was disappointed, but the idea was dropped all the same. The truth of the matter was that the set-up at Visva-Bharati did not take kindly to outsiders or to such changes as Bose proposed to bring about.

The very fact that he wanted to change things brought him into direct conflict with the Executive Council. He wanted to put things into order, to impose some sort of discipline on to the extremely casual manner in which classes were held. He wanted registers to be kept and regular attendance recorded. He was told that Gurudev did not want it. There would be no class if there was a shower of rain. This too was supposedly done according to the wishes of the Gurudev. At every step he was made to feel that he was an outsider, who did not understand the spirit of the place!

What annoyed the custodians of Visva-Bharati most was probably the shifting of the venue of the convocation from the mango grove to the Uttarayana, one of the buildings in which Tagore lived. Nehru, the Chancellor of the University, also expressed his mild displeasure in the following words: 'Let us not go further away from the grove, I do wish that we go back to the grove.'

For Satyendranath, however, such functional changes seemed no sacrilege. He was concerned with bringing about other improvements, which he in all sincerity thought to be in keeping with Tagore's line of thought. The indications were there in his convocation speech:

> It will not be out of place here to append a few comments on the need of introducing scientific studies of a fairly high level in the Visva-Bharati. It cannot be denied that a certain amount of essential scientific knowledge does enter into the mental make-up of a modern man of culture. Gurudev himself had recognized it, and had advocated its teaching in Visva-Bharati. It would be a pity if science is deliberately left out in the endeavour to bring about a synthesis of the various kinds of culture of different lands and races in an ambitious programme of giving a whole education to the modern man and woman.[11]

In 1956 Visva-Bharati offered science courses only up to the Intermediate level. Satyendranath drew up a scheme of science teaching and research up to the post-graduate level. He wanted to build up an Advanced Centre of Science here. This scheme was sent to Nehru who referred it to S R Das, then Chief Justice of the Supreme Court, and who was later to become Vice-Chancellor of Visva-Bharati. S R Das was not in favour of expanding the science stream. The main purpose of Visva-Bharati, according to him, was cultural exchange. Besides, setting up of science laboratories would be expensive. The matter was also referred to Leonard Elmhirst, an old friend of Visva-

Bharati, for his comments. He too discouraged the entire idea, adding that only subjects like soil science, botany and agriculture which were more in keeping with the overall atmosphere, may be introduced. So the scheme was nipped in the bud, although the Sur Industries had agreed to finance it. Surprisingly, all the ideas of Bose were implemented after S R Das took over, and eventually proved to be such a success that in the words of one of the Upacharyas, 'if Visva- Bharati can be proud of anything now, it is the Science faculty.'[12]

This was the only period of Bose's life when he had to function as a full-time administrator. By temperament he was a teacher, and it was unfortunate for the students of Visva-Bharati that they could not see Bose at his best. His house at Calcutta was affectionately called the five-point crossing, to which everybody had access;Bose could change his wave-length at will. But at Santiniketan he had fewer visitors. His friends, apart from those who came down from Calcutta, were Annadasankar and Lila Ray, Syed Mujtaba Ali, Sudhin Ghose and some foreign scholars. He had excellent relations with Pratima Devi and Indira Devi Chaudhurani. But his informality was often misunderstood by many. There was always an undercurrent of strain. He was often cross and irritable, which was totally contrary to his nature.

Nobody understood him better than Annadasankar. The memory of those humiliating days provoked him to write this piece thirty-five years later.

> Men around him did not understand him. He was nobody's partisan in Visva-Bharati affairs. Partisans of both camps made his life miserable. But he stood his ground and did not resign rather than court dismissal. One day I heard that he had been kicked upstairs by the Government of India as National Professor on Rs 2500 a month plus perks. He was to be his own master. He was drawing Rs 1500 as Vice-Chancellor with free quarters. I went to congratulate him on his luck. He was morose : 'Annada, they have driven me out', he moaned in reply to my greetings. I tried to console him. 'Now you will be able to go back to Calcutta and train some research scholars of your choice. You need not waste your precious time here where none is worth your strife.' He justified his preference for the Visva-Bharati appointment. 'I have been guiding research scholars all my life,'he said. 'I had some educational ideas which I wanted to work out as Vice-Chancellor.' He was frustrated. [13]

During this period more recognition and honour came his way. In 1957 three Universities — Calcutta, Jadavpur and Allahabad — gave him D Sc honoris causa; Calcutta on the occasion of its centenary celebrations. Honours which were long overdue followed one another in quick succession. In 1958, belatedly, he was elected Fellow of the Royal Society of London along with Professor S K Mitra. In 1959 he was appointed a National Professor, the highest honour which the nation could confer upon a scholar. In 1961 after he had left the place, Visva-Bharati conferred upon him the title of 'Desikottama', the University's equivalent of a Doctorate honoris causa.

During his tenure an important event was the visit of the Chinese Premier Chou En-Lai, on 30 January 1957, when he was given the honorary degree of Desikottama

at a special convocation.

His growing frustration can be sensed from the reply he gave to the felicitations on his being elected an FRS : 'I remember my teacher the late Jagadischandra Bose who was the pioneer in scientific research in our country. All my bits of endeavour in life have been in that direction of scientific research . . . Now at the fag end of my life I wish I could be useful to you in our recently sponsored general science classes, although here at Visva-Bharati we could not afford to have much of science as yet.'

It was a pity that Bose was not much of a success here. The prophetic words of Tagore pointed to such a possibility.

> It is no use deluding ourselves that this institution of Visva-Bharati will continue its success through the ages in the same manner. Let us work to the best of our capacity, without expectation, without attachment. God demands work from us, but carefully keeps the accounts away in case we demand rewards for our labour. He does not insult our effort by prompt payment. Besides, the vows we have taken today may not be repeated in future — that is contrary to nature's law. All we can do is build a way to the future but we certainly can't make the destination absolutely static by today's taste and standard. If blind obsession leads us to this it would be reduced to a graveyard of our dead aspirations.[14]

The destination had become static, but Satyendranath had to move on.

9

The Last Lap

1959-1974

On 18 December 1958 Bose wrote to the Secretary, Ministry of Science and Technology, expressing his desire to be relieved from his duties as Vice-Chancellor of Visva-Bharati from the afternoon of 24 December 1958. He wrote that he wanted to devote himself entirely to scientific research from January 1959. The Calcutta University had allowed him the use of a fairly large room at the Science College to enable him to hold his weekly symposia. Regarding his future research plans he mentioned an intensive study of theoretical developments in nuclear physics in order to obtain greater insight into the nature of fundamental particles and the laws of interaction. He also wished to complete his earlier investigation in organic chemistry and the structure of some organic compounds of industrial importance.

His budget was quite modest with a staff requirement of two senior and two junior scientists and contingency expenses, the total financial implications of which came up to Rs 40,000 per year for a period of five years.

Since the grant came to him through the IACS Bose shifted his office and laboratory there. His group consisted of Nripendranath Ghose, Purnangshukumar Roy and for a brief period Professor N R Sen; and Salil Roy and Partha Ghose, younger scholars working for their doctoral degrees. The Visva-Bharati interlude marks a negative phase in the career of Bose. In a more congenial atmosphere he might have been able to contribute in some way or other, but even then a basic infrastructure for scientific research did not exist in Santiniketan then. So whatever Bose may have said to Annadasankar Ray in desperation, it was in a way the best thing under the circumstances, to come back to the mainstream of research with his honour unimpaired.

This final phase in his career is marked by a distinctly different style. His domain was now larger than the Khaira Laboratory, his interests could range over a wider field.

> He started in a new style, of a very special character, acting and talking as an equal in a group of young and enthusiastic scholars undergoing training in diverse branches of science. Tempered by the years , with his luxuriant crop of white hair, he came to be regarded as a sort of father confessor, giving counsel on any problem on which anyone sought his guidance. To him may be applied what Francis Bacon said about himself: 'I have taken all knowledge to be my province'. [1]

With the passage of time his outlook had also undergone a basic change. The problems which engaged his attention had less to do with mathematical physics — they were more related to the application of physical principles in areas extending beyond the ivory tower of academia. One such project was the extraction of helium from the hot springs of Bakreswar.

The existence of helium in the hot springs of Bakreswar was first detected by Professor Shyamadas Chatterjee of Jadavpur. However, it was only through the efforts of S N Bose that a field study unit was set up at the site to collect the gas which was later purified at the National Professor's Laboratory. Bose fully realized the importance of helium as a strategic material. Later other such springs were also found at Tantloi close by. After Bose's death S D Chatterjee handed over the project along with the workers to the Variable Energy Cyclotron Centre, Calcutta, of the Department of Atomic Energy. This unit has done very good work and the helium extracted from the spring has been used to produce high energy alpha particles at VECC, Calcutta. Bose left Visva-Bharati in 1958 even as nation-wide preparations had begun for the coming centenary of Tagore. Three years later when the celebrations were in full swing, Bose was at Calcutta. The experiment at Santiniketan had evidently touched a raw nerve; it was not just a personal frustration, but the aftermath of a clash of attitudes.On the one hand was a group who had put Tagore on a pedestal, deified him and assigned to him a rigid set of values, and on the other were people like Bose who understood the poet and his philosophy better, knew how to reinterpret them in the light of changing needs.

In 1961, when the Calcutta Vice-Chancellor Surajit Lahiri raised doubts about the desirability of putting undue emphasis on the teaching of science on the supposed authority of Tagore who was supposed to be against science since it preached a material outlook which was opposed to a truly Indian stand, Bose gave a hard-hitting reply. His lecture could not be transcribed from the tape, and so Bose re-wrote it in the form of an article in which he comes very close to the philosophy of Meghnad Saha. Bose, however, did not get into a debate as Saha was forced to do with the philosopher Anilbaran Ray of Pondicherry over science and religion;[3] but for someone as gentle as Bose it was quite a scathing attack on the traditionalists. He said that our old philosophy has done very little to improve the lot of the common people. Only a proper application of modern science can do this. After all we have some obligation to our fellow beings!

complex by characterizing their attitude in the now famous phrase 'we-have-everything-in-the-Vedas'! Satyendranath too would often refer to this as our main psychological block. He attacked the problem, however, from a different angle. In his public speeches and writings he kept on harping on one theme, viz. that the reason why science has failed to make a headway in India is the barrier of language.

A foreign language, he said, encourages cramming among students and discourages creativity. The imposition of English has made our education wasteful, and has stunted the mental growth of our students.

Bose's crusade for the use of our mother tongue as a medium of education gathered momentum after his return from Dhaka. But he had always been aware of the weakness of our education system and was highly critical of Sir Asutosh for not having given the vernacular its due emphasis. In the tussle between the advocates of the 'national' school of education and the English System, Asutosh had won. Asutosh preferred to open our windows to the west, and as later events went to prove, his policy did yield some good results. But as a teacher, Bose felt differently. In 1963, in his convocation address at Ranchi University he elaborated on the defects of our education system :

> As an old teacher who had occasion to come in contact with hundreds of students, I am convinced that considerable amount of time could be saved and a better foundation of scientific knowledge prepared in the minds of our students, if at every seat of learning teachers and students cooperate and frankly discuss their problems and work together to solve their difficulties. However, if a foreign language comes in between them there is bound to be considerable hardship. [4]

Later when he visited Japan he was impressed by their approach to the question of language. He found that physics was being taught in Japanese, research papers were being written in Japanese, even at seminars all discussions were conducted in Japanese. He returned from Japan firmly convinced that only the use of one's mother tongue can bring out the best in every individual.

Perhaps the realization came gradually, with time. When he was a young teacher at Dhaka his class lectures were usually in English. In those days he hardly made any public speeches. But all these changed as he moved to Calcutta. He not only gave class lectures in Bengali, he even gave the Saha Memorial Lecture on a very abstract topic in fluent Bengali without the use of English scientific terms at all. He demonstrated that it could be done. But somehow the reception was lukewarm. Many were of the opinion that if all the states in India took up their respective languages as the medium of instruction, social and academic mobility would be at a standstill. Bose always had a ready answer to every such charge. He spoke at length on this at the 'Angrezi Hatao' [lit.Drive English away] conference in Hyderabad. He spoke endlessly on this topic.

It was not as though he was a lone Don Quixote. Bengali literature has a tradition of science writing, dating back to the nineteenth century. There were science journals too, though most of them were short-lived. In 1931 a science association was formed by eminent people like Sunitikumar Chatterji, Prasantachandra Mahalanobis and others, with a very ambitious programme of popularizing science, even establishing

industries. Bringing out a journal was part of this programme. But this project did not come off. About sixteen years later S N Bose was more successful in founding the Bangiya Vijnan Parishad [the Bengal Science Society] with a more limited objective.

On 18 October 1947 at a meeting held at the University College of Science, a resolution was taken to form a society with the sole objective of promoting and popularizing science through Bengali. The Parishad was formally inaugurated on 25 January 1948. The circular printed for the occasion ran as follows:

Bangiya Vijnan Parishad
92 Upper Circular Road, Calcutta 9

We need science at every step, but our system of education does not prepare us for it, so that we are not able to utilize science in our everyday life. The main obstacle so far was a foreign language through which education was being imparted. Today the times have changed. New hopes and aspirations are emerging. Now it is the duty and responsibility of our scientists to popularize science through the medium of vernacular and thus help to create a healthy scientific attitude among the people.

As a first step to this effort it has been resolved to form a 'Bangiya Vijnan Parishad'. It was mainly through the inspired leadership of Professor Satyendra Nath Bose.The primary objective of the Parishad would be (*i*) the creation of a scientific outlook among the people, (*ii*) to publish school and college texts in an easy style, yet keeping the scientific content, (*iii*) to publish and provide texts of available books on science, (*iv*) to enrich popular and children's literature with scientific knowledge, (*v*) to help popularize science in Bengali. The Parishad would conduct conferences and seminars and arrange exhibitions and popular lectures.

We know we have a limited capacity, still we have come forward to take up this responsible job. We consider this a national duty and we count on the cooperation and active support of our learned friends. We believe we shall not lack help or goodwill. We particularly bank on the support of the Universities of Dacca and Calcutta because all of us are connected to these two glorious institutions either as student or as teacher. We hope the Bangiya Sahitya Parishad will help us. We also hope for the cooperation of the Visva-Bharati because Rabindranath had dedicated his first book on science *Visva Parichay* to Satyendra Nath Bose who is one of us.

We have decided to formally inaugurate the Parishad on the 25th January 1948. May we request our patrons to kindly send their subscriptions and be members and attend the session.

Please send you membership fee along with name and address to

Dr Subodh Nath Bagchi,
Secretary, Bangiya Vijnan Parishad,
92, Upper Circular Road, Calcutta-9 [5]

Publishing a Bengali journal was the most important task of the Parishad. It also undertook the publishing of books on various scientific topics. The journal *Jnan O Vijnan* had a very able editor in Gopalchandra Bhattacharya (1895-1981), a biologist trained under Jagadischandra. S N Bose was able to collect several prominent men of science around this venture. They included Dr Priyadaranjan Ray, Dr Bireshchandra Guha, Dr Purnendu Bose. Rathindranath Tagore contributed an article to the July issue of the first year on the ever-increasing food problems of the country. Articles on energy, organic ingredients of soil, land development, water resources and hydro-electricity and problems of industrial development appeared in its pages.

In a limited way *Jnan O Vijnan* was a vernacular version of *Science and Culture,* the organ of the Science News Association initiated a decade earlier by Bose's friend, Saha. Since *Jnan O Vijnan* was in Bengali, it did not get as wide a readership. The limitation was self-imposed because S N Bose chose to concentrate on Bengali-users only, and carry out his experiment on the expansion of science literacy through the mother tongue.

The early issues had thought-provoking articles. One member of the core group, the satirist Parimal Goswami (1897-1976), wrote an article on the general Bengali apathy towards science. An important issue was raised by Binoykumar Sarkar in the very first number — 'Rāmendrar Path Nā Jagadis – Prafullar Path' [The way of Ramendra (i.e. Ramendrasundar Trivedi, 1864-1919) or the way of Jagadis and Prafulla (i.e. J C Bose and P C Ray)] — raising the question of popularization *vis-à-vis* scientific research. His idea was that the Parishad should do both. But the journal drifted towards popularization which is really what S N Bose had in mind.

Gopalchandra Bhattacharya, as editor, introduced a new style in science communication. Not only was his language close to common speech, his approach too was experiment-oriented. The journal also had a section for children. Very soon its circulation went up, reaching the remote corners of rural Bengal. Kaushik Bhattacharyya, a post-graduate research student of Calcutta University, has done a coverage analysis of this journal which is given below .

JNAN O VIJNAN : COVERAGE ANALYSIS, 1948 – 90

Sl. No	Topics	1948–1972		1973–1990		Total	
		No.	Percentage	No.	Percentage	No.	Percentage
1	Agriculture, Food	177	5.8	161	6.5	338	6.1
2	Health, Medicine	277	9.2	325	13.1	602	11
3	Fundamental Sciences, Physical, Biological etc.	1913	63.6	964	38.9	2877	52.5
4	Science, Man and Society	511	17	615	24.8	1126	20.5
5	Electronic Media	125	4.2	282	11.4	407	7.4
6	Environment	6	0.2	131	5.3	137	2.5
		3009		2478		5487	

The continued success of the journal over the years has brought it considerable recognition. The National Council for Science and Technology Communication has cited it as a model for similar organizations. The Parishad now has a building of its own. It was possible through help from the State and from donors like Nirendranath Ray.

All through his later years Bose was getting deeply involved with the Bangiya Vijnan Parishad. It was a platform from which he could reach the common people easily. Significantly, almost all his writings and speeches — beyond the strictly scientific papers — barring one or two exceptions were written after 1964. Thus this last phase marks his greater involvement with larger issues. It was not just the question of popularizing or teaching science through the mother tongue, but the social and political scene after the Partition. He was growing gradually disillusioned, his earlier idealism was still there though it found expression more in bitterness and frustration at finding the old values being steadily destroyed.

Although Bose for all his advocacy of the mother tongue never wrote a book in Bengali himself, he did act as a catalyst in many instances. One of his earliest converts was Quazi Mutahar Husain of Dhaka. Bose had asked him to write a practical course book for physics because most of the students did not follow the instructions in English and consequently made mistakes. That book was written, shown to Bose, but unfortunately the manuscript was lost.

Undeterred, Quazi Mutahar wrote another book in Bengali — this time on statistics. He showed it to Bose who was so pleased that he showed it to all his research students. Later, the 'introduction' was published in the journal, *Jnan O Vijnan*. Quazi Mutahar kept in touch with Bose all his life.

> Just as I was his student in the 24 years he stayed in Dhaka, so I remained all my life. For a long time I used to visit 203 B T Road Calcutta where Mahalanobis's Indian Statistical Institute was housed — I went there every year as an external examiner. Then I would visit Satyen--Babu consecutively for a few days. He too used to come to the institute. . . I had made a Bengali translation of Satyen-Babu's Quantum Theory article. This was published in the Calcutta journal, *Prabasi*. Satyen-Babu spoke highly about my simple and natural style. . . This elder brother of mine has helped me, enlightened me in so many ways that I cannot possibly pay him back.[7]

Science textbooks in Bengali have flooded the market since, but in those days writers needed a lot of persuasion to embark on such a venture. We have this account from one of the pioneers in this field :

> One day in 1951, I had dropped in to see Satyendranath in his room at the Science College, where I found his colleagues, Jnan Mukherjee and Pulin Sarkar among others. They were discussing the feasibility of teaching science through Bengali. Pulin-Babu and the others felt that teaching in Bengali would be inconvenient. It would also bring down the standard. Satyendranath of course felt differently. His emphasis had always been on teaching everything through one's own language. That was why he had

founded the Bangiya Vijnan Parishad and published the journal, *Jnan O Vijnan*. He was trying to convince his colleagues that teaching in Bengali was not difficult at all. In fact it was very important to take the step. He saw me come in and asked, 'Rakshit, do you lecture in Bengali or in English?'

— Well, Presidency College has English for its medium of instruction, besides there are non-Bengali students too. I teach in English.

— Can't you teach chemistry in Bengali ?

— Certainly I can. But there are two problems. It wouldn't be easy for the students to listen to Bengali lectures in class and read English texts at home. Secondly, all the teachers have been educated through English, and are used to teaching in English. It would mean an extra effort for them to train themselves up to teach in Bengali. Only if they are willing to co-operate it can be done.

— You are very right. Well, why don't you write a book on chemistry, beginning at the I Sc level ?

— Me ? Would I be able to do it ?

— Why not ? Try. You have been teaching them in English — can't you write it down in your own language ?

I left them. An order from Satyendranath was an order. I began writing from the next day.[8]

The book — a secondary level treatise on chemistry — was published in 1952, and ran into editions. Then others came forward with similar books in Bengali in physics and other subjects. In the words of Rakshit, 'Thus Satyendranath's dream began to be translated into reality.'

Chapter divisions made in a book are for the sake of convenience alone. No life can be neatly pigeonholed into phases as we have been trying to do with the life and career of Satyendranath. The analogy of the Ganga used by one of his research students is true in more senses than the one he meant. Ideas and convictions are more like flowing currents though there may have been evolutions, even transformations.

So when in 1959 we try to picture Bose in his new role of a National Professor we are merely stating the obvious — that now he was given a pedestal of honour and plenty of freedom to work, think, write, do as he pleased. Apart from the scientific research in which he was personally involved, or the projects through which he guided his students, he now came to take considerable interest in the Indian Statistical Institute and the Bangiya Vijnan Parishad.

The origin of both can be traced to a much earlier time. His contact with Prasanta Mahalanobis, founder of the ISI, goes back to their college days. Mahalanobis passed his B Sc from Presidency College in 1912 with honours in physics. He left for Cambridge then, but was back again at Presidency after his Tripos, this time to teach.

Satyendranath had been close to Prasantachandra Mahalanobis from his college

days. In Dhaka he continued to be in contact with him and remained all through a great supporter in his effort to build the Indian Statistical Institute. The ISI was started on 17 December 1931, with Sir R N Mukherjee as President and P C Mahalanobis as Secretary. The Institute immediately started publishing *Sankhyā*, the Indian Journal of Statistics, the first issue of which came out in June 1933. A Statistical Publishing Society, established in 1935, handled the publication.

Satyendranath's papers on D^2-statistics were published in the second and third volumes of *Sankhyā*. The subject was closely related to the work being done by Mahalanobis and his student, Rajchandra Bose. In 1945 when Bose returned to Calcutta, C D Deshmukh was the President of the ISI. Mahalanobis had to be away from 1946 to 1948 serving at the United Nations and other places. At his request S N Bose took charge of the administrative work of the ISI as its Secretary. The sincerity with which Bose carried on his work can be seen from the correspondence reproduced below.

Statistical Laboratory
Presidency College
Calcutta.
27 February, 1947

Dear Prasanta,

Your letter of 16th February, 1947. You must have received in the meantime a copy of Sargent's letter to Deshmukh as well as a telegram from the Institute requesting you to return before the annual meeting. I sincerely hope that it will be possible for you to be personally present when important decisions regarding the Institute are taken. In case you definitely decide not to return, kindly go through Sargent's letter and give me your opinion on the points raised by Sargent. It would help the discussion immediately if at least your authoritative opinion is before us even though you may be unavoidably absent. I have written to Deshmukh requesting him to discuss the proposals which the members of the Reorganization Committee and the Council may put forward after perusal of the government letter.

The letter had been circulated according to the suggestion of the President to the members of the Council.

Government gave a reminder of the Purchase of Printing machineries to which I have given a reply as enclosed.

The old agreement of the Institute with you regarding the use of your building will terminate on the 31st of March. I shall put forward your point of view regarding the housing of the Institute before the Council and shall support it.

I hope the staff will keep you regularly informed about the developments that are taking place at this end. We have not yet heard anything definite regarding the continuance of the crop survey though many hopeful rumours are current. I shall certainly let you know in case anything tangible and definite is proposed by the Bengal Government.

What about Mrs. Chatterjee's admission to the Rockefeller Institute ?

In case you are in New York and Brooklyn, will it be possible for you to meet Prof. and Mrs. Mark (Director, Brooklyn Polymer Research Polytechnique Institute) and give them my greetings and the greetings of the family.

With kind regards to Rani and yourself,

Yours sincerely
S. N. Bose

Prof. P. C. Mahalanobis, F. R. S.
C/o Chase National Bank,
Rockefeller Centre,
New York.[9]

S N Bose's deep involvement in the affairs of the ISI extended to everyday details, as evident in this letter. It was a critical juncture in the history of the Institute because R C Bose and S N Roy had already migrated to the USA and C R Rao had accepted a research post in the University of Cambridge. While Mahalanobis was away, S N Bose and his family were asked to move to the Mahalanobis residence of Amrapali, so that Bose could be more readily available for ISI work.

The Indian Statistical Institute Act (No. 57 of 1959) was passed in April 1960, and the Institute was authorized to start B Stat and M Stat courses and doctoral work. Dr Deshmukh remained President of the Governing Council till 1967, with S N Bose as Vice-President. Bose became President in 1967 and remained in the position till his death in 1974. In recognition of his services the ISI awarded him the honorary degree of Doctor of Science at its first convocation held in 1962, when the following citation was read out :

> Professor S N Bose, National Professor of India and Fellow of the Royal Society, is an eminent theoretical physicist who introduced a new concept in quantum statistics which, with the collaboration of Albert Einstein, was later developed into a basic principle of quantum mechanics known as Bose-Einstein statistics, and elementary particles conforming to this principle were given the name 'Bosons'.
>
> In recent years he has made important contributions to the unified field theory in relativity.
>
> He has been an inspiring teacher and has helped successive generations of students with fertile ideas not only in theoretical physics but also in chemistry, biology and other sciences.
>
> He has been taking a leading role in the dissemination of scientific knowledge through the medium of his own language, Bengali.
>
> He has been deeply interested in the cultural developments and in literature and music, and served for some time as the Upacharya of Visva-Bharati, the international University formed by R N Tagore.
>
> He has been actively helping in the work of the ISI since its foundation and for a considerable time immediately after the War shouldered heavy administrative responsibility as the honorary secretary of the Institute.

Professor Bose's participation in the affairs of the ISI till 1972 are recorded in the volumes of *Samvadadhvam*, the house journal of the Institute.

That Bose was the guiding spirit behind the statistical school at Dacca University has not been properly recorded. The origin of the department is recorded in the memoirs of Quazi Mutahar Husain, a young assistant lecturer in the physics department in 1921. Impressed by Quazi's flair for mathematics Bose wanted to know if he would like to study statistics. Bose was already thinking about offering a course in the subject in Dacca University for which teachers were needed. Quazi has given an account of how he was drawn towards this new science and the help he received from Satyendranath.

> He asked me, 'Would you like to study statistics ?' I replied, 'Yes, provided I get a chance'. Promptly he said, 'Suppose I give you nine months' paid leave, add to it the three months' vacation you normally have in a year'. When the summer vacation began he took me to Calcutta and introduced me to Dr Prasantachandra Mahalanobis. 'He is like my younger brother, please teach him as much statistics as you can in one year. I think he is capable, that is why I am leaving him in your charge.'[10]

The plan worked beautifully. From 1939 Quazi Mutahar Husain began teaching statistics. Gradually it grew into a fullfledged department, though not in Bose's time. But the germ was sown by him and encouragement given and it was an indirect compliment to him when in the 1950 convocation his young colleague, Mutahar Husain, was given high praise, for having 'carried out in this University independent research work in statistics,' and 'produced a most outstanding dissertation on which he has been awarded the degree of Ph D in Science. It is gratifying to note that one of the examiners of his thesis, Dr R A Fisher, an international authority in Statistics, has spoken in very high terms about the quality of his thesis. Dr Husain has been appointed Reader and Head of the Department of Statistics.'[11]

The 'independent work' mentioned here had however been done when Bose was in Dhaka. At every stage Quazi Mutahar consulted Bose who was initially sceptical about it, but Quazi Mutahar persisted. Research was then going on in the ISI on Balanced Incomplete Block Design. Quazi Mutahar wanted to do it by the trial method. His entire thesis was later 'corrected' by Bose.[12]

Bose had three terms of five years each as National Professor from 1959 to his death. His eighteenth year coincided with the golden jubilee of Bose Statistics. International seminars were held for the occasion, attended by celebrities from many countries. A long and fruitful life was drawing to a fitting climax. Perhaps Bose had a premonition ; because replying to one of the felicitations, he expressed satisfaction that his work was being recognized at long last. Many later scientists have wondered why Bose was denied the Nobel Prize which should have come to him, but Bose had no regrets. He had lived a full life and won the love of his countrymen.

He died on 4 February 1974, only a month after he had completed eighty.

10

The Complete Man

Bose was married early by present standards, but in his time twenty was considered a marriageable age and there was nothing unusual about a young man still in college marrying a girl of eleven. In fact, Bengali fiction of the early twentieth century is full of brides pining away in their village homes for their college-going husbands in the far away city of Calcutta.

What was unusual in this case was Satyendranath's insistence on not accepting any dowry. His father-in-law was a rich doctor who owned many houses in the neighbourhood and Ushabati was their only child. But the idea of taking any help from his father-in-law (he could very well have financed Bose's trip to Europe) never entered his mind. Even in his old age when the salary of the National Professor was often delayed in the red-tape causing him considerable strain, Bose never used Ushabati's money to tide over the crisis. Bose never took recourse to the prevalent custom of using a moneyed father-in-law as a stepping stone to success. In fact, in the days of scarce scholarships it would often be the father-in-law who sponsored the sons-in-law on their trips abroad for higher studies; sometimes as part of fulfilling the stipulations laid down for the marriage.

Ushabati went to Mahakali Pathshala and Duff School, thanks to the liberal influence of her mother who came from the enlightened Dutt family of Rambagan.

The Dutts, like the Tagores, were well known for their enlightenment and culture. One branch of this family was converted to Christianity. This line produced two brilliant daughters, Toru (1856-77) and Oru (d. 1874), both of whom published poems and translations in French and won critical acclaim. Both died in their prime. An even

better known member of the family, Romeshchandra Dutt (1848-1909), civilian and novelist, resigned from the colonial Civil Service in 1897, to take a leading role in the early national politics, and came to write some of the classics of Indian economic history, e.g. *The Peasantry of Bengal* (1874), and *The Economic History of British India* (1902).[1]

In Satyendranath's family, however, there was resistance to the idea of sending the bride to school. So Satyendranath took upon himself the role of a teacher. For some time he gave her English lessons, but in a joint family with many responsibilities this could not go on for long. Besides Ushabati was a very simple and submissive type and lacked the determination to carry on her studies. Her husband was understanding and friendly; and, unlike traditional husbands, did not wish Ushabati to be tied down to household work. If she slept late, Satyendranath would take care of arranging breakfast for the children, prepare for himself a couple of toasts and tea and then rush off to college.[2]

A beautiful friendship had sprung up between them as the following letter shows :

June 8, 1958
6 Rue de la Taur Paris 16th

Usha,

The plane took off from Calcutta at 11.30 in the night of the 6th. I arrived here yesterday on the 7th. We landed at Orly at 9 in the night. All the planes coming to Paris land at Orly.

The Malakars came to receive me and Jacqueline. Malakar had booked a room in this hotel — we all went there in Jacqueline's car. By the time we reached it was 10.30. Had a bath at about 12 midnight. Woke up very early. I had wired them before coming.

Nothing much happened on the way. There were not many passengers in the first class. I had a window seat, the next seat was unoccupied till the last. I could not sleep very well — but there was plenty of room to be comfortable. At the press of a button the back of the chair tilts and a foot stool appears from below.

Today I'll have a chance to rest. There is no sign of any trouble on Sunday. Malakar assured me that all the frightening stories only come out in the newspaper. The people here are carrying on as usual. So, you need not worry on my account.

I will fix up the plans today. Tomorrow I will visit the office of the person responsible for inviting me here. I'll finish the work of Mark, spend a few days here before going back. Jacque was sorry that my stay was not for a longer period.

How is summer at Santiniketan? In Karachi I saw dark clouds — perhaps it was followed by a shower. But before I left there was a good hailstorm. Did you feel any of it? Jaya and Debprasad came to visit in the afternoon but their car broke down, they couldn't come to Dumdum. Many of the family went to the airport in an ISI bus, though I don't know what they saw. I was out of bounds from one o'clock. With my love

Yours
S

Bose had a large family. Of his nine children two died as infants. Five daughters and two sons survived, viz. Nilima, Aparna, Purnima, Sova, Jaya, Rathin and Ramen.

His eldest, Nilima, born after he had completed his Masters, was the closest to the father.

Ushabati never made her presence felt; and Bose, not an indulgent father in any way, was considerate and caring. So the children grew up in an atmosphere that was open and relaxed. All the girls were sent to school and college; the boys too were given proper education, both spending some time in the US, Rathin for his training and Ramen for his M S.

The only advice Bose gave to his children was to be honest and sincere. These values were cherished in the family. Satyendranath himself was influenced by his father to a great extent. Although there were other external factors, strong family ties created a bond between the inner man and the champion of larger causes. To the last he remained unassuming, friendly, easily accessible. Common people had no understanding of the scientific achievements that had placed him in the front rank of world physics, yet the public expression of sorrow at his death was so overwhelming as though he was a political leader.

For all his international renown, Bose retained the spirit of his boyhood, the spirit of 1905, the days of nationalist zeal in Bengal. Bose was then a boy of eleven. The boycott of British goods, or bonfires of British mill-made clothes had the excitement that could naturally fire a sensitive boy's imagination. Then there were patriotic songs, a lot of choral singing in the streets. The leaders of the movement had made a ritual of the *rākhi-bandhan,* a popular festival in North India in which a sister ties a coloured thread round a brother's wrist as a mark of protection from all untoward dangers. It was given a different orientation in Bengal by the Tagores when they redefined it as a solidarity pledge; and Hindus and Muslims exchanged the *rākhis,* to the singing of songs glorying in the heritage of the motherland and upholding communal amity. The celebratory dimension soon gave way to a sterner line, and then took a violent turn. The agitation swept through Bengal.

In Dhaka Bose's contemporary, Meghnad Saha (they were to meet six years later in 1911 in Presidency College), was in school. The school he went to was a government institution — the Dacca Collegiate School. As one of those who came to school bare-footed and boycotted their classes in protest against the visit of the British Governor Sir Bampfylde Fuller, Saha, then a boy of twelve, was expelled from school and taken off the scholarship list, causing him immense hardship.

With a more secure social standing and a very strict father at home, Bose's response to the surge of patriotism was different. His father would not allow his son to spoil his chances of a good career by getting actively involved in the Swadeshi movement. His father is supposed to have exhorted him to stand by his responsibilities as the only son in a large family. Bose was a loyal son. Hence active involvement was out of the question. But all through the Swadeshi period Bose was in contact with the underground, sometimes acting as a courier, sometimes providing shelter to fugitives from 'justice', or helping them out with money; with the police trailing him at one time and making enquiries about him.

There was a phase when he was active in the organization of night schools for

common people which however did not last long. To quote from his own reminiscences :

> Like the more recent 'Down with English' movement with which I got involved, we were taken up with the slogan 'Down with the British' in those days, and got carried away in various directions by various currents of thought. Some of us were convinced of the need to educate the masses of our country, to create in them a sense of belonging. But this feeling had to grow to be part of them through a thorough education, and not because somebody in authority like a guru or a zamindar said so. With this in view we had organized some mass education centres in Calcutta but these night schools were short-lived because the British suspected they were training centres for bomb-making. So they had to be closed. That chapter is likely to have been forgotten by now.[5]

Girijapati Bhattacharjee corroborates : 'A night school for labourers was opened in Keshab Academy on Maniktala Main Road, through the efforts of some patriotic people. Satyen had connections with them. He took Harish, Niren and myself along and we were involved in the teaching.'[6]

When he looked back more than fifty years later, Bose spoke a little critically of the people who believed in the power of the bomb, and hoped that everything would be fine and dandy once the British were made to go! In this he was very much in the company of his friend and classmate Saha, who was able to impress upon Subhaschandra Bose, the nationalist leader, the need for scientific planning.

There were three clearly marked phases in Bose's slowly evolving sense of social involvement. In the first phase ideas were nebulous and the response emotional. In the second phase, roughly between 1916 and 1945, a period which coincides with his leaving the University and extends over Calcutta-Dhaka, Paris and Berlin, he was actively involved in helping a group of revolutionaries planning armed uprising against the British, the most prominent of whom was Abani Mukherjee (1891-1937).

Abaninath was part of the group that sought to smuggle American arms into India with the help of the Germans during the First World War. He had a very colourful career. He was sent to Japan by Bagha Jatin (Jatindranath Mukherjee) to contact Rashbehari Bose (1885-1945), a Bengali revolutionary who had found asylum there. In 1922, Sunitikumar Chatterji (1890-1977), the eminent linguist and scholar, came across Abani in Germany and recognized him as they were from the same neighbourhood in Calcutta.

Abaninath was then planning to return to India to organize the underground revolutionary activities there. Sunitikumar tried to dissuade him because since Abaninath had left the country ten years ago, the emergence of Gandhi had changed the political scene in India. But Abaninath paid no heed to this advice and reached India in late December the same year and contacted Sunitikumar who was not directly involved with politics and so did not know how to help him, and thought of Satyen Bose.

> I thought I could take Satyendranath Bose into confidence. So I asked

> Abaninath to come and see me the next day towards the afternoon, and I thought I must exert myself to find some way out. I managed to see Satyen at his residence and told him about the case of Abaninath, and I said he must come with me to Dilip's place, and there we would take Abaninath and jointly discuss what to do. Dilip Roy, as I knew, had come back to India, I think, just a couple of months before me, and he was *au courant* with what was happening among some of the revolutionaries in the continent of Europe; and Satyen and Dilip being good friends they would put their heads together and help Abaninath out.
>
> The next day Abaninath came to me in the afternoon and Satyen was there, and Abaninath, Satyen and myself went to Dilip's place . . . At that time, . . . I think he was stopping with his maternal uncle — Dr Jitendranath Mazumdar . . .
>
> We did not have anything to do with Dr Mazumdar. But Dilip, Satyen and myself as well as Abani had a little serious consultation at a corner of the room, and we came to a plan of action. I wanted them, if they could, to find out the name of some persons who belonged to the Anushilan Samiti, the revolutionary nationalist party . . . Satyen offered to take Abani immediately to some of the Samiti people whom he knew, and do something for him right then ; and that was beyond my power to do . So we started from Dilip's uncle's place. I returned to my house, and Satyen took Abani to the Anushilan Samiti gentleman who would be able to look after him.[7]

The matter did not end there. Satyendranath returned to Dhaka where soon Abani surfaced only to be betrayed by his friends, and had to leave the country. Satyendranath took a photograph which he needed urgently for his passport and made other arrangements for him.

His next encounter with Abani was in Europe. This time he was supposed to hand him over some money sent by his brothers from India.

Once again it was Sunitikumar, who would send that money to Satyendranath

> in his address in Germany, and this was to be the code to tell him the purpose of the money — I would write to him that this money which I was sending to him was to buy for me some Sanskrit books. He would then understand, and pass the money to Abaninath. His brothers gave me about Rs 300 and they knew that I was sending it to him through Satyen Bose. This money reached him all right.[8]

Sailen Ghosh, another contemporary of Bose's, and a favourite of Sir Asutosh's, had close links with one of the secret revolutionary societies, and had to go underground and escape to Europe when the police caught up with him. Bose in his recorded reminiscences has nothing to say about Ghosh's revolutionary connections, and he appears only as an extremely resourceful and popular young man. Years later when Ghosh had made his peace with the Government and returned to India, he came to his good friend Satyendranath for help. Bose was instrumental in getting Ghosh a position

at the Jagannath College, when the latter was finding it difficult to carry on as principal at B M College at Barisal. Ghosh's daughter, Mariam, in her reminiscences,recalls with love and affection 'the many friends from Dacca University, and Satyen Bose.'[9]

Dhaka in the twenties was a hotbed of underground revolutionary activity —

> The Hindu students of the Dacca University had a notable share in the formation of the Revolutionary Societies which believed in the use of violent methods to liberate the country from the foreign yoke and carried on their activities in secret . . . Many students of the University including women joined the societies . . . Under the cover of social welfare services, they secretly conducted their programme of revolutionary activities.[10]

Satyendranath and his contemporaries in the academic community were not unaware of these activities. Since the keyword was secrecy, documents and records are not easily available. But there are indications that most of the Dhaka teachers supported the revolutionary groups financially, though they had to be careful about it.

The third phase in Bose's social and political thinking was marked by a sense of disillusionment. Though he had seen the direction things were taking when he left Dhaka, the partition of the country in 1947 cut him to the quick. After partition Bose's house at Iswar Mill Lane in Calcutta had streams of visitors from erstwhile East Pakistan (now Bangladesh). Like many of his nationalist contemporaries, Bose never quite accepted the division of the country and the formation of another state.

In despair, his sense of patriotism took the form of an intensely Bengali nationalism. Earlier he had ridiculed P C Ray's excessive concern for the Bengalis — the Bengali Brain and its Misuse was one of his common themes. In college Ray would often lovingly point to Satyendranath and remark, 'Here goes another example of my theory'. Ironically, in his late years S N Bose grew more like P C Ray in his exclusive concern for the Bengalis. His crusade for the mother tongue is another manifestation of this sentiment. He was growing cynical with advancing years. The young nationalist with a vision was now an old man who did not see any future because Bengal had been mutilated and freedom from the British had not brought any respite.

One particular Bengali word is always used, often disparagingly, in association with Bose, that he was too fond of *āddā*, almost addicted to it! It is difficult to find an exact English equivalent for *āddā* which covers a broad range from small talk to serious discourse, but without predetermined purpose or direction. This predominantly Bengali habit is indeed wasteful, but at the same time can be intellectually stimulating depending on the quality of the *āddā* which in its turn depends on the intellectual richness of the participants. In its best form the *āddā* can be an opening out — the mind opens up and reaches out to others with absolutely no regard for gain or material profit, in a leisurely trustful spirit. A very clear understanding of this activity can give one a clue to the apparent paradoxes in Bose's character.

In a sense his entire career was an extended *āddā*; for instance, when a young student or colleague came to him at Dhaka with a problem and then spent the whole day trying to solve it, the day passed into night and they were still at it. We have a moving description by Quazi Mutahar Husain :

> Let me relate one incident. It was early morning, a little before sunrise. I reached his room in the department armed with books and notebooks. He was already there busy solving some problems in mathematics. What was I doing there so early, he wanted to know. 'I can't get this equation right, I have been trying for two days,' I told him. He had a look at it , 'Well, this has been solved by Maxwell, it is about mutual induction.' He started working on it — but no solution could be reached. The morning passed into noon. Afternoon came, yet no solution was in sight. We had our breakfast there, then lunch, then tiffin. Our classes were taken by other teachers. When it was late afternoon he said, 'The attendants have to go home, let us go to the Teachers' Common Room.' We moved there, had our tea, then dinner, but the problem couldn't be solved. When midnight struck he suggested we go over to his house. 'Come on, I have a copy of the Maxwell book.' We proceeded to his house. He rushed upstairs and brought the book. The proof in the book contained an experimental data. None of us had thought of that. We were struggling in vain with incomplete information. His advice to me was — 'Listen, Quazi, one must keep on trying without losing patience. Sometimes a clue comes in a flash — then solution becomes easy, perhaps this is called inspiration.'[11]

This is generally not done in academic circles. Nobody works out others' problems unless he gets credit for that. One always thinks in terms of number of publications, laurels. Einstein would not think of it — though Niels Bohr came very close to a high level *āddā*. The capacity to stimulate others to creative expression— this is the supreme quality of an *āddā* addict. One need not question why, or to what end. Many of Bose's true friends lamented the fact that he had too many distractions. But Bose could not help it — it was intrinsic to his nature. He was exploited by some, even cheated, but that made hardly any difference to him.

By nature sociable though never in an aggressive way, Bose's *āddās,* or rather his role in the *āddās* in which he joined, have acquired legendary values. The earliest ones used to be at his friend Girijapati Bhattacharjee's place. Both the brothers, Girijapati and Pashupati, were good friends of Bose; what is more, their home at Haralal Mitra Street provided the young Bose with all the necessary food for his receptive mind. They had a good library and the parents encouraged the *āddā*. Bose had his first musical training in this house. An esraj was procured. They spent hours on the terrace, discussing, singing, composing new *ragas*. They also brought out a hand-written journal called *Manishā*. One of Bose's earliest known pieces appeared in the pages of this journal — it was about his childhood impressions of Assam. Another favourite haunt was the Hedua— a park with a swimming pool — now known as Azad Hind Bag. It was known as Cornwallis Square in those days. They used to meet here after class — the same friends who frequented the Bhattacharjee household — Haritkrishna Deb, Nirendranath Ray, Harish Sinha.

They also met at Haritkrishna's place. From the following excerpt it is evident why Satyendranath was drawn towards Haritkrishna :

> Although much of their old wealth and grandeur was gone, the family still retained an ambience of musical and literary culture in their ancestral home. His grandfather, Upendrakrishna, wrote social novels in Bengali and they were much appreciated at one time. Haritkrishna along with his other grandchildren used to listen to his many amusing stories. That is how probably Harit's sense of irony and humour was nourished and developed from childhood. His father, Ashimkrishna, had an excellent collection of books on the ancient history of many countries of which Harit made good use. He had formed his own ideas about Greece, Rome, Arabia, India and Egypt. From time to time he would present his views to his friends in his salon, but he had no taste for the hard work and research required to earn a reputation among the cognoscenti. The Shovabazar house had a long tradition of private concerts of music and dance and discussions on literature and drama. Ashimkrishna was a great connoisseur of music, and an excellent harmonium player himself. Many excellent singers and musicians used to call on him and famous musicians performed in his house.[12]

Both Satyendranath and Haritkrishna were regular visitors to the *āddās* at the Bright Street residence of Pramatha Chaudhury. Better known as the *Sabujpatra āddā* after the name of the periodical Chaudhury edited —*Sabujpatra*, literally 'The Green Paper' or 'The Green Leaf', with a pun on the word *patra* — the group included some of the finest critical minds of the period including Atulchandra Gupta (1894-1961), jurist and scholar in ancient Sanskrit aesthetics and rhetoric, and Dhurjatiprasad Mukherjee (1894-1961), economist, sociologist and music critic. It is at the *Sabujpatra* circle that Bose made friends with Dilipkumar Roy (1897-1980), composer and singer and later religious guru, after the latter had sung a Bengali song set to the classical *rāga* Mālkous. In his memoirs, Roy acknowledges :

> It was Satyen who initiated me in several different ways into the mysteries of the art of forming the closest friendships. I picked up from him several techniques of how one can bring close to oneself people who have been unfamiliar . . . He would . . . ask me : ' Can you tell me what is the greatest gift that schools and colleges have to offer?' I would reply : 'Go ahead, tell me.' Then Satyen would say : 'Coming into contact with a wide range of classmates. In my life as a student I at least gained more from new and still newer friendships than from books, or papers, or lectures. It is just from these that I gathered most of what I needed as resources for my journey through life. Those who are good only at studies may have access to all that lies in the category of fact or knowledge, but they go without the gift that stirs the mind to life from the priceless contacts with the many. In adolescence and youth, our young minds need friendships more than anything else to awake to life.'
>
> It is nothing very original maybe. There would be many who could say : 'What's so great about a statement like that that it needs to be recorded?'

> But one who has tasted the honey of Satyen's profoundly satisfying friendship and unostentatious familiarity would read the essence of this statement in the charm and magic of that taste of honey. They are sure to realize that he could give so generously only because he had acquired so much from his mixing with people. Such giving and such receiving are the privilege only of those who can give and receive in a rhythm of detached ease . . . Satyen could let himself loose and plunge into a carnival of sheer joy born of love and affection. How easily he could enter into relationships! . . . He had infinite patience and curiosity reserved for his friends; he would always have time to listen to their most trivial complaints . . . Once he accepted someone as his trusted friend, he would keep track of him forever.[13]

Roy recalls Bose praising Chaudhury's 'mastery' of a racy, colloquial, rational language. Bose, in his memoiral piece on Haritkrishna Deb, recalls: 'Mr Chaudhury wanted the younger generation to come forward, to express their own views about history, philosophy, poetry, science and so on in their essays. It is he who inspired Harit and some others to write in Bengali.'[14] Pramatha Chaudhury believed in Bacon's dictum, 'Reading maketh a full man, conference a ready man.' In a letter to Satyendranath he wrote :

> Our literature is sadly lacking in criticism and scholarly writing. For everything one cannot and should not depend on stalwarts like Rabindranath and Bankimchandra — besides great minds like them are always rare. We have to manage with the second best. This is where we come in. I consider it the duty of the educated people like us to make our fellow countrymen share in the knowledge.[15]

Though Satyendranath gave him moral support, he did not contribute anything to *Sabujpatra*. But one can almost hear the voice of the more mature Bose in this declaration. Perhaps Bose was not yet very sure about what he wanted to do and writing in Bengali for the common people had not yet become a mission with him. Perhaps it was his inherent modesty which prevented him from taking such a stand.

He was still very young. In 1921 he moved to Dhaka, where there was less distraction and some irritants. Soon he was busy working out problems in science and producing papers which made him famous overnight. Then he went to Europe — to France and Germany. His two-year stay in these two countries brought him into close contact with some of the best minds of the period and he had a good deal of intellectually stimulating exchanges. Since they did not necessarily lead to the publication of papers, one could term them as *āddās* par excellence. In his article in the Physics Reunion Souvenir Bose has written about the wonderful time he had in Europe. Anyone can enjoy Europe provided he is not bothered by thoughts of finishing his thesis.

From all accounts it seems Europe was the ideal place for Satyendranath. Why did he not extend his stay or look for employment there? He would not have had any dearth of offers certainly. But then along with this predilection for the *āddā*, he had another trait which urged him to learn the latest scientific techniques. There was the entire

student community waiting for him back home. Yes, he too had a mission — the nationalist mission. He must absorb the best and pass it on to the students in India. They had nothing — so his one overriding desire was to teach them whatever was in his capacity to teach.

Bose went back to Dhaka and almost literally lived for his students.

Meanwhile in Calcutta in 1931, when Sudhindranath Datta (1901-60), the eminent poet, initiated a new Bengali quarterly, *Parichay*, its first editorial board comprised of Charuchandra Dutt (1876-1952), a retired civilian and nationalist, Girijapati Bhattacharjee, Nirendranath Ray, Dhurjatiprasad Mukherjee, Prabodhchandra Bagchi (1898-1956),scholar in Buddhist and Tantric studies, Subodh Mukherji (a friend of Girijapati's), Sudhindranath Datta, and Satyendranath Bose, who happened to be in Calcutta for his summer holidays when the journal was being planned. Girijapati later recalled :

> We had problems with Satyen. We were determined to have an essay by him in the very first issue. When all our persuasion failed, we had no other way but to apply the ultimate Fascist method. We shut him up in a room and told him that we'd not release him until he had written his piece. Satyen is a born man of science. Under the accumulated personal pressure he came up with an impersonal essay — *Vijnan-er Sankat* [Crisis in Science]. That was his first and last essay for *Parichay*. Whenever he came down to Calcutta he never failed to turn up at the *Parichay ādḍā*. But we did not have the power to make him write a second piece for us . . . But Bengal should not forget that Satyen Bose, who now leads the way in the exploration and dissemination of science in Bengali through the periodical *Jnan O Vijnan* and the Bangiya Vijnan Parishad, had written on the most abstruse ideas of science in Bengali for the first time in the pages of *Parichay*.[16]

Dhurjatiprasad Mukherjee, in a rejoinder to Hirankumar Sanyal's short history of *Parichay*, published in *Parichay* in the early fifties, recalled at least one instance when Sudhindranath Datta and Satyendranath had joined forces to slash a 'massive piece of a great scholar' to edit it to size, and had 'lain low in fear and embarrassment' for a long while before and after publication of the issue.[17]

Edward Shils, in his introduction to *The World of Twilight*, Sudhindranath's incomplete autobiography, quotes a writer in *The Times Literary Supplement* in 1936, who wrote that *Parichay* 'marks the definite passing of Indian literary criticism into the modern world . . . it has set itself far above anything else in Indian periodical literature, in catholicity, range, intelligence and freedom from prejudice.'[18]

The first issue of *Parichay* appeared in July 1931. For several months before that the *Parichay* group was meeting for their *ādḍā* every Friday, at Sudhindranath's house. As Hirankumar Sanyal later recalled : 'The purpose was work, not *ādḍā*. In other words, selection of items for publication, making arrangements to collect contributions, and meeting people for the purpose. Sudhin provided for light refreshments at every session. But what had begun as a society gradually turned into a mere *ādḍā*.'[19] Bose's participation or presence in some of these *ādḍā*s is recorded in a diary maintained by

one of the regulars :[20]

10 April 1936

Turned up at the session and found Satyen Bose speaking about his experiences on tour through north-west India. He was deeply moved at Bodhgaya . . .

Satyen Bose, Dhurjatiprasad, Charu Dutt, even Mallikda prodded me to tell absurd tales . . . Satyen-babu asked me all about how the Sinha brothers Jogish and Harish, and Somnath Maitra were, as for their health. He is equally attached to all . . .

14 August 1936

We met at Prabodh Bagchi's place today. The room was quite full when I walked in. Besides most of the regulars, there were Satyen Bose, Priyaranjan Sen, Suren Maitra and Pramatha Chaudhury. The discussion centred around the clash between the Muslim League and the Krishak Praja Party . . . Then a member suggested that the introduction of the Roman script all over India would bring the provincial languages closer to one another. Prabodhchandra Bagchi, Priyaranjan Sen and Jibanmoy Roy protested loudly. Susobhan Sarkar defended the suggestion. I find his argument unassailable

Why not Urdu? Asked one. Well, shorthand would be even better — retorted another.

Satyen Bose who was listening quietly now joined in. He said, Urdu does not have short vowels. Besides ability to read Urdu requires a good knowledge of grammar.

7 May 1937

Satyen Bose and Dhurjatiprasad were present today, saw Kiran Mukherjee [1880-1940, outstanding classical scholar and university lecturer] after a long time . . . When I came in, people were talking about Rabindranath's charitable disposition.Dhurjati-babu said, the poet often could not help obliging a persistent suitor, and gave away testimonials to people without having properly read their works. After having cited a few examples, he expressed his serious disapproval of having Rabindranath's name heading a list of progressive writers.

Dhurjatiprasad tends to exaggerate in his strain of mockery. I found him in devastating form today.

When he failed to draw any response from Satyen-babu he turned to Saratchandra [Saratchandra Chatterjee (1876-1938), perhaps the most popular Bengali novelist still].

4 June 1937

Found Satyen Bose and Jnan Mukherjee among the scientists today. Since

> the meeting was at Prabodh Bagchi's place today, Pramatha Chaudhury represented the writers. Besides the regulars there was Suhrit Sinha the psychiatrist. The room was full. I sat in a corner and listened to Girijapati speaking on C V Raman's plight. Niren Ray said that Meghnad Saha had told him that he was not responsible for Raman's fall from power. Yet he seemed to look quite happy. Somebody seems to have said that Syamaprasad Mookerjee has had a hand in it.
>
> I didn't hear either Satyen Bose or Jnan Mukherjee make a single comment on all this. Somebody said Syamaprasad would not try to influence the Central Government in the matter. The real reason was Raman's executive inefficiency.

Bose could attend these meetings in the summer holidays, when he came down to Calcutta. What is evident from the diary is that most of the time he kept quiet, never criticizing others even under great provocation. He was far too refined for that.

But he had a strange power to draw like-minded people. In Dhaka he formed a small club of twelve members. When his friend, Dilip Roy, came down to Dhaka they had musical soirees. Even in Santiniketan he was successful in forming the International Club but as he was forced to leave the club too was short-lived.

After coming back to Calcutta he used to have gatherings every Saturday at his Iswar Mill Lane home. There were discussions on all possible subjects from the history of Calcutta's street names to Mohenjodaro scripts — detailed accounts of which have been given by many of his Bengali biographers.

His versatility could only be properly understood by the highly gifted. He made some useful suggestions to Sunitikumar Chatterji when he showed him the file copies of his magnum opus *The Origin and Development of the Bengali Language.* These suggestions were incorporated in the final version, and acknowledged in the preface.[21]

Describing Bose as 'the king of the *āddā*', Hirankumar Sanyal(1899-1978), eminent writer and photographer, wrote:

> There was a time when he would travel from one end of Calcutta to another, tempted solely by the charm of the *āddā*— sometimes perhaps even walking all the way. When in late life he could no longer walk distances, the *āddās* took place in his own house, every Saturday afternoon, over tea and *ghugni*. [light snack made of peas or Bengal grams]. Matchless *ghugni* prepared at home. They could of course be described as literary *āddas*, and *āddās* devoted to science, philosophy and history; for there was just no subject that was not taken up in these *āddās*, and when Satyen-babu commented on any of these, it seemed as if he was an authority on every field except science, for there used to be little debate on scientific matters.[22]

Bose's interests took in the widest possible range. Dhurjatiprasad Mukherjee recounted how, when he had gone temporarily blind just before his M A Examinations in History, and found it difficult to cope with the course in Egyptian History, it was Bose who gave him the briefing that enabled him to answer the three questions set for the

examinations with a competence that aroused the wonder of his examiner, who told him one day : 'Dhurjati, I was quite surprised when I went through your script. You never came to my class. You never seemed to have studied the Ancient East. Yet you wrote so well!' Dhurjatiprasad answered : 'Satyen taught me.' The professor said, 'Oh, Satyen, that explains it.'[23]

Bose was present at the *Parichay* session where Jamini Roy (1887-1972), the eminent painter, joined for the first time. On Roy's second visit to the *Parichay āddā*, on 10 June 1938, the entire discussion centred on Indian painting, when Sudhindranath turned to the painter and asked him, 'Jamini-da, last day you evaded answering a question. You have to face it today. Tell us which particular branch of Indian art you hold in special regard — Rajput, Mughal, Kangra, Ajanta —' Satyen Bose smiled and said, 'Let's hear now.' Jamini-babu turned to Satyen Bose and said that he had no special regard for any of them. When Sudhindranath referred to the Ajanta fresco of the female attendant turning round that he found 'magnificent', Jamini Roy argued that there could be exceptions 'where hundreds of artists were at work', but if 'one took an overall view of the Ajanta frescoes, one would see faulty, unsightly lines, inconsistencies in perspective, incompetent attempts to create perspective by stretching the corners of the houses. If the artists were really great, how could they make such ridiculous mistakes?' He took the firm position, 'Either everybody would be perfect, or entirely imaginary and unreal. The two cannot be mixed in a creative frenzy. What's needed is integrity.'

The diarist reports, 'So long Satyen Bose had been running his fingers through his unruly hair, and had not said a word. The smile had vanished from his face. He suddenly said : "Jamini-da, I have found the aetiology of your art." Jamini Roy spoke with an urgency in his voice, "Tell me." [Satyen Bose said] "You've found a working theory for your surrealism, a shelter from where you can defend yourself from the critics of the world." At Satyen-babu's words Sudhindra broke into a loud laugh.'[24]

One of the most familiar photographs of Bose (reproduced elsewhere in this volume) shows him playing the esraj, his favourite musical instrument. Sunitikumar Chatterji testifies, 'Satyendranath had friends among musicians and singers, and he had picked up a rare accomplishment — he is a very good esraj player.'

Dilipkumar recalled :

> I recall . . . how we searched together for the houses of the great ustads to hear them sing. Once in Dhaka we sought out together the house of Renuka Sengupta who later became famous all over Bengal for her song 'Gokulachandra Brajey na elo . . . ' Later when I went to her place to give her lessons in music Satyen would often accompany me to hear her angelic voice. He was a real connoisseur of music — he had the perfect discernment to recognize good singing in a flash. He learnt to play the esraj, to follow music better. This was the only field where I could explain things to him with authority, and feel a little pride.[25]

It was Bose's fascination with French literature — 'watching him read French plays and novels' — that drove Dilipkumar to start taking lessons in French from a French

woman staying at the Grand Hotel!

All through life Bose remained more interested in absorbing ideas than expressing them in print. A voracious reader, he was completely at home in French, German and Sanskrit, though French remained his special favourite. His own output in the vernacular was slender, but there seemed to be a belated flowering in his seventies. In the last ten years of his life he took to translations as an intellectual pastime allowing his mind to wander from the intricacies of mathematics. He translated mostly from French, but also from German and Italian. For obvious reasons English renderings of these translations have not been included in this volume but his absolute command over the language and flexibility of style take the reader by surprise. These works contain three stories by Joseph Agnon, one by Heinrich Bohl, excerpts from the diaries of Romain Rolland, André Malreaux's account of his interview with Mao Tse-Tung, and an account of the last seven days of Tolstoi's life from Henri Troyat's biography in French. The shadow of Einstein looms large over his translations of scientific writings. They include a lecture given by Einstein in 1931, Pauli on Einstein at a conference in 1955, a conversation which took place between Tagore and Einstein in 1931. Bose published a summary in Bengali of an article published in *Le Monde* on the theory of Hoyle and Narlikar. Most of the translations are 'free renderings', not 'strictly literal', as Bose himself warns in his prefatory notes.

Bose's love for and familiarity with French literature owe a lot to his good friend, Mme Jacqueline Eisenmann, whom Bose met for the first time during his first visit to Paris; and they kept in touch till Bose's death. Mme Eisenmann never visited India but remained a close friend. It was she who had gone on supplying Bose with the latest books and periodicals published in France.

Bose's zest for life was unbounded, his appreciation of a good thing was that of a true connoisseur. He was a man who will perpetually elude classification. B D Nag Chaudhuri records :

> I recall many years ago a conversation in the company of several others. Satyen Bose loved good conversation laced with lots of tea. Bose was arguing, in part with himself, not quite sure whether it was his expanding horizon in science which led him to a wider perspective of life, this desire to achieve a fullness of life, to seek a certain completeness within him, or whether it was the other way round that a certain intuitive and satisfying concept of the total man led him to develop progressively wider perspectives. His own view was that probably both grew in him simultaneously. [26]

Notes

1 Childhood

1 Jawaharlal Nehru, *The Discovery of India*, sixth edition, Signet Press, Calcutta 1956, p. 332.
2 B C Pal, *Memories of My Life and Times*, Calcutta 1932, pp. 425-28, as quoted in Benoy Ghose, *Vidyāsāgar O Bāngāli Samāj* [Vidyasagar and Bengali Society], Orient Longman, Calcutta 1973, p. 310.
3 Ganendranath Tagore, in his secretary's address to the second session of the Hindu Mela, held in 1868, quoted by Ghose, op. cit. p. 315. Translated.
4 Nirendranath Ray, 'Professor S N Bose (an Impression of his Personality),' *Satyendranath Bose Seventieth Birthday Commemoration Volume*, Part I, Professor S N Bose Seventieth Birthday Celebration Committee, Calcutta 1965, p.6.
5 Ibid. p. 6.
6 Ibid. pp. 6-7.
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3 S N Bose, 'Āmār Vijnan Charchār Purākhanḍa' (in Bengali), originally published in *Sāptāhik Basumati*, special puja issue, 1374 [Bengali era, corresponding to 1967], included in *Rachanā Sankalan*. Translated in this volume as 'The Early Phase of My Explorations in Science'.
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5 S N Bose papers, Dhaka University, no. 7.
6 S N Bose papers, Dhaka University, no. 4.
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8 Correspondence : S N Bose to M N Saha, Meghnad Saha Letters. Courtesy Nehru Memorial Museum and Library, Delhi.
9 Correspondence : S N Bose to M N Saha, Meghnad Saha Letters.Courtesy Nehru Memorial Museum and Library, Delhi.
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10 The Complete Man

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S N Bose : Lectures and Addresses

1

Science in Education

Mr President, Ladies and Gentlemen,

I confess at the outset that I had great misgivings in choosing as the subject of this evening's discourse, the place of Science in Education. My difficulty arose on the ground of my ignorance as to what this conference really stood for, what it exactly aimed at; whether it sought to change the present educational system radically or was merely an attempt to revise the arrangements in vogue.

I am aware the Hon'ble Minister of Education took pains to convince us that the present system of education was by no means an ideal one, especially as it was indirectly responsible for the steady increase in the unemployment of our educated youths; and we are grateful to him for his vigorous attempts to devise plans to remedy some of its glaring defects. To be candid, if I had not been assured that you all desire to revise and reconsider the present system, I would not have wasted your time by pointing out to you that science is one of the crying needs of our present educational system. For one can do very well without much science, or any real scientific teaching, if all that is wanted is somehow to impart literacy to the masses. But if we propose to look on education as the means of effecting adjustment between man and his environment, we cannot but recognize the supreme importance of the study of science in schools and universities. I need not remind you how science has hitherto played a very insignificant part in our educational programme. It had no place in schools when we were boys, and though during the last two decades the remodelling of the university courses has offered increased facilities for scientific studies to the college students, the schools are still practically without it. I do not make this point with any controversial

bias, and I may be mistaken in my apprehension of facts. But if any plan proposes to go to the roots of our educational needs, it must, you would admit, allot to science and scientific teaching a more important role, not only in the curricula of the universities, but also in those of the schools — I therefore take this opportunity to express my gratitude to the organizers of the week, for extending to me the privilege of pleading before this distinguished gathering of educationists, the case for science in education. I need hardly add that I am not one of those who would dispense with either science or education in life altogether, simply because neither our universities nor our schools have so far succeeded in solving the economic and cultural problems of our lives. As a matter of fact, this failure is writ large on the face of almost every institution and organization in the province. But the remedy for this state of things does not lie in renunciation of all organized social efforts and a return to wilderness and asceticism. We need not put on sack-cloth and ashes and adopt some simplified form of existence hardly distinguishable from primitive patterns of pastoral life. Lack of direction in our educational efforts has, to my mind, brought about this deplorable state of things; and the amelioration lies not in the abandonment of hopes and obligations but in the adoption of a conscious and consistent plan, and a united will to realize it.

The charge that our educated youth is an economic misfit has been often repeated. Assuming that he is so, is it fair to put the whole blame on him alone? What is really responsible for the fact that our young men cannot utilize their learning in any practical way and find for themselves suitable places in the economic arrangement of the country? Is it because we have not got even a decent amount of trade and industry here, which can offer them occupation? How is it then that hundreds and thousands of strangers make their easy pile in this land? Does it not look strange that side by side with this amassing of large fortunes by others, our young men, who spend the best years of their lives and a considerable portion of the slender resources of their families in acquiring academic distinctions, should be practically starving for want of employment? These questions cannot be answered without going into the historical conditions under which the present educational policy was inaugurated. It is an open secret that the policy was neither scientific nor economic in its origin; its object was not to provide education of a type best suited to the social needs, and scientific considerations did not come within the purview of the educational authorities of the bygone days, in determining the interests of the people concerned. But this is not to deny that it sought to implant an ideal in the minds of the young men of those days, and to serve a useful purpose: for youths of the country were necessary to interpret a new civilization and a new culture, and act as cheap but efficient links in the long chain of administration flung far and wide over the land. There was in consequence sufficient employment for them and the question of fit or misfit never arose. The educational experiment, on the contrary, was so effective, that its range expanded by leaps and bounds, as years went by. But changes gradually came over the whole country, and were accentuated during the last two decades. Imperative social and economic needs made the parochial conception of a hyper-literary education obsolete and useless, and the old system, at best suitable for a leisured and affluent few, continued to cater for the toiling millions.

What else should you expect in the circumstances except a misfit? And should you not in all fairness apportion the blame of the sad situation between the authorities and the students, instead of criticizing the students alone? Should we not desist therefore from further complicating the issue by introducing the question of race, congenital inferiority and all such inane erudities? It is, however, futile to discuss who is or who is not to blame for this deplorable state of things. The important point is to devise such changes as will adjust the educational system to altered conditions, and to do this we must approach the question in a dispassionate and scientific manner, which alone will reveal not only the magnitude of our task but also the methods that we have to adopt to surmount our peculiar difficulties. The importance of a critical survey of the actual conditions prevalent in the country cannot in this respect be overemphasized, and in an examination of ways and means, the essential must be separated from the inessential. It is no doubt, for instance, desirable for a budding teacher to know the height of the highest mountain and the length of the longest canal, as well as the intricacies of the lordly game of cricket and the hierarchy that holds together cabbages and kings. But it is more than perilous to miss the interdependence of anopheles and malaria, and forget the connexion between cholera and drinking water. The life of the community depends on a detailed knowledge of its natural resources, and in an agricultural country with an increasing birth-rate the modern methods for improving the yield of the land deserve a much greater attention than they have hitherto received. It is needless to remind you again that in this land superstition is still the prime mover, and no amount of book-learning will overcome it sooner than an intimate contact with the modern miracles of man-made machines. Thus will the supremacy of law and order even in inanimate nature become implanted in our minds and events will cease to be ordained by stars or determined by a malevolent fate. The distinction of science from idle speculation, and its connection with common sense has been too often stressed for me to emphasize anew. Your philosopher is surely justified in dilating on the epistemological difficulties of accepting any recognized cosmology and may even dismiss time out of court for want of what he considers to be logical evidence; but we men in the streets are denied such luxuries and if we fail to provide for the morrow we shall surely starve. The study of ancient history and archaeology, of literature and theology have, no doubt, their ennobling influence and make us conscious of our past heritage. But science with its causal laws and technical mastery over nature is essential, if our aim is to harmonize life with its surroundings and to seek to harness elemental forces to our will and service. The sooner we impress upon the minds of the coming generations that individual destinies are amenable to human control, the nearer we shall be to whatever be the goal.

It is rather late in the day to extol the achievements of the Japanese; and their phenomenal rise in the political and economic sphere is a tale that has lost its point by endless repetition. It is, nevertheless, important to remember that their contact with Western science and culture began not much earlier than ours. Yet Japan today has not only left us behind in the race of progress, but many of the pioneers of European industrialism are unable to keep pace with her. This has been brought about by careful

and conscious planning and not by cultivating a fastidious accent in foreign tongues. So we may laugh as much as we like at their mistakes in English and their woeful lack of expression in languages other than their own; but their adaptation of educational methods to national needs should become an object lesson for those who are still opposed to any sort of planning, and adhere to the medieval ideal of a cloistered education. But education and scientific planning can no longer be dissociated with impunity in the modern world; and even in the country which, we heard, won the battle of Waterloo in the playing fields of Eton and Harrow, education tends to become more and more technical and utilitarian and less and less of an expensive luxury. If the paramount importance of planning in an efficient education is now recognized in a conservative country like England, the part it plays in progressive America or Germany need not be stressed here when it is well known that in these countries many a wonderful discovery in science have, in recent years, resulted from well-planned efforts to supplement national resources.

I am aware that planning in a vague sense is not quite absent in our educational system; and many improvements have been or are about to be effected in providing technical education for grown-up college students. But in order that such plans may bear fruit, it is necessary not only to organize the final forms, but also to think out carefully all the steps leading thereto; and unless the ground is assiduously prepared from the initial stage, such efforts are likely to remain sterile. Success in technology is a triumph of the scientific spirit, and a mind nourished on purely literary ideas is less likely to absorb it than one on which classical moulds have not left their indelible mark. Training in the scientific mode of thinking should, therefore, begin as early as possible, preferably from the primary schools. This, however, must not be confused with either technical education or vocational training, but boys should be familiarized with the elements of natural sciences and the principles of hygiene and biology. I have been told that these subjects are touched upon by some of the recommended textbooks and teachers speak on these topics in their classes. But that is not enough. Pictures and fairy tales about science and scientists are worse than useless; and students, if they are to form any idea of the subject should have direct experience of the scientific method which is not, in reality, a dexterity with instruments, nor an initiation into the mysteries of mathematics, but a training of the senses in direct contact with Nature. Experiments are, of course, essential. But they do not require, in the early stages, expensive instruments. Nature herself should be the laboratory; and an ingenious teacher can find a lot to talk about in the everyday world familiar to the student from his birth. The natural setting of the primary school is the village; and for an agricultural country no better background for an educational system could be imagined. For all the fundamental forms of life are assembled there: agriculture and its possibilities, marketing and its various difficulties, public health and its deadly enemies, machines and means of transport in various stages of development, all are there ready for objective study. If therefore the right sort of teacher is forthcoming, the elementary economy of the Bengali village will be no handicap to the youthful seekers of scientific knowledge.

The teacher is all-important in such a system and his example and sympathy will counterbalance the deficiencies in equipment. He must, therefore, be carefully trained, and acquire not merely book-learning in methods of teaching, but a real enthusiasm for his work. For it is useless to cite examples from foreign textbooks when young children are concerned; and in order that knowledge may become vital to them more attention should be paid to local conditions and instruction should be through materials in daily use. The medium of instruction is, therefore, of foremost importance, and it is not enough that children should be taught, as they are now, in their mother tongue; but that the teachers themselves should learn to think in their own language. This, it is needless to point out, requires the adoption of Bengali as the medium of instruction throughout the whole of the educational system. At any rate, so far as the teaching of science is concerned, there is no other alternative, if in ordinary scientific education we should aim at the cultivation of a scientific attitude rather than proficiency in scientific and pseudo-scientific catchwords. I am aware of the immense difficulties involved in this desirable change in the existing order of things. But remembering the solid achievements of a young university like the Osmania, I believe my suggestions are neither unworkable nor incapable of realization in the near future. What is indispensable for success is the goodwill of the whole country, the cooperation of universities and educational institutions, aspirations of the teachers and determination of the educational authorities. . .

I repeat again that the soundness or otherwise of every educational system depends entirely on the teachers, especially in the primary stages, where foundations of all future activities are laid in extremely impressionable soil. They, therefore, should be carefully recruited, furnished with the necessities of life, freed from material anxieties to devote all their energy and time to the realization of educational ideals and made alive to their great responsibility and power for good that they might easily become. For this purpose such meetings as this conference should become a regular feature of our national life. At the same time less complicated methods to foster closer cooperation and promote more frequent exchange of ideas among teachers must be devised. For Bengal is a vast country with insufficient means of communication between distant parts where isolation of workers is more the rule than the exception; and personal meetings, even if not expensive, are impracticable here. An obvious way out of these natural difficulties would be establishment of a periodical through which teachers would state their problems, receive suggestions, keep in touch with modern educational movements, gather information about current achievements in science and to some extent make up for the lack of usual library facilities. So that its appeal may be universal, the journal must perforce be in the vernacular. There should also be state-aided central libraries feeding different groups of schools, judiciously distributed throughout the province, and stocked with books of reference which are ordinarily beyond the means of primary and secondary schools.

But a forum for teachers and co-ordination of primary schools alone are not enough to ensure a sound scientific education. The secondary schools as well as the subsequent institutions have also to be suitably planned. Here the personality and sense of

responsibility of the teachers alone will be insufficient, if the teaching of science is to be taken out of its rudimentary state and made suitable for the complexities of modern life. Provision of properly equipped laboratories where students with aptitude can work in natural and biological sciences becomes imperative at this stage. It would perhaps be a Utopian dream to expect such resources in every secondary school, but some selected institutions must be established, one in every district at least, if science is to be of any practical utility in national life. It nevertheless is unnecessary to make teaching of science compulsory in secondary schools as it is suggested to be made for the primary stages. On the other hand, in case of students who choose science as a special subject, some fairly rigorous test of fitness is desirable. For it would never do to repeat the mistake that has resulted in the overcrowding of some of the professions; and as the minimum of science indispensable for the purposes of daily life is already provided for in the early stages, the available resources should not be wasted indiscriminately, but utilized for the really deserving student. The process of selection should not therefore end in mere written examinations, but be supplemented by such practical tests as will determine his talent for application. For science, divorced from pragmatic considerations, is even worse than speculative philosophy.

I am confident that if these suggestions are given effect to sufficiently early, the universities and advanced technological institutions need not be handicapped for want of suitable human material. On the contrary, there is bound to be a sufficient number of capable students who could both be adequately trained and trusted to carry out the programme for higher studies, and this will serve the real need of the country sufficiently if all the other institutions, like the universities, have already chalked out their programme on the basis of a survey of our national needs. For after all, it is not a question of only producing the right sort or number of students in as economical and efficient a way as possible but also of forming a plan on the actual survey of the country's needs. It is not for me to talk about the university methods, the higher studies, in this gathering. But just as it must be emphasized that we cannot be too careful in training our students, it must be equally obvious that the programme of higher training must be directly and intimately in touch with the actual needs. If we really want to solve the problem of unemployment and make our educational institutions sufficiently effective and responsible, we must so build up our universities and schools as if they were expanded departments of our social existence. It follows from this that science cannot be made to face social problems adequately unless and until universities are linked up with the industrial and commercial activities of the country, and co-ordinated with such Government concerns as make practical use of scientific investigations. What we have to bear in mind is that the question of scientific training and that of the industrial and technical activities in the country do not fall apart; on the contrary they are intimately bound up with one another. If, for instance, we do not utilize the myriad centres of industry and agriculture for giving our students an opportunity to work out their more or less theoretical knowledge, we would be leaving their training half finished. Our experts, at best, will be more of the academic type and therefore not sufficiently useful for practical commercial purposes. If, on the other

hand, commerce and technical departments have to recruit all their specialized staff and skilled labour from abroad and not draw upon the indigenous sources, they are bound to suffer from the point of view of economy as well as the country as a whole from the point of view of efficiency. What is, therefore, of supreme importance is that we should provide for as direct an interchange of services as possible between our educational institutions and the technical and industrial centres with assurance that they stand in a relation of interdependence and form between them the social order as a whole. I need hardly add that in the nature of things, there must be an unrestricted give and take between the educational institutions and non-academic undertakings. We should never really be ashamed of going down to the mines, factories and railway lines to finish our scientific training, nor should industry fight shy of our universities on the false ground that they produce only theoretical and academic men.

I have talked sufficiently long and it is time for me to wind up. I am not at all sure if I have been able to convey to you what value I attach to science as it is, or its place in education. But I do not want you to carry away the idea that I am one of those who teach science and prepare its principles on purely pragmatic grounds — and hold that science has nothing spiritual about it — that it is altogether utilitarian in its scope and serves no end that is not rooted in what may be called the mundane existence. I have insisted much too often on the place and the value of science in the economy of our social needs. As a matter of fact a good portion of my lecture has gone to explain how from the primary to the university stage science is bound up with its environment, how the village, in the case of the primary, and the town and the city in the secondary and university training form the necessary background for an efficient and all-round scientific education. But in all this it never was for one moment my object to draw any line of distinction between the spiritual and the material, and in so far as I am convinced of the essentially liberating influence of the scientific spirit, and the profoundly humanizing tone of its investigations, I believe also in its intimate contact with the utilitarian and the mundane. After all it is not the universe alone, but human life too, that is one whole; we do it harm and injury by dissecting it. What we should always bear in mind is that in so far as science is ministering to the economic and social needs of the human race, it is really working out its deep spiritual purpose.

Lecture delivered at a conference held in connection with the Bengal Education Week in February 1936 at Calcutta. From the Proceedings of The Bengal Education Week, *1936, vol. I, pp.56-64. Obtained by courtesy of Rathindranath Bose.*

2

Search for New Sources of Power

Ladies and Gentlemen,

I deem it a very great privilege to be able to address you on the occasion of the birthday of Sir J C Bose. I am among the fortunate who were able to sit at the feet of the great master for their first lessons in modern physics ; and I still recollect the thrill of intense delight which we all felt, when he modestly talked about his striking discoveries on electric waves in his class. His own life was a flaming example of devotion to science ; and the fact that many of the students of our period had deliberately chosen science as their calling, at a time when the facilities for such studies were rare, had been in no small measure due to the inspiring examples of those great pioneers of research in Bengal, Sir J C Bose and Sir P C Ray. May their memory live long and continue to inspire successive generations of students in our land.

I have chosen 'Power' as the subject of today's address ; we are all interested in quick and extensive development of our power industry by the utilization of India's natural resources. Our ultimate source of energy, the Sun, is apparent as an incandescent disc which subtends an angle of about 32 minutes to an observer on the earth. In reality it is an incandescent globe of vast dimensions, 1.39×10^6 kilometres in diameter, but very far away from us, 1.49×10^8 kilometres. Seen from the Sun, the earth, our little globe, will appear as a speck of dust in the vast space. In fact our earth collects 0.5×10^{-9} fraction of the total energy radiated by the Sun at every instant. This small fraction nevertheless amounts to a constant reception of 1.6×10^{14} kilowatts, a tremendous amount distributed at the level of the stratosphere or 1.35 kilowatt for every square metre at sea-level.

Ages ago, our little planet was born as the result of a cosmic upheaval. Originally an incandescent mass had separated out of the materials thrown out from the Sun, and had gradually cooled down, through about 2,000 million years from an incandescent state to what it is now today. Deep crusts have now formed over the once molten mass, and land rocks, continents and oceans have been formed.

Life appeared at one stage on our planet, and thenceforth through its various manifestations has unceasingly worked on and produced far-reaching consequences on earth.

Under its ceaseless thrusts, rocks have crumbled to soil, vegetations have covered bare continents. We do not yet understand life but we realize that the power necessary for such tremendous transformation has been ultimately derived by life from the energy that the earth continuously receives from the Sun. It is the radiation from the Sun, which provokes evaporation from the sea ; rain and snow reprecipitate this moisture and water flows back ultimately to the sea, through thousands of rivers. The Sun's heat is also the ultimate cause of atmospheric circulation. The plant world traps the daily flow of energy by the photosynthetic process, and stores it as food, which ultimately sustains the whole animal world and builds the plant body with energy-rich carbonaceous material. This process has gone on for ages, ever since life has appeared on this earth and though endless generations have been born and have died, the results of life's photo-synthetic activity has not been all lost. It subsists in the deposits of coal and oil, which form the raw materials for the generation of power for the present age.

These natural processes, (1) the circulation of water from the land back to the sea, which provides the basis of hydroelectric power, (2) the photo-synthetic process which determines the growth of plants, are however not very efficient in the sense that only some thousandth part of the actual energy received from the Sun is utilized in these processes. Most of the radiation that we receive is ultimately scattered back into space. There is thus room for speculation about means of better utilization of this abundant power we daily receive for the ultimate good of man.

Enduring achievements can only be brought out by large concentration of power devoted to the purpose in view. Before the age of steam and coal, man had relied on large scale employment of human and animal labour. Food and comfortable surroundings were then the principal quests and agriculture was the principal industry which engaged the attention of man. Other needs of the human society, its garments and its shelter, were also met then by unaided human skill. The development of the mechanical sense however has gradually transformed the course of human efforts.

Human ingenuity had been devoted to the discovery of labour-saving devices, and the growth of scientific knowledge had aided materially to bring about the industrial revolution. The tempo of progress has increased enormously with the discovery of the steam engine and later by the understanding of electro-dynamical processes, and the manifold uses to which electricity can be put. Modern civilization is now based on large scale uses of natural resources and means, whereby convenient concentration can be directed on any object, and human labour does no longer play a preponderant role in

all human efforts, especially among nations who are at present in the vanguard of human progress.

The extent of electric power development in a country can now be regarded as a positive index of the economic prosperity and the standard of living of its people. As an Indian my thoughts naturally turn to my own country, and here I sadly note that India is still a long way behind the industrially advanced countries.

In spite of magnificent ancient achievements and contribution to human civilization, the present day India ranks among the underdeveloped countries, where efforts will have to be made now to utilize the natural resources that lie buried in the land, or to utilize the natural advantages which its position and geography have lent to this country. It is clear that the future development must be carefully planned and a careful survey of all our resources for the generation of electric power should be undertaken immediately.

The three chief sources of generation of electric power are oil, coal and water-flow. Our known mineral oil resources are not very significant. For our consumption we have still to rely on foreign imports, and though the recent talks about the probability of oil deposits in Bengal have encouraged us to dream of a blissful prosperity in the near future, much yet remains to be done and explored before we can really take oil into account in formulating our future plans. Coal however is apparently plentiful. Proved natural resources of coal here according to a recent Government publication is about 16,000 million tons, and probable total reserves may be still higher, say about 60,000 million tons. Much of it however is added with inconveniently large ash-content, or probably contains harmful ingredients such as sulphur in its composition. We have also to remember that large scale industrial developments will require development of extensive metallurgical processes in this country which would principally require high grade coal. Use of coal for transport and power development would have to be carefully thought out in a manner which permits the most economic use of our natural resources. This has, unfortunately, not been considered so long and much of our valuable deposits have been thus frittered away or wasted unnecessarily.

It is now comforting to think that recently greater care is being bestowed on our reserves, and our future developments will try to conserve our good coal and put our low-grade coals to increasing uses.

It is perhaps relevant to remark that other more highly industrialized countries have thought about the uses of low-grade fuel and have evolved methods by which they can be efficiently and conveniently utilized. Efficient methods of combustion have been worked out and extensive researches have been undertaken on the problem of gasifications. One hears of electric supply in big cities elsewhere (in Russia for example) being now secured by utilizing combustion of peat and lignite and low-grade coals and attention is now mainly directed towards attaining a better efficiency ratio, by using higher pressures in boilers and more efficient generators. It is a depressing sight in our country to see coal freely burning in open hearths, whereby useful gases which could have been utilized in developing chemical industries are being carelessly wasted

away.

The heavy and dense smoke that hangs about now in the evenings in the streets of Calcutta is indication of how careless we are in our daily practices and how urgent is the necessity of intelligently tackling our common-day problems.

Turning now from coal to water power : We hopefully observe that large-scale developments of our hydroelectric resources are on the eve of taking place. There have been significant developments of water power in South and West India, where in Bombay, Travancore, Cochin, Mysore and Madras increasing uses of our resources in water power are being made now. We hear of Bhakra-Nangal, Hirakud and the D V C undertakings and we hear of Kosi and Teesta surveys being undertaken for the development of power.

A large-scale development of hydroelectric power in our country has evidently much to recommend itself. Here as in all other countries we have to remember that once the costly undertakings, barrages and installations are over, we utilize resources that nature annually gives us free and our supplies in dams being annually replenished by precipitation are perennial sources of power, which would not mean any progressive and quick impoverishment in natural resources as would happen if we relied on the burning of coal or oil reserves. Other countries have begun to think of their coal resources, and have been seriously exploring alternative means of generation of power, which may ultimately replace the gradual exhaustion of their resources. Even in countries which have no plentiful water power such as Canada or Scandinavia, people have turned their serious attention to the quick and efficient development of water power. Indian engineers however seem to be more cautious, and even where there is water-flow and hydraulic head, the problem of transport of heavy machinery seems to them to be occasionally a very deterrent factor for ultimate utilization.

It is clear however that once we have fixed our plans, no difficulty need deter us. In other countries such difficulties of transport and comparative inaccessibility have been tackled in various ways.

We may for example think of building our units in *situ*, instead of having the full-fledged units transport over long distances. In all such matters the old adage that 'if there is will there is a way' still remains valid to a great extent.

During the recent war, we heard of tremendous happenings during which heavy war implements were transported by animal power over inaccessible mountain-barriers and such events had happened near the eastern borders of India. If one can achieve success by concentrated effort during war-time, what hinders us from thinking that such intense efforts will be lacking during peace-time when by such endeavours we will be making our country's future secure for once and for good ? I feel that more stress should be laid on water power development and all-out effort is needed to develop the resources to the fullest extent possible in our country.

It is clear that all large-scale development in any one direction means very often a simultaneous development to a high grade efficiency in other fields. For example, our industry should be ready to furnish the raw materials that may be needed and home

industries should be equal to the task of furnishing all steel, cement and other metals that may be needed. Our resources in other fields make us hope that they can be tackled, once our mind is made up about the matter.

The next five-year plan of future development is now on the anvil. Let us hope that an adequate and careful consideration will be given to the problem of adequate development of water power in India.

During my recent visit to Europe as a delegate to the International Conference on Crystallography, I had the good fortune to be able to study how France has been tackling its problem of development of electric power. After the war electricity in France has been nationalized. Large-scale hydroelectric developments have taken place after the war, and different centres of hydroelectric projects in the Central Massif, Pyrenees, Alps and in the Rhone Valley, have been developed to such an extent that France is now producing approximately 50 per cent of its total power output from its hydro-electric installations. When we remember the area of France, which is approximately 1/6th of India, its comparatively fewer rivers and its moderate precipitation, we have an objective demonstration of how much can be achieved by intelligent planning. We have also to remember that the annual power production of France stands at 40,000 million kilowatt hours, which is approximately ten times our present output, and 50 per cent of the output gives a figure which will exceed many times the projected output in our country by the hydroelectric schemes during the next five years.

The industries of France are able to consume fully the power that is thus developed. Her water power is however not able to tackle all the industrial problems and a simultaneous large-scale development of thermal stations has also taken place. I mention this only as an example of how a developed country has tried to conserve its rather slender supply of coal and has gone on for large-scale development in water power.

I have mentioned in the beginning how inefficiently we have been able to utilize the constant flow of solar energy. The tempo of modern developments has necessitated such large-scale expenditure of power that people have begun to think of discovering other ways of utilizing the solar energy which is now mostly scattered away. Whether solar energy can be trapped conveniently so that it would provide a cheap source of power is still a problem of the future. It is an enticing problem, and it may be interesting to note that the eminent Indian physicist in whose memory this lecture is being delivered had thought very early about the probable means of utilization, and perhaps that was one of the reasons which turned his attention from physics to biophysical problems. The role of chlorophyll always fascinated him and he had thought of utilizing in some way the entrapped energy other than the way the plant actually utilizes it. In his diary he writes :

> *5th March, 1885.* I have been long thinking whether the vast solar energy that is wasted in the tropical regions can in any way be utilized. Of course trees conserve the solar energy. But is there no other way of directly utilizing the radiant energy of the Sun ?

> Taking advantage of the heating effect, there have been attempts to construct solar engines which is merely a heat engine. We may also get thermoelectric current by heating one of the junctions. But such thermoelectric batteries are practically of not much use. Great amount of energy is also lost by the wasteful conduction.
>
> Now, I have been thinking whether we could not directly convert the energy of light into that of electric current.

However this problem still remains largely unsolved. Though recently news has come through of the achievement in America, where a significant progress in the development of photo-voltaic cells has been reported. In the absence of more detailed information, I am unable to report on the actual achievement, though we are all eager to know the full details of the discovery.

In the tropics where the Sun shines for more than two hundred days in a year, the problem of utilization of sunshine is always a fascinating one. We utilize energy not only for industrial purposes, but also for the enhancement of comforts and we at once remember the problems of air-conditioning and refrigeration which are so important here, as in all tropical countries. It has been reported that by means of heliostat and paraboloid mirrors this problem of utilizing solar energy for refrigeration has been successfully tackled in Tashkend, in the Soviet Russia in 1916.

A cement paraboloid 80 metre (which can probably be turned to follow the daily motion of the Sun) has been covered over by small mirrors of silvered glass which thus concentrates the Sun's heat on a boiler which is connected with a refrigerator which generates the cold by the ammonia cycle.

This news is interesting to us situated in the tropics as we are ; it opens out a prospect of so regulating our installations that we can comfortably endure our otherwise tiring summer seasons. The National Physical Laboratory of India has developed a few types of convenient solar cookers, water boilers, and it is hoped that further researches there will enable us to discover better ways of utilization of solar energy.

In France itself by means of huge paraboloid mirrors which are skilfully made to turn by means of photoelectric control, the tremendous concentration of solar energy has been utilized in a rather novel fashion. Extremely high temperatures are reported to have been obtained in solar furnaces and extreme refractories like zirconium oxide and alumina have been reported to be conveniently melted. They have also been utilized for metallurgical purposes and high purity ingots of several metals have been obtained.

Exciting news of photosynthetic biological activity of Chlorella have electrified the biological world. By its rapidity of growth and by its satisfying food-value, Chlorella promises to be a valuable aid in tackling the difficult food problem, and demonstrate at the same time possibly a more efficient use of solar energy.

I conclude my present discourse by reporting on the prospect of utilization of Atomic Energy for peaceful purposes. We have heard that an Electric Power Station has already been installed in Soviet Russia and that in England by 1970 there is a prospect

of Atomic Power Stations being installed which would supply power and electricity at competitive rates. While much of the necessary technical developments still remain secret, enough has been ventilated to show that this development is bound to occur in the near future, in the first instance in the countries which possess a convenient Uranium supply.

We have not as yet discovered any large source of high grade Uranium in our country, and we may safely presume that for the next twenty-five years, we would have to depend upon the old and conventional mode of generation of electricity, i.e. steam and water-turbines for our power supply.

This does not mean that the Atomic research in our country should be discouraged or that there are no ways of peacefully using the moderate sources of Atomic Energy that we may develop in India in the near future.

The recent Conference at Delhi has examined the problem from all points of view, and it is satisfactory to report that we are now understanding better our limitations and our immediate problems.

I have endeavoured in this brief survey to indicate the present-day trends in the search for sources of power. In India as elsewhere people have become conscious of the necessity of such development which will improve the lot of the common man, give him valuable and cheap mechanical aid, so that it will be easy for the society to give to each individual member sufficient leisure for the development of those human qualities which make life worth living on earth. Cheap power, abundantly developed and delivered at the door of every human house-dweller, is the *sine qua non* for such a result. I am an optimist and believe such a day is not too distant to dawn in India.

The sixteenth Acharya Jagadischandra Bose Memorial Lecture, delivered on 30 November 1954. Obtained by courtesy of the Bose Institute.

3

Visva-Bharati

Friends,

It is my proud privilege to welcome you all in our midst on the eve of our Annual Convocation. We are deeply touched and glad beyond measure that our Acharya and our beloved Prime Minister, Sri Jawaharlal Nehru, has consented to conduct the proceedings of this year. We all know how the great tasks of promoting peace and goodwill among nations and of leading our country in the path of progress towards prosperity absorb all his time and attention. His visit is an encouragement and its memory will remain a cherished source of inspiration to all workers of the Visva-Bharati.

Cordial greetings to our Pradhana on her first visit to this University. Our memories go back to the days when Santiniketan was a place of pilgrimage :— the abode of our illustrious founder, the great seer and poet who sacrificed so much for the sake of his experiment to find a place where the people of the East and the West will be welcome as equal partners and labour for the synthesis of all that is good and beautiful in the culture of many lands and races. It had been his dream that humanity will ultimately be able to rise above petty strife and mean selfishness and realize the glorious destiny which the Poet had seen in his beatific vision. Thereafter many had come and drunk at the fountain of inspiration, among them your great mother, the illustrious daughter of Bengal, the great Sarojini, who had walked with the soldiers of freedom working out a bloodless revolution. We hope that this visit will renew an old tie of friendship and that you will continue to extend to our activities at the Visva-Bharati your sympathy, encouragement and benevolent support.

To my friend and the Chief Guest of this Convocation, Professor Prasantachandra Mahalanobis, our grateful thanks for having consented to speak about the ideals and the aims of Visva-Bharati. You have been a close associate of Gurudeva when the idea of the Visva-Bharati took shape in his hands. Our old workers still remember you as one who, as the first Secretary of the Visva-Bharati, toiled ceaselessly to nurse the infant association and guide its first footsteps. We hope that what you will say to us today will remind us of the noble beginning of a great enterprise and help us to remain steadfast to our ideals.

It is also gratifying to me to an immense degree that in this first Convocation that I attend I am able to present three of our illustrious pioneer workers for the award of the highest honour that Visva-Bharati may offer in recognition of merit.

Pandit Haricharan Bandyopadhyaya has passed his long life in pursuance of the one great task which he had set before himself. His monumental work, the *Bangiya Śabdakosha* will endure as a magnificent contribution to the study of the Bengali language. His simple life at the Guru-Palli is in harmony with the great tradition of our land where Gurus lived simply and in dedication to truth and wisdom.

Our Pronetri Indira Devi Chaudhurani had long guided our artistic departments. Her sure instincts and profound knowledge had been our mainstay in many of our recent artistic endeavours. She had also been our Upacharya during a short but difficult period of the University.

Pandit Vidhusekhar Shastri will be long remembered in Santiniketan for his assiduous scholarship and for his association with the linguistic studies at Visva-Bharati. He initiated here the studies of Buddhism and Tibetan. He has trained a long line of workers, here as well as in Calcutta, who continue to follow his footsteps and reap harvest in the rich fields of linguistic and other studies.

The names of our pioneers remain in Visva-Bharati as shining examples of what can be achieved by pure devotion and valiant efforts. In honouring them we pay homage to true scholarship and single-minded devotion to the high ideals of Visva-Bharati.

I pause here a moment to remember the great-hearted scholar who literally sacrificed his life to the cause of Visva-Bharati — Professor Prabodhchandra Bagchi. He had organized research in Sino-Indian Studies in Visva-Bharati. He had won laurels in his own particular field and made the name of Visva-Bharati an honoured one in the field of Sino-Indian Studies. He consented to be its Upacharya at a critical time and, I repeat, literally sacrificed himself in endeavouring to pull the affairs of the institution out of the confusion into which it had unfortunately fallen.

Only five years have elapsed since the Visva-Bharati has been recognized as an institute of national importance and the Government has agreed to take it under its protection as a centrally administered University. We are as yet in the initial stages of the new development. Much still remains to be done. However, generous aid has begun to flow in.

Out of Government grants we have been able to put up a new building for a

Vidya-Bhavana and a new hostel for students. The block grants-in-aid of 4.5 lakhs, with supplementary aids to cover our deficits, have helped us to meet our recurring expenditure. Additional grants have helped us to stabilize our Tibetan Studies and to procure valuable books in our library. We have now been able to acquire the Uttarayana properties which will be used by the Visva-Bharati to house its Rabindra-Sadana, a Museum entirely devoted to the memory of our illustrious founder and preserve the priceless treasures that the poet has left us : manuscripts, paintings and other objects hallowed by their association with Gurudeva.

During the present year Visva-Bharati has continued to function in the normal manner in spite of great difficulties. Our scholars have continued to contribute to Indological and Sino-Indian Studies. Valuable Bengali manuscripts have been edited and published. The task of village reorganization and social service continue in Sriniketan. The Basic Training and Crafts School has been converted into a Multipurpose Junior Certificate School. We have organized an Adult Education Centre and refresher course for village teachers at the Vinaya-Bhavana. We have extension services for adult-education. We co-operate in the Community Development projects by allowing our experienced workers to take active part in National Extension Service. From December 1956 we have a Higher Rural Institute in the Sriniketan area which promises well to develop into a Rural University.

With the confidence that we have gained from the first five years' work in the new set-up as a centrally administered University it is now necessary to think of our future. It is best perhaps to reiterate our ideal as set forth in the first schedule of the Visva-Bharati Act. We do not want Visva-Bharati to be an exact replica of the other Indian Universities. We have faith in the programme set forth for Visva-Bharati by our illustrious founder and we do believe that one can achieve a high standard of efficient and all-round training by having a close association of the teacher and the taught. We may have to reorganize our studies in the various departments and emphasize the intimate relation between the disciples and the preceptor still more. Visva-Bharati of the former days had faith in this essentially Indian view of education. However, owing to the stress of circumstances it had temporarily to abandon its programme and had endeavoured to fall in step with the other Indian Universities. It had hoped thereby to escape financial embarrassment by catering to the needs of the students in a way familiar to the student community at large.

To accept this as a permanent feature of teaching in Visva-Bharati will be, however, to accept defeat and to declare our original programme of education to be essentially unworkable. We have recently given to this problem our careful consideration in the Academic and Executive Councils of our University and have chalked out a programme of reorganization of studies which has been approved by the Samsad of the year.

To carry out this programme in future we may have to approach the Government for a more generous grant. This will be one of the items on which we shall base our demand for aid during the next five years. However, we seem to have gained the public confidence. Demand for admission to Visva-Bharati is on the increase. Unfortunately,

we have not much spare accommodation to offer. Students from distant and foreign lands continue to come but our residential quarters are often not suited to their needs. To maintain our humanistic studies at a proper level we have to organize our existing schools of studies and offer facilities for new studies. This means bigger maintenance grants and larger aids for capital expenditure on more hostels and staff-quarters. A new Library Building will have to be set up and a proper Administrative Block is to be erected to set free the Udayan for a Museum. The problems of sanitation and water-supply will have to be solved satisfactorily.

All these have gone into our demand for development grants for the next five years. It will not be out of place here to append a few comments on the need of introducing scientific studies of a fairly high level in the Visva-Bharati. It cannot be denied that a certain amount of essential scientific knowledge does enter into the mental make-up of a modern man of culture. Gurudeva himself had recognized it, and has advocated its teaching in Visva-Bharati. It would be a pity if science is deliberately left out in the endeavour to bring about a synthesis of the various kinds of culture of different lands and races in an ambitious programme of giving a whole education to the modern man and woman.

I understand that the Grants Commission will consider the demand for the Library and Administrative Block sympathetically. They have already come forward with a handsome grant to help us to solve our problem of water supply. I understand they have in view a Master Plan for Visva-Bharati. I hope that all we have asked for will ultimately go into the final picture. We welcome the Master Plan, but we pray that the Centre does not prescribe a Master-mould for educational experiments here. A demand for conformity to prescribed courses will limit the scope and hinder the spirit of freedom which is so essential to normal growth of Visva-Bharati.

Friends, I have briefly told you of our hopes and fears. I now request our Chancellor to declare the Convocation open.

Convocation address as Upacharya or Vice-Chancellor delivered at the convocation of Visva-Bharati on 15 January 1957, addressed by Jawaharlal Nehru, Prime Minister of India, and Padmaja Naidu, Governor of West Bengal.

4

Higher Education for Us

Madam Chancellor, Mr Vice-Chancellor, Ladies And Gentlemen,

I am grateful to the authorities here for having asked me to deliver the Convocation Address. This University is my Alma Mater — and I strive to be one of her faithful sons. Here were spent my student days as also the early years of my apprenticeship as a teacher. The invitation of our Vice-Chancellor is thus almost a command and I obey him with very great pleasure. But what to say on an occasion like the present? Words of advice would perhaps be deemed out of place — an infliction — and nowhere would they go far into young minds! I have, therefore, decided to narrate before my young friends bits of past history : of this University as it then was — when as fresh graduates, as they are now, we had ventured on a long and arduous trek in search of knowledge. Few had cared to traverse this unknown road before them. They had many hurdles to cross; they had numerous failures to contend with. But there were also a few successes which encouraged them for still further efforts. Luckily, the old difficulties have now almost disappeared. But the story may be interesting; it will enable you to compare the old days with the present and I confess outright before my friends that they go out now very much better equipped for their lives' work than we were when we set out quite ill-equipped for our ambitious journey. Those were the days before the famous Sadler Commission. The University had not yet opened the PostGraduate Departments, and the old regime continued. More than a hundred years ago Raja Rammohun had written to Lord Amherst asking for the introduction of the study of the Western Sciences in India. The University, a few years after, came into existence in due course. But for the first fifty years in the curricula of the University the principal stress was laid on the

teaching of English. English was adopted as the medium of instruction of all Arts and Sciences in colleges and in high schools. That was also regarded as the best way of advancing the cause of learning in the country. Our foreign masters had wanted intelligent Indians to help them run their administration and their offices economically; and so for many guardians living in the cities, it seemed that the broad way to soft jobs and easy comfortable lives for their wards ran through the portals of the University. This was clearly not a quick way to disseminate knowledge as the statistics of the progress of literacy in a hundred years would reveal. Many years before, the Catholic Missionaries of Serampore had thought of a better alternative. They were the first to erect a Bengali Printing Press and published mainly religious tracts. But they also helped spread literacy among the people. They also published textbooks of Science and Mathematics in Bengali for the beginners. Bengali books on Medical Science were current in schools and were regularly used by the students before the advent of the University education. The task of spreading knowledge was soon taken up by the Indian educationists. Iswarchandra gave us the textbooks and the keys to the traditional lore of Sanskritic learning, Akshaychandra revealed the wonders of creation, and there was soon no dearth of Bengali books in all conceivable subjects. In creative literature, after vain efforts to write in a foreign tongue, Madhusudan and Bankimchandra realized that to gain approval of the people and an easy access to their hearts one has to write in one's own heart's blood — in that native language which springs from the age-old yearnings of the subconscious soul and nourishes the conscious efforts of our people. If things had moved on this rising tide of national effort, if this University had then proclaimed the principle of dispensing learning through the medium of the mother tongue, perhaps our dreams about the advent of a new era in this ancient land would have come true much earlier. But the old Syndics and Senators were not convinced. They left teaching in the charge of colleges and schools, and busied themselves with the maintenance of standards by the rounds of inspection and examinations. But the spirit of times moved ceaselessly on, and soon came the rude shock of the National Movement. In 1905, people wanted economic independence. Lost trades had to be revived and new industries had to be started. But the University studies were not oriented to that end. It could not meet the sudden demand for knowledgeable men in Science and Technology. Immediate reorganization also was not possible. But feelings ran high : the University was condemned as a slave-factory in public meetings. Attempts were made to reorganize education within the province on a broad basis independent of government control. A National Council dispensing the knowledge of Arts and Sciences through the medium of the vernacular came into existence. Funds were also available from patriotic donors but beyond establishing a College of Technology no lasting results could be achieved. Gradually the political struggle moved on to other arenas and the task of reorganization of education independent of government control remained unachieved for many years. The old system of education stood condemned. But there was no satisfactory alternative to replace it immediately. However, the situation was saved and the popular demands were met to a large extent by the genius of Sir Asutosh who was then at the helm of affairs of the University. He

understood the urgency of introducing more satisfactory methods of teaching in schools and colleges. The syllabus and the courses of studies were revised, and the teaching of the experimental sciences was given a prominent place within the University. Before 1908, only a few colleges taught sciences. As a result of the reforms, many colleges, both in the city and in the mofussil, undertook to open laboratories and provide the necessary funds for scientific training for which there were such urgent demands. While the schools and colleges tried their best to meet the demands of the undergraduates of the new age, the University had to come forward herself to shoulder the burden of higher education in Arts, Sciences, and Law. From a merely examining body, the University was thus gradually transformed into a vigorous centre of teaching and research. Sir Asutosh was the principal organizer and the leading spirit of the new movement. All went on very well for a time. The government approved his programme, and the people had confidence in his wisdom and foresight. The legal studies were re-organized in the University Law College. Funds were forthcoming for the installation of several University Professorships (e.g. in Mathematics, Economics, History and Philosophy). These few years of constructive efforts restored people's confidence in the University. Large endowments came from two Bengali lawyers, Sir Taraknath Palit, and Sir Rashbehari Ghose — land, building and funds. Sir Asutosh now took a great step forward and laid the foundations of the University Colleges of Science and Technology. There is however a peculiar condition attached to the Palit and Ghose endowments. The Professors have to be Indian scientists. Such suitable persons could not be found immediately for all the new University Chairs at the Science College. C V Raman, elected for the Palit Chair in Physics, was then working in the Finance Department of the Government of India and wished to have some time allowed him to make a clear decision. He was anxious to pursue his own researches in his leisure hours at the Indian Association free from all worries of a teaching post. Sir P C Ray agreed to be the Palit Professor in charge of the Chemical Laboratories only after his retirement in 1915. Professors D M Bose and Agharkar selected for the Ghose Chairs of Physics and Botany respectively wanted to be sent to Germany for their own researches. The University would have thus to wait for some time before the scheme for the higher teaching of science could mature. In the meantime dispute arose between the University and the government regarding the control of schools. Sir Asutosh took a fearless stand against the government in matters affecting the academic freedom of the University. In 1915 he was no longer the Vice-Chancellor but still remained Chairman of the Boards of Trustees of the Palit and the Ghose endowments.

The National Movement had thrown up many idealists imbued with a spirit of adventure. There were a few young graduates who had resolved to devote themselves entirely to Science and soon after 1915 M Sc examinees had approached Sir Asutosh for advice and guidance. The name of one of them had been sent for the post of a Lecturer in a government college in a different province, but was not considered on the ground that his qualifications were too good for the post. He wondered how he could possibly get along with his programme of higher studies. One of them had even the temerity to suggest that the University might now undertake herself the teaching of

the various courses in Science which were prescribed but were not taught in the government colleges. It was difficult for the youngsters to guess what the great Sir Asutosh thought about their proposals. However, some got special scholarships sanctioned and direction to study certain special branches of science which had come into prominence. He asked another particularly to prepare a report on the possibility of procuring the necessary equipments locally in view of the World War — he was also to report about the probable expenses necessary in the first year for fittings and other equipments in case the University decided to open classes in Physics. Soon it was evident that the University would be teaching science subjects. Sir Asutosh had won over his colleagues who had doubted the wisdom of this hasty act. They would have preferred to wait till C V Raman joined, or till the interned Professors returned from abroad. But ultimately they were persuaded to support Sir Asutosh. Thus the Postgraduate Departments in Chemistry, Physics and Mathematics were opened even before the Sadler Commission had submitted their report. The members of the Commission came and went on rounds of inspection when regular class work was going on in all the departments, and apparently approved what was going on. It was a risky venture. But the young graduates of the University worked hard to make the programme a success. In Physics the details of the programme were worked out by the Lecturers. C V Raman joined a few months later. He found the routine work going on satisfactorily. He consented to deliver a few lectures but kept himself busy most of the time at the Indian Association where the Palit grant was spent for research purposes. Here some Lecturers worked under his guidance and earned their well-merited Doctorate degrees.

I have told only a part of the story, relying mainly on my personal knowledge. But in all subjects in Arts and in Science, at Ballygunge or at the Darbhanga Building, the beginning of higher teaching took practically the same course. After the Postgraduate Departments had come into existence the young Lecturers carried the principal load of the new task everywhere. The leader had seen the propitious moment arrive. He had confidence in the young scholars, in their capacity to shoulder the heavy responsibility of hewing new pathways to progress. His call had gone forth and the young people responded. By their devotion, and their enthusiasm, they made the scheme a complete success.

The government had not looked on this new venture with much favour. No grants were forthcoming for the maintenance of the Science Departments and the University had to rely on her own fee fund and on the endowments received from the generous donors for this purpose. But the transformation into a teaching University met the urgent needs of a nation growing every day conscious of its own powers. Students came in increasing numbers; they went out trained, and were either absorbed in the growing industries or they started themselves new and successful ventures. Original contributions began to appear. Within five years, young Indians had made their mark in the scientific world. Ghose's papers on the Conductivity of Electrolytes were published in 1919. Saha had his famous theory on the Temperature Ionization of Stars ready by 1922. Numerous contributions on different subjects followed in continuous stream. Sir

Asutosh's hopes were realized, and the name of the Calcutta University soon occupied a respected position in the scientific world. This position has been maintained, the good work has been continued by successive bands of devoted scholars for about half a century. This has not always been an easy task. Public opinion had undergone violent fluctuations about the need and the importance of University education. During the years of struggle for national independence education enjoyed a very low priority value. Whenever needed, and the call came, students ran out of schools and colleges. Teachers were no longer regarded as reliable guides and they preferred to follow the leaders of the national agitation who assured them that once independence was secured all difficulties would disappear and all problems would find quick and easy solutions. A new era of freedom has finally dawned. For Bengal however the chalice of bliss is tinctured with many sorrows and bitter memories. The old land, united by the traditions of centuries of a common culture, now stands divided. This has occasioned movement of people on a tremendous scale. The problem of the rehabilitation of refugees taxes heavily the resources of our administrators. There has been simultaneously a great demand for technical and scientific training. Schools and secondary education are no longer under the care of the University. Large-scale reforms of secondary education are under contemplation. But, for efficient working of the new plan, trained teachers would be necessary and the University has ultimately to supply a large number of well trained teachers to the schools. Besides, a large number of graduates demand higher training at the University. Their training and test also raise problems of great complexity. Many are refused admission for want of accommodation. And almost every year we hear of unseemly disturbances in the examination halls : an unsympathetic examiner has set a particularly stiff paper, or the questions asked are outside the prescribed syllabus. All these problems require a careful review of all University activities. A closer understanding among the teachers and the taught is eminently desirable. And now more than ever has arisen the need of adopting the language of the province as the medium of instruction in all classes within the University. This had appeared perfectly feasible to the leaders of thought in the province more than fifty years ago. It should again be considered carefully by our Senators and our legislators now. There are now a little over thirty universities in India and many more are bound to follow. As higher education is a Central concern, there is a school of thought which urges an approximately uniform standard for all the universities. They demand the working out of a master plan; they desire that all universities should adhere to it as closely as possible. The need of the same medium of instruction follows, according to them, as a necessary corollary. These planners thus urge the continuance indefinitely of English as the medium of instruction in universities as at present.

I have always spoken against unwise adherence to past policies. This foreign language has been a real hindrance to the rapid spread of literacy in the country. In an educational institution, it encourages cram and effectively damps all creative efforts.

It is time now to introduce the regional language as medium of instruction at every

level of the University for all subjects. Fifty years ago, this appeared a viable proposition to our leading thinkers. Now members of the University Senate and members of the State Legislature have to take up seriously the issue of giving to the vernacular its rightful place in education.

Convocation address delivered at the Calcutta University in 1962. Obtained by courtesy of Rathindranath Bose.

5

Dr Mahendralal Sircar

The Indian Association for the Cultivation of Science has organized a meeting in the memory of Dr Mahendralal Sircar. 23 February is the anniversary of his death. I feel myself honoured to be here with you to discuss his achievements. I have also been asked to say a few things suitable to the occasion.

Mahendralal was born on 2 November 1833. That was roughly the time when Raja Rammohun Roy had urged that science education be introduced in India following the western practice. Our country would not regain its former glory if we continued to look back to the past and remained engrossed in the study of Sanskrit alone. In a letter addressed to the Viceroy Amherst, he wrote of the need to introduce a new system of education to bring about a change of attitude suitable to the times. The letter echoed the aspirations of the common people generated by their contact with foreigners.

Many students were already turning to the study of science. The poor and orphan Mahendralal had joined the Hindu College on a Junior Scholarship. He was very good in studies and showed great proficiency in English literature and philosophy. His teachers spoke highly of his aptitude and his acute discernment. He earned a Senior Scholarship too, but since he was keen to study science, he got himself admitted to the Medical College in 1854 in disregard of all pleadings to the contrary. Preparations were then afoot to establish the Calcutta University.

He showed great aptitude in both organic and inorganic chemistry, as also in surgery and nursing. I have heard that Bengali was then the medium of instruction in the Medical College. I have heard that appropriate science books had also been written in Bengali which are now hard to get. Bengali disappeared from the colleges with the

opening of the University. By the time Mahendralal began practising medicine after obtaining the L M S in 1860 and M D in 1863, English was in full swing in India. Though English provides the key to the repository of scientific knowledge, it does not necessarily create a scientific temper. The qualities needed for this are a sound intellect and a fearless passion for truth. Before English was introduced in this country there were many free and open-minded seekers of truth who can be regarded as scientists even by today's standards. There are many such examples in our history. They recorded the findings of their efforts in the native languages. For example, there is ample evidence that Indian astronomers had studied foreign texts but they wrote down the results of their well thought out experiences for their countrymen in their native languages. The spirit of servility, the false pride of knowledge and obsession with a foreign language had not shrouded the perception of truth that our countrymen still cherished. Our own history like that of other countries bears witness to the fact that observation and experiments are the foundations of scientific progress.

Mahendralal never acted contrary to his own convictions. This attitude characteristic of a true scientist came through in his character. A scientist finds out truth by making various kinds of experiments. He cannot always depend upon the experience of others. A scientist finds immense pleasure in discovering new truths through analysis of experiments and his discriminating judgement. He never relinquishes this habit and communicates his new discoveries to others. He also leaves behind meticulous accounts of his method of arriving at the truth. Mahendralal had this kind of unique experience in his medical career. He was an established doctor, trained in the allopathic system. When he realized that there was substance in homeopathy, he heartily welcomed it. He felt that his resolve to cure diseases would perhaps be better served in this way. Fear of disgrace or monetary loss could not deter him.

He has himself written about this change of opinion in the July 1902 issue of the *Calcutta Journal of Medicine*. I cannot resist the temptation of quoting a few lines from the translation of it made by his biographer, Sri Manoranjan Gupta. Mahendralal was treated as an outcaste by the rest of the medical community for his advocacy of homeopathy. His teachers and friends urged him to return to allopathy again. But, he wrote : 'I have the same answer for my adored teachers, well-wishers and friends: I cannot give up truth. My profession may flounder and I may have to take up some other work, and would perhaps go hungry. Come what may, I will declare to the world with all the power at my command that I believe in whatever I have come to know as true.' These few words reflect the character of a self-confident, determined scientist and fighter.

After a temporary setback, Mahendralal was restored to his earlier position of honour. His reputation spread far and wide. The University meanwhile had entrusted the colleges with teaching the students. Very few colleges had the facilities to teach science. In the meantime the need for science was beginning to be felt by our people. Students were firmly convinced that in order to change the conditions in the country and to eradicate poverty, we must proceed along the path that enabled the West to reach the acme of progress. It was around this time that the practice of going abroad for higher studies in science began. The government was initially sympathetic to these efforts and

even gave scholarships to a few meritorious students. Competitive examinations for the private Gilchrist scholarship were also introduced around the same time. Thus among the hundreds who went abroad to seek good jobs or to become barristers there were a few students who went for higher studies in science. Among them were Jagadischandra, Prafullachandra, Pramathanath and many others. On their return some of them took up the responsibility of teaching science. A new era in education began in this country after 1882.

Mahendralal never went abroad. But with the countrywide poverty and ignorance in mind, he felt the need for a national association to enable Indians to practise science. He started his movement in 1870. His appeal began to appear in different newspapers in English and Bengali. It said, 'Cultivation of science commanded great respect in this country in ancient times. The ancient Hindus sowed the seeds of mathematics, ayurveda, chemistry, botany, psychology and *ātmatattva* and many other scientific subjects. But, unfortunately, most of these have been forgotten. Science has become an imperative for the Indians, and, for this purpose, a proposal has been mooted to set up an association named the *Indian Association for the Cultivation of Science* in Calcutta.'

The campaign continued in this way. Many luminaries like Vidyasagar, Justice Dwarkanath, Krishnadas Pal, Rajendralal Mitra and others upheld the proposal. Mahendralal's efforts bore fruit. Donations totalling about Rs 80,000 were already collected by 1875. On 29 July 1876, the Indian Association for the Cultivation of Science was inaugurated at 210 Bowbazar Street under the chairmanship of the Governor, Sir Richard Temple.

We are filled with wonder and respect when we think of the foresight of this selfless and determined patriot. Neither Jagadischandra nor Prafullachandra had embarked on research in 1876. Who could think then that Indians would earn worldwide acclaim through their successes in fundamental scientific research, and that the country would once again see prosperity return through industrial progress? That dream of his is about to come true. Professor Raman has won the Nobel Prize for the discovery he made in the laboratory at 210 Bowbazar Street. The Association shifted to Jadavpur in the post Independence period. A huge building has come up with assistance from the Central Government, and now many scientists have opportunities to carry on their research in their own subjects in a congenial environment.

Many things were said in favour and against the practice of science in this country in relation to the proposal to set up the Science Association. Traditionalists were afraid that the natural good qualities of man would disappear with the popularity of science. Mahendralal tried to refute this argument in different forums. His religious beliefs and perception of social matters were to a large extent free from superstition. He would often strongly protest against certain social conventions then in vogue at public meetings. In later life he pondered over the future of science and wrote about these questions in an article written in English in 1901, two or three years before his death. The question was : 'Can physical science enlighten man as to his destiny?'

Mahendralal's concern was the spiritual destiny of the individual. He expressed in

this article whatever he realized through his faith, imagination and judgement. More than fifty years have elapsed since then, and science has achieved wonders. No hurdle appears insurmountable to it now. Science has taken man beyond the earth's atmosphere. Scientific instruments have given us information about stars and nebulae that are more than hundred thousand light years from us. Radioactive materials that do not occur in nature have been brought into existence; even questions as to what was it that impelled the primary elements to produce the wonderful world of molecules and to create the mysterious stellar world have begun to be explored by science. How life has evolved from non-living matter is also being discussed. On the other hand, a self-centred, religious faith based on a thousand years of rites and practices is facing a crisis. At the same time, vast knowledge has not always been geared towards the welfare of man. In this very half of a century there have been two world wars. Even those nations that had cooperated in the development of science were engaged in the destructive game of war on the pretext of national welfare, and competed with each other in savage destruction. The lessons learnt from science were brutally appropriated to exterminate mankind. Today the wars are over but an uneasy peace prevails, with terror at its core. That is why the question raised by Mahendralal is once again finding an echo everywhere. Where will the practice of science lead mankind? All over the world structures of civilization have been erected on the foundation of science. But good sense has not yet developed in man, and the race to create the ultimately destructive weapon still continues, to determine which nation will rule the world as the sole power. Will science ultimately incite hatred and destroy civilization? Lovers of science in all countries have to answer this question. The members of the Indian Association for the Cultivation of Science are also faced with the same question. Moreover, we have a tradition clouded over with the dense smoke of elaborate religious rituals. They might have to answer the question as to whether it would be necessary to hold a new kind of ritual to counteract scientific knowledge.

But the emphasis of Mahendralal's question lies elsewhere. The concern now centres on the collective instead of on the individual. Discussions are going on about the future of man and man is looking for solutions to the crises of civilization in religion, philosophy, sociology and spirituality. He is no longer content with the hope of an after-life. Many are wondering if science can find a way out of these crises. I would like to conclude with a few things that can be said for science, as my limited knowledge and discrimination tell me.

The reality of the flow of time was not reflected in the four-dimensional geometric concept of the relativity theory because the measure of time was given the same footing as the triad of axes required to define space. Through the context of an ever-expanding universe, the unique nature of time has now made a deep impression on the mind of the scientist. Time flows in one direction from the past to the present and beyond. Within it were created in due course nebulae and star clusters. Several radioactive materials have evolved and decayed within this span of time also. Through an analysis of the principles of this process we can get a proper measure of the changing time and the direction of its tide.

The creation, existence and destruction of stars, planets, suns and all inanimate elements in the universe are now subjects of scientific research. The theory of evolution and gradual development in the material world is now well established. The position of the earth among those planets which revolve around the sun and the nature of its ultimate end are today subjects of intense investigations. Philosophers like Bergson had begun thinking about how in the remote past life evolved from non-living matter through the continuous process of creation. Today scientists are thinking about the same problem and searching for a fundamental law underlying the process.

In the opinion of the scientists all substances of the visible world — organic and non-organic — were not created at one particular moment. Nor is it true that evolution has proceeded everywhere in the universe with the same rhythm. So historical methods have been adopted in science which is based on experiment. Scientists are convinced that there are reasons behind the creation and development of every object and the quest for these reasons is obligatory for scientists who believe in change. If we were to analyse historically the manifestation of life on earth, we would at once be astonished by the diversity of the animal kingdom. Animals of different kinds and shapes have been born at different times and have at the end become extinct. When were these born within the uninterrupted flow of time and what was the environment then in the world? Would discussions on these matters reveal the fundamentals of the theory of evolution? Controversy over these matters continues, and all scientists do not share the same belief. The outlook of scientists depends a lot on their personal viewpoints.

The eminent French scientist and Christian missionary, Pierre Teilhard has recently thrown some hopeful light on this difficult problem. His recently published book, *Phenomenon of Man*, has attracted worldwide attention. I will try to explain his ideas briefly. While discussing the evolution of life, he has said that if we were to observe the evolution of various living organisms through time, our attention would be drawn towards the nervous system in the body. It seems as if life is trying to manifest itself in higher states of consciousness. Taking a cue from this a discussion of evolution would easily lead us to a certain conclusion : a rapid development of the brain took place along with the tremendous development of the nervous system of the vertebrate along the same branch of the tree of evolution. Human beings appear at the last stage of this branch. Man, a relatively weaker being, evolved among so many large and powerful animals of the past. But today he has assumed sole authority in the world by subjugating all other animals. This did not happen in a day, nor by intelligence alone. Once he learnt to think, his ascent accelerated. Let us think of the whole of mankind rather than the fate of an individual. Once human thought processes spread throughout the world, it became possible to hand over to succeeding generations the experiences and knowledge gathered by man in the past.

Successive generations continued over the ages to further add to the repository of knowledge by drawing inspiration and guidance from the past. This is how the present civilization has evolved, and man is gradually set to acquire proper enlightenment. What vast opportunities have opened up before him! To reach the higher stages of evolution the life force has adopted a particular method — that of cooperation. Initially

life was feeble, confined to a single cell. It was only by being multi-cellular that life gathered strength. Tens of millions of life cells in the bodies of the higher animals perform their functions in close harmony and in full cooperation with each other to make their lives more complete.

The Christian missionary thinks that the same seed of cooperation that the scientist has found in the body of a living organism is also at work in the development of society. In it he also found the guidelines for future evolution. Man's future is in his own hands, and if he adopts an attitude of compassion and cooperation towards all without discrimination, then all present day conflicts and violence can be overcome. Only then will a civilization of universal humanity emerge. Otherwise, just as the gigantic animals of the past have become extinct leaving their skeletal remains to bear testimony to their existence, so the same sad fate may well await human civilization in the future. We find an echo of this vision of science in the teachings of many of the sages of our country. Science has nothing in common with the sort of religious belief, social policy and caste division which lie at the root of envy and conflict. Rather, one can say it has emerged from ignorance. The way in which freedom of thought has manifested itself in human civilization is distinct from the expression of instinct that a study of the insect world reveals. It is based on this instinct that insect societies have evolved. We are amazed by the discipline and harmony of their lifestyle. But there is no future in this direction and Pierre Teilhard believes that there is in nature an indication of this in evolution. If man is able to build up a universal society in the future that is consistent with individual freedom, it would mean in one respect the victory of life. Today we cannot even imagine the glory and magnificence of the edifice of progress that man would thus be able to establish. At present the progress of science can only reveal a faint glimmer of that possibility. Hence it is not true that man will find salvation by turning away from science and by channelling his thinking, intelligence and inspiration to other directions. Man will have to go ahead and cross all hurdles bravely. The future civilization will have to be built irrespective of caste and religion. All men will find a place in it. We get this message of hope from the scientist. We need to develop and establish love, a scientific outlook and cooperation instead of enmity and conflict. This is the directive of the history of evolution. Science is the road to victory. Man will not attain ultimate success by rejecting science and reverting back to the past.

Behind this theory of the French scientist lie many years of introspection. His life is one of unique sacrifice and love of mankind. I have spoken about his experience because it has deeply touched me. Many will find a similarity between the words and teachings of this Christian missionary and those of India's mystic saints. They will also recall the words of the valiant Vivekananda : 'Nāyamātmā balaheenena labhya'. (This self cannot be acquired by the weak!)

Address delivered in Bengali at the Indian Association for the Cultivation of Science, in March 1962, at a meeting held to commemorate the death anniversary of Mahendralal Sircar (1833–1904), and published in Jnan O Vijnan, *November 1963, and reprinted in Satyendranath Bose,* Rachanā Sankalan, *Bangiya Vijnan Parishad, Calcutta 1980. Translated from Bengali.*

6

Man in the Scientific Age

Friends, first I have to record my gratefulness for having been invited to come to this symposium, and to be among you, all young scientists eager to explore new roads and to tackle all kinds of difficult problems by new methods.

So, I shall begin with an apology, the apology of an old scientist, an elderly person, who, when he began his studies, was educated mostly in the old school. Determinism still had its sway and, as our teacher used to say then, we were not so anxious to define our concepts as we were eager to get results. He used to say jokingly that when he was a student in Cambridge there was a joke like this : 'What is matter ? Never mind. What is mind ? No matter.' You know the *Principia* of Newton. Well, there he tries, not only very profoundly, to define the concepts, but introduces to us valuable methods. And with them, at first, we were getting results and we were happy. We thought that perhaps all the world phenomena would be investigated and analysed in the same way : starting from a set of axioms and following it up by methods of deductive reasoning, basing our views on certain differential equations with certain initial conditions ; thus we could, in principle, successfully explain all physical phenomena. But very soon, when we were still students, the quantum theory cropped up. Physicists began to know that after all there were many events for which the usual description on the basis of the law of cause and effect could not be applied and newer methods had to be devised. Ultimately we gave up Newton's determinism. At least physicists became more humble and gave up the old idea of giving a casual description of phenomena but satisfied themselves by a statistical way of analysing the results. I say this because certain questions have been raised here regarding the difference between the study of material

phenomena and the study of human behaviour. I say even in the world of material science we have now given up the older idea of being able to follow more closely the unit processes and very often because of the reason that we do not possess sufficient intimacy with the processes concerned. We are directly concerned with results. And therefore, certain methods have been developed and older ideas have been given up. This new and statistical method not only permeates the whole of physics but also gradually finds its way into all the sciences.

There was formerly always a quarrel between the materialists, or in other words the natural scientists, and the social scientists or the philosophers. As somebody said here, we are very often talking about things we do not know what they really are. Instead, we have got to reconcile ourselves with the idea that even in the material world we are not so much concerned with the essential nature of things, as with how to record the results of experiments and our interpretation and applications of the results that we get. Our standpoint has been more or less utilitarian ; therefore, I feel, while admitting that there are essential differences between the M-concept and the P-concept, that the scientists naturally are concerned only with results. They need not even, in the case of social sciences, think of the correlation, as somebody wanted to find out, between the M-concept and the P-concept, but devote themselves exclusively in the P-plane as Mr Davidson has said, recording, at first, the description of the circumstances and also nothing than the results. So, I mean to say that we are not so very different : the natural and the social scientists after all in the sense that when we try to analyse things we are more or less concerned with what we think about them. The older idea of getting a sort of knowledge by more intimate understanding has now been dropped.

Of course, we, orientalists, are reputed to have a sort of a different way of getting knowledge, a sort of intuitive knowledge and if you want to understand how the tree grows, you try to think yourself at one with the tree, and then once having brought yourself in tune with a particular phenomenon, you will perhaps be better able to understand it than when you simply observe it from outside. I suppose in some biological sciences you still try to explain the behaviour of the cells in a similar way, bringing down to this elemental plane the ideas of human relationship with which we are familiar : attraction, desire, and volition etc. All these things come in. But while it remains always something personal with the scientist, his results are always expressed in terms of measured quantities similar to those of natural scientists.

When the problem of free will or attraction crops up, there are some persons who seem to think that in the elementary attraction between the atoms and the molecules there is just the beginning of a certain tendency which in the higher beings develops into a feeling of attraction or seem to see in the quantum laws a vestige of the free will. But whatever they might be, I am not very competent to talk about them. But what I want to insist on is that, so far as scientific method is concerned, they have taken a similar turn, by which not only the physical but also the biological sciences are now studied in the same way. Even when it is a question of studying human behaviour in the social sciences, we follow more or less the same statistical methods. Well, in

statistics very often you have got a large number of elementary acts in which some will succeed, some will go wrong, and there is always a measure that we have to devise in order to record your impression about the large number of individual events that you study from a certain standpoint.

It was interesting for me when the speaker here talked about the judgement of various observers regarding certain events. Here a statistics of the observers would follow, as far as I have understood him, in which by well-thought-of statistical analysis the possibility of eliminating personal equations in some cases had been recorded. However, in such cases in intelligence tests, in the appraisal of student work, there remains always a larger margin of uncertainty. And during the long years of my experience as an examiner we often felt that when there are more than one examiner it is very difficult to correlate the diverse judgements eliminating the personal equations in the process. In our University of Calcutta we had a peculiar rule : if two examiners differ, refer it to the third examiner ; and then, when the third examiner's results are different from both, take the mean. But you see how uncertain is the possibility of eliminating personal equations or personal prejudices in such cases in this way. However, I am glad that attempts are being made, and very soon a way might be found out, suitable specially for your country, because the methods of appraisal of intelligence or evaluation of results differ in different countries. So every one will have to think out, basing his general ideas on the principle of statistics, as to how to proceed in any particular case.

We are thus in a position to say that, in contradiction to what we thought before, in the social sciences we may also study from the same scientific point of view, thereby meaning that once we have devised a method of measuring certain things, we can always subject all our results to the same statistical treatment as in the case of observations of material phenomena.

Well, naturally I was interested to note that the future of the society depends largely here, as elsewhere, upon the education of your graduates, and the problem of the relation of the essentially scientific and technological teaching with the humanistic one is the problem that has been raised here. But I have not been able to gather exactly the reaction to it of all the members of the symposium. In our country, also, the same problems trouble us and some of us felt that early specialization is a hindrance, and, as far as possible, during the earlier years education should be broadly based, neither too technological nor too literary. All the students should participate, as far as possible, in a general course of education in which eyes and hands should be trained. Endeavours should be made that gradually students master a way of expressing what they feel. As such a certain amount of mastery of the language will be necessary, just as they should be required also to have a broad and unprejudiced view of the physical and biological processes that are going on around us. Naturally that raises the question of the role of scientists, as to what the scientists may do in the future society.

I think that the scientist now claims himself to be a person who has got a greater knowledge and more intimate acquaintance with nature. He not only relies upon his

personal experience but can fall back upon years or centuries of recorded experience of the human mind. And if we believe in evolution, then we may say that through thousands of years there has been a simultaneous development, a slow process but essentially the same in all the countries, whereby man has gradually woken up to his responsibilities and now, in the twentieth century, feels responsible enough to take his position as one who is able to guide his destiny.

Naturally the social scientists have got a bigger responsibility in such matters. However, the scientists should take up the standpoint that properly belongs to them ; they belong to the class of intelligentsia who have to be consulted whenever grave decisions are taken regarding education, regarding steps that should be taken to reorganize our economic or political life. This is a big claim, and I wonder whether the persons in power will agree to admit the scientists in the council of advisers. If, however, you, young scientists of Japan, contribute to study nature and human behaviour in the same way that we are doing now, I have great hopes that you will be showing to the world a novel way of reorganizing society.

The problem of education is a matter about which I always feel a bit confused, though I have been teaching for well nigh forty years. But the problems of communication of knowledge, the relation between the pupil and the teacher is to me yet something mysterious. How does your pupil learn from you ? He perhaps tries to imitate you. Your example will act as a leaven which produces the necessary ferment, but the pupil must find himself in the sense that as he grows up from boyhood or from childhood through adolescence into manhood, he should be able to utilize all the elements that the society provides for his development.

And here there are different ideas. I personally feel that in such matters, as in all matters, the process of giving as much freedom as possible is perhaps the best thing to do in the long run. You have talked about machines : now, by suitably coupling the influence with the result by means of a feedback mechanism you can make the machine almost simulate a conscious behaviour ; your missile may follow the enemy through his intricate military evolutions or your automation in the factory may regulate the supply of raw materials and at the same time test the quality of the products that are ultimately produced. It is something remarkable. But in all such cases, if one thinks a little, the best results are always obtained when the machines are so conceived as to allow maximum amount of freedom.

In this symposium, you have taken up very big problems. I was at first at a loss to find out what reactions would be. But being among you I have gradually educated myself and I am telling you of my reactions. I think the object of this symposium is not that you should have a sort of lecture *ex cathedra*. But in intimacy and in close collaboration, we will be able to come to a closer approximation to solutions of problems. Maybe your contacts and discussions will bring out new angles of vision of difficult problems and I hope that in your discussion, after these discussions are over, the active scientists, the politician, the social scientist and the legislator, they will all have a sort of intimate understanding of how things look from different viewpoints.

Naturally, society and the place of the scientists in society bring up the question of the ultimate future of the society : what purpose will we be working for ? There, as an optimist, I believe that the scientists of Japan will be able to educate their countrymen to the point of view that ultimately the best thing for the nation is also ultimately the best thing for mankind. You can always note the evidence of history : how shortsighted ways of looking at things, though they have given quick results, may ultimately lead to disaster. It is not only an experience of Japan alone but practically of many countries and we hope that people will become wiser after such events and they will set up a common brotherhood of scientists of all countries who, naturally as they possess a more intimate knowledge of affairs, are able to guess better about the probable results that may follow any course of action. They may thus be able to guide all nations towards a new era, an era when the different nations, though they may be on different levels of material progress, yet ultimately will be united in one common bond of brotherhood and unity. I often think that if we can really compare humankind to a family we should admit that all the members of the family may not be endowed equally well, but ultimately to the mother or to the head of the family they are equally loved and they have got equal right to the affection of the mother, though the one who is backward may not rightfully claim to have such a treatment. The different nations are in that position, and if the same feeling of goodwill can be cultivated among nations as generally is reputed to hold among a brotherhood, we can hope for the best and all our endeavours and our scientific studies may be directed towards a common purpose not only for the good of the motherland but also for the good of the whole human race. Thank you, gentlemen.

Address at a symposium on Science and its Position in Society, in Tokyo, Japan 1962.

7

The Mother Tongue

I have been involved with education for the greater part of my life. As students we nurtured in our minds the excitement that science, the cultivation of which has enabled the west to progress so rapidly, would soon be introduced in our country and that we shall dedicate our lives to bringing such things here.

Nearly sixty years ago when the Swadeshi Movement began, the great men and patriots who gave leadership to the country hoped to set up an indigenous system of education, at least within the province of Bengal, through national schools. Several years have elapsed and still education remains confined to a limited sphere in our country.

One thing comes to view when visiting other countries. The attempt everywhere is to evolve a system of education rooted in the mother tongue — a language that everybody understands. This policy is followed everywhere. In medieval times, however, a different language was used to impart education; only a few educated men had access to it. The limitation of this system was that the common people did not understand the language. They required interpreters, and whatever the latter understood was conveyed to the people.

But this system resulted in extremely slow dissemination of knowledge. Today people are attempting to make the country march ahead with the help of knowledge and science, to provide the common man with a wide range of material benefits; at the same time they have realized that the pace of progress will be slow if knowledge remains confined to a certain class. Now that we have achieved independence, we have to think out carefully how quickly education can be spread within the country. All those

who have studied history know that if efforts are made, ignorance and illiteracy can be eradicated from this country.

I am reminded of my experience during my visit to Japan. I shall tell you briefly about that.

It is about a hundred years ago that Japan, reeling from the blows struck by the West, resolved to acquire all the knowledge and technology which have made the West so powerful. It is less than a century now since then and everyone is already aware of the achievements of Japan. Twenty years ago when Japan was defeated in the Second World War there was no end to her misery, but today, if one visits that country one can hardly imagine that she had to go through such a time.

The question that naturally comes to mind is, how could Japan rise to this present state of extreme affluence within such a short time? Education and progress are inextricably related. I was therefore curious to know the details of their system of education. In Japan children are sent to school for a minimum period of nine years. School education is free and is compulsory for almost all children. At the primary level the cost is borne by the municipality or the department concerned in the district or the state government as in our country. They share the major part of the expenses. The Government runs all institutions that teach the fine arts. In addition at higher levels the universities are mostly funded by the government.

At first I was under the impression that in Japan science and arts were taught in a foreign language. But on arriving there I found that I was wrong. I collected a large number of books, most of them in Japanese; hence I was not able to make out what they were about. Of course I received a few English books too.

The Japanese had convened a conference where educated people — philosophers, scientists and teachers — gathered to discuss what men should do in the modern age of science and what the future of mankind would be. Except for one or two foreigners including myself, the conference was attended mostly by the Japanese and the proceedings were in Japanese. Some of them occasionally spoke in English, but what followed was conducted entirely in Japanese. It will be incorrect to say that they do not know English, because when we spoke in English most of them understood us. However, they were not confident that they could express their thoughts clearly in English. So all the Japanese scientists and philosophers present there expressed themselves in Japanese. It was apparent that it was possible for abstract philosophical theory or advanced ideas of contemporary science to be expressed in Japanese and the people were keen to do so.

Compared to our Indian languages, Japanese has certain problems. Those who are better informed are already aware of it. In our languages writing and printing are done using a handful of letters of the alphabet, but this is not possible in the Japanese system. Apart from their own letters there are three thousand letters of the Chinese alphabet. Those interested in pursuing higher education have to learn them all. In our country a child can pick up the vernacular within a year or two, whereas an average Japanese child takes about six years to learn Japanese. Despite these problems the

Japanese language has reached a stage where every Japanese scientist or philosopher can express his thought in Japanese. This proved to be an advantage when compulsory primary education was introduced in the country, and there was no dearth of teachers —teachers who had already mastered science or philosophy or fine arts in the Japanese language and were competent enough to teach them.

This has contributed to the rapid spread of education in Japan. As a result, Japan could easily acquire all forms of knowledge. If we take the cases of Japan and Germany, the two nations in the world who have made rapid progress from a low level of knowledge to the highest position today — more than ninety per cent of the people in both countries are literate.

If we take a country to stand for the people of the country, not just the educated or the leader class, but the entire mass of the common people of the country, then only if they are educated can we call the nation fully developed. And then they would hold in their hands all the power to defend the country from any danger. Our country has a number of languages. But in every language, specially those recognized by the Constitution as major ones, the total number of educated people will be more or less the same as the population of Japan.

I feel that to start with if we confine ourselves to building our own homes without thinking of the entire country, that is, if we prepare the system of education for people in our respective provinces in such a way that every child gets compulsory education, then the new problems we are facing will easily disappear.

There is an idea in the world today that the nations will unite to build up a greater human community. But nobody thinks that to achieve this each country should adopt a specific language as the medium of education. After all, linguistic barriers do not count when there is a mental affinity.

It has often been said that without the use of the English language there will be no unity in the country, that we became conscious of our national identity only when we lived in subjugation.Hence that sense of unity should be maintained through the English language. But it is not correct to say that we had no sense of national identity before our enslavement.

Those who are at the helm of affairs in our country are perhaps not sufficiently aware of the strong feeling of unity within the country, or how firmly the bond of unity binds the people together. Only when people from various states of this country meet abroad, they note how striking the similarity is in their mental make-up, customs and manners and how different they are from a foreigner's. I have a strong conviction that the unity of the country will never be at stake if education is imparted totally through the regional language of every state.

In our recent history there have been some glorious as well as some shameful episodes. An analysis of the history of these shameful episodes reveals that the persons who were responsible for bringing disgrace were all educated in English.What I am trying to say is that just as there are instances in our recent history of which we can be proud equally there are instances which we should regret and be ashamed of. But

if one were to look for the root of such regrettable events, one would find in many instances the activities of people who were arrogant, selfish and educated in English.

There I feel that English has never been a unifying factor. The root of unity resides in mental affinities. If we are responsive to the feelings of others, then we can be friendly with foreigners. On the other hand, lack of brotherly feeling can only go on to build another Pakistan. And the two men who built these two nations came from the same province and both had English education. Attempts were made to awaken the mass through English for many years but they have not succeeded.

Later when the father of our nation came to us, he chose to speak in a language understood by the common people though he could speak and write English better than many. With independence we have now a new responsibility. The fruits of freedom should not be confined to a handful of English-educated people; they must reach one and all.

Those who think that the doors to knowledge and freedom will be shut if English is taught less, are grossly mistaken. They think that India should be surrounded forever by high prison walls and that light will enter from the top to reach only the upper echelons of the educated people and they will transmit that light as they receive it to the ignorant people at the lower strata. It is difficult to achieve progress in this way. It is not advisable to impose the responsibility of the entire population on a small section. People of the country should bear their own burden themselves. We have not made much headway in achieving this in the last fifteen years.

I think that your demand of banishing English from the prime position it has occupied so long should be given serious consideration. Think carefully how this proposal can be implemented in different provinces. My arguments for removing English arise purely from an urge towards self-defence. I strongly feel that this extraordinary emphasis on English would impede the spread of education in the country. Those who choose the vocation of a teacher must make a lifelong effort to see how best one's thoughts can be expressed in one's mother tongue.

We appeal to the educated people. Let them try hard to usher in a new era in the country. Let them figure out what the new medium of instruction should be. Let them introduce such a system in every state as would remove illiteracy and ignorance from our country. A word of caution is needed for the fanatics. Our call for a new era will never materialize simply by substituting one language by another.

Our priority is to educate our countrymen. What I want is that knowledge which is useful to them, which keeps them free from disease, makes them prosperous and provides them their daily bread, should be made easily available to all.

This calls for a sustained effort. The problem will not be solved simply by taking part in discussions. I hope the participants speaking different languages will keep this in mind when they return to their respective homes. The success of this conference will depend on our realization of two points. First, we can achieve our ends quickly only if the mother tongue becomes the medium of instruction; secondly, our regional attachment does not depend on language, it is something rooted in our minds. If you can carry

these two observations back with you, then I will consider that your efforts have been worthwhile. Through mutual interactions we hope to become aware of two more facts. First, our unity is not superficial but lies deep. This has not come about by speaking one language. Secondly, there is no problem that cannot be solved through unified and earnest effort.

As I have said before, the integration of our country cannot be brought about through a special language. It is necessary to make it clear that the unity is based on mental affinity. If we wish to build something, the feeling of mental closeness among the workers is essential. Otherwise, what we will try to achieve in different provinces will not hold together as parts of a single structure.

We have tolerated a foreign language and a foreign rule long enough. If this was the way to establish unity, then it could have been attained a long time ago. As it did not occur, voices are now being heard against the acceptance of the constitution and its directives.

Our national heritage is the result of centuries of endeavour and no matter how old it is, we wish to cling to it with our life. Respect for tradition is the foundation of national unity. At least from my own experience I have felt that anybody willing to subdue his self-interests and travel within the country can see that this mental kinship existed though it may not be present in the political sphere. The sages of our country, our pilgrims, our ordinary Hindu boys and girls and even the people of different religions living in the villages have all realized the existence of a common bond that is not to be severed easily. However, sometimes conflict of interests can overwhelm the sense of unity. To keep alive the spirit of integration requires special dedication. Unity cannot be achieved through words alone.

How does one pursue this dedication? It seems to me that people in different provinces have to use their vernaculars to try and understand the nature of our heritage and the common bonds which held them together in the past. The diversity of nature is such that every moment man is faced with a demand to create something new. So the harmony of our country that we wish to build today has to be more complex, profound and powerful than the harmony that was felt in the past. This is my conviction.

We can hear the clarion call : the time has come to know thyself. Let the people from different provinces draw close together and come to know the taste of freedom, what we had lost and what we have gained today.

We have to follow the paths which other nations have adopted to reach the peak of development and move fast. To do this, we need to take the principal message of the modern age to the minds of our people. Prehistoric ideas have to be discarded. The victory of man over nature and the consequences of it that have been felt in many countries have to be realized not only by those whom we regard as educated but conceited but by everybody.

If this is not done, then we cannot keep up with the rapid pace of progress in other countries. And if we are really moved by the term 'Bhāratvarsha' and if our hearts bleed

for our countrymen, then we do not need the aid of any foreign language to strengthen the bond between us. As a matter of fact, by the time the English-educated leaders who had once fought to drive a foreign power away from our country themselves gained power, a tendency to drift apart was clearly evident.

It should be easy to create a sense of unity and overcome the separatist mentality through the use of vernaculars. It is not enough to preach national integration from a regal platform. We have to talk this message of unity from every home. Let us not rest content with the fact that we have provided for education — everyone has to realize that it is our common responsibility! This is why we need our mother tongues, we need arduous practice and we need to keep our minds pure. Instead of clashes and conflicts, let us talk about love.

Those of you who realize this, do carry on your work patiently within your own limits. In our childhood we used to say, let us carry on as we have God above us and we have nothing to fear. If you have a clear vision of unity in your minds, then you have nothing to fear. Let the movement that you have started continue to grow. Let our people be conscious of the degree of their responsibilities. Let us hope that compulsory education will be implemented soon so that every child, every young man and woman will have the scope to develop as complete human beings. Let us realize that in the present age into which we have been born, there is great danger in 'little learning'.

I have really nothing more to say, because speaking is not always a good way to reach others. As a writer once said, the aim of language is to conceal our thoughts. There is a belief in our country that if we think of one another's problems intently, we shall find the right answers from within ourselves. I hope that will happen, and your movement will be successful in every sense.

Address delivered in Bengali at the 'Angrezi Hatao Conference' [Banish English Conference] at Hyderabad in October 1962, included in S N Bose, Rachanā Sankalan, *Bangiya Vijnan Parishad, Calcutta 1980. Translated from Bengali.*

8

Education and Science

Mr Chancellor, Vice-Chancellor and Friends,

I am very glad to be with you all on the Convocation day and to take part in your rejoicings. I congratulate the new graduates of the year and look eagerly forward to know about your young University. It is a wonderful idea to have a residential university at Ranchi. Its beautiful surroundings and its mild climate have always attracted visitors. It has been famed as a haven of the East for mentally ailing and depressed people. A vigorous Agricultural College has now started in its neighbourhood and a research institute of lac has been functioning for some years and you will soon have a factory for heavy engineering tools not many miles from the centre of the town. All these are important developments and will certainly benefit your graduates. However, these important institutions are all of very recent date. Fifty years ago there was only the University of Calcutta to take care of higher education in all the eastern provinces of India. It prescribed courses of study, held examinations and granted degrees to successful candidates. The actual teaching-load, however, was left to the care of college and school teachers. Our recently acquired independence has now increased the tempo of progress. We have now five Universities : within its own jurisdiction, each University superintends and organizes higher learning and research. It selects its own teachers and with the resources placed at its disposal by the State it tries to obtain the best possible results. However, it is not always wise to have mere replicas of older institutions. Every time we have occasion to start afresh a new venture, it is desirable to take account of the results achieved so far and to assess how far the older Universities of India set up after western models have been able to bring

about essential improvement and achieve enduring results. I have been all along a University teacher. It had always distressed me to watch the low yield that our earnest endeavours produce in the field of education. If we take an honest and frank stock of our achievement we have to confess that since the introduction of University education in our country, in about a hundred years we have not been able to produce any spectacular results. A large percentage of our people are still illiterate; and in all our attempts of rapid industrialization we have still to lean heavily on foreign support. We have now arrived at a critical moment of our country's history. We have to remember that we can no longer afford to linger peacefully in our nation-building work, but we have to reorganize ourselves quickly if we wish to preserve our self-respect and our recently achieved independence. It is therefore necessary to note the results which have been achieved in a hundred years in other eastern countries and try to analyse the causes which have worked against our quick progress. It will be needless to say that as a nation we are vitally interested in the programme of education. The future of our country depends upon its citizens. The universities are working for the evolution of properly trained men and women. We cannot assert that the advancement of learning and knowledge are the only ideals of our universities. A modern university in India has to think of and to work out a sure way which will enable them to evolve a method to train our future citizens, strong in mind and body, who will be able to resist the influences of disintegration, conserve their forces and intelligently utilize the natural resources of this land for the progress and upliftment of our countrymen. I have mentioned illiteracy which still continues to torment us. Recently there appear other disquieting symptoms which have developed within the universities. Our students and our teachers have often been at loggerheads and the authorities of various universities often pass through days of torment and agony due to a widening gap of misunderstanding and a lack of discipline amongst students.

Students are now sent every year from India to different countries, to universities, factories and industrial establishments in foreign lands. They go abroad for studies in the various branches of mechanical and electrical engineering. But very often the Indian apprentice is found to be lacking the preliminary training which would enable him to absorb rapidly the modern methods of work. An illustrious Indian educationist who has returned recently from America reported that Professors and experts there had remarked to him that our boys are often considerably nervous for their unpreparedness when asked to handle modern engines and machineries. In our own country an analysis of the results of the university examinations would reveal a large percentage of failure in every test. What a tremendous waste of our national effort! I have often thought that all these shortcomings and failures are not really due to anything essentially lacking in the quality of the materials we handle in the student community. On the other hand, given proper guidance and facilities Indian students do shine and show their aptitude and efficiency in all directions and in every field. I feel this loss of time and effort can be avoided if only we analyse and discover the shortcomings in our methods of teaching, and set about rectifying them resolutely in our universities. Some time ago I had occasion to suggest that time has now come when we should revise our

method of teaching and utilize the mother tongue as the medium of instruction at all stages in schools, colleges and universities. As an old teacher who had occasion to come in contact with hundreds of students, I am convinced that considerable amount of time could be saved and a better foundation of scientific knowledge prepared in the minds of our students, if at every seat of learning teachers and students cooperate and frankly discuss their problems and work together to solve their difficulties. However, if a foreign language comes in between them there is bound to be considerable handicap. Often an enquiring student is not able to express himself properly nor is the teacher sure that he has been able to make his pupil understand all that he wanted him to do, if he always has to use English as the medium of conversation and teaching. I am convinced that this present method encourages cramming, does not work for a proper understanding of the essential facts of the subjects taught. My opinion expressed with a certain amount of reserve had however raised controversies all over our country and there are many who still think that it would not be to the best interest of our country if we abandon English as the medium of instruction in the universities. A very eminent educationist has put forward the view that if different languages are used in the different universities of India it will hinder 'academic mobility' and thereby work against a free exchange of thought and ideals which alone can eradicate aggressive provincialism. Though our chain of slavery has slipped away, it has left behind the golden thread of the English language, which, as many venture to suggest, has worked for our integration and has helped to re-discover our souls.

These people still pin their faith on the medieval ideals of the University. According to them, the material needs of the hour should not be magnified unnecessarily and oblige us to change the character of our universities or our principal curricula. They think that we should continue to give prominence to the eternal verities, to the philosophy which our forefathers have advocated, and continue to depend upon the study of the classics, on Plato and Aristotle, on the humanities, languages, and jurisprudence as the principal items which should provide the universal background of knowledge to all our students. The importance of scientific studies should not be urged beyond a certain limit. They recognize the usefulness of science. But a modern scientist is a suspect. They believe that science has nothing to offer which would replace the old ideals. Scientists are useful perhaps as builders of efficient tools which would help men to establish welfare states; but they still maintain that culture and purposeful life in a welfare State can best be built on ancient philosophy which, many maintain, meant principally non-violence and *ahimsā*.

In a democratic country anyone is free to have his own views about the lessons of history, and read and interpret them in his own way. Thus it is also permissible to have an alternative interpretation of the decline of Indian civilization.

The 'other-worldliness' which was consistently recommended in our seats of learning, in Nalanda and Taxila, and which formed the backbone of our ancient culture, set a high premium on the sleepless contemplation of the eternal verities — the individual would thus realize the knowledge of self and recognize the world as a very temporary halting place, and this realization would make him yearn for release as soon as possible

from the trouble and trails of this life. However much we may extol the philosophic heights of these speculations it cannot be denied that a persistent propaganda that the world is a place of temporary stay for the individual who should think of his own salvation as the principal aim in life has its repercussion in creating a carelessness in all mundane matters. The Indian thereby soon lost grip on the affairs of the world, and was soon overcome and subdued by the barbarians.

The lessons of the centuries should teach the modern Indian not to lay too much emphasis on individual salvation. It is better perhaps to look on life in a different way. The weal or the woes of the individual should not be made the principal object of philosophy, but the men in any country, closely knit by a common heritage, may be regarded as taking part in a relay race, where the advancement of the country and the advancement of the national flag is more important than the fate of the individual runner. Though successive generations toil, and break down in their effort to raise the standard of life, to conquer poverty, disease and death, the important thing that matters is how far such efforts have succeeded for the betterment of the country and the world. It may be urged that this alternative view is also Indian, though it may ultimately deny individual soul at the ultimate reality, and preach the sovereign rule of *Karma*. The 'materialistic' philosopher may also urge that this 'other-worldliness' provides a good breeding place for inefficiency, selfishness and greed. If a man is anxious for his own salvation he will take refuge in a cavern, and go away from the abodes of men. He will thus be saved, and allow the devil to snap up those who linger and toil against earthly ills, and try to procure material prosperity for the community and his country.

While this philosophy flourished in our country, the neglect of a serious acceptance of life by our first-class thinkers brought second-rate petty people into prominence who gave lip-adoration to philosophy but actually indulged in jealousy, squabble and internal strife. As a result foreign invaders came, often by invitation, and established in India centuries of slavedom. However, it is a hopeful sign of the times that we are waking up. There are alternative solutions, and a harmonization of the worldly duties with the other-worldliness may be brought about. Our country has recently celebrated the centenary of the great Swami Vivekananda who bravely put service of suffering humanity even before salvation. May we all remember his message, and gird up our loins to raise up the wheel of the car of humanity from the mire in which it has got stuck and sunk!

I had recently an opportunity to visit Japan, where practically at about the same time as in India efforts were made to introduce the western methods of education, and Japan has now transformed itself into a modern State and its progress causes universal wonder and admiration. I therefore eagerly accepted the opportunity and went to attend a symposium in Tokyo about the place of science in modern life, arranged by the Physics department of the Tokyo University. Here I met many scientists, mathematicians, physicists, biologists and philosophers, mostly Japanese, but also a few foreigners, e.g. a philosopher from America and a mathematician from Yugoslavia. I had thought that in such symposia we would utilize a foreign language but when I arrived

I was told that while most of the scientists of Japan understand English and possibly a few more foreign languages which they often read, the training throughout the length and breadth of the country is based upon the Japanese language and I should make myself ready to hear Japanese principally spoken in the symposium. I should however have the benefit of an interpreter who will interpret the principal points urged by the different speakers in the symposium and when my turn would come the Japanese. Evidently this method worked and I was surprised to see that a most intricate and abstract discussion about the application of science in a modern State was carried out easily in Japanese and when we the foreigners spoke they understood our thoughts perfectly well and put forward their pointed criticism evincing approval or dislike of our theses. I also found Indian students in Japan. I visited the universities where most modern things in Physics were being taught in Japanese side by side with many daring and original experiments rigged up by the Japanese students. In the universities the research scholars discuss science among themselves in their mother tongue. Obviously they use many loan words but they are not ashamed of this habit. I discovered that a book on the effect of the nuclear explosion which was written in English by two Indian scientists has been translated in Japanese and I was told that this book has sold well, about three thousand copies in six months. Ordinary Japanese who perhaps can read only Japanese are anxious to know about the effect of nuclear explosions and would perhaps trust the impartial Indian view more than others. However, in our own country these eminent scientists have continued to write in English and thereby keep about 80 per cent of their own people in ignorance about the danger of nuclear fallouts. I have been also in places where the Japanese students work and try to rebuild what had been destroyed during the last war. The war has been terrible for them. The victorious allied soldiers have broken down their tools of research, and thrown them into the sea. For a time all research institutes were closed where anything with even a faint suggestion of nationalism could be detected. But all these changed after all, and they have now found opportunity to reconstruct during the last seven years what Japan had lost. I was told that most of the research instruments and even the nuclear reactors are manufactured and erected locally by the Japanese themselves. In research institutes students practically build all the apparatus and equipments they need with parts manufactured in their own country. This was a refreshing sight for me who always believed that wonders could be achieved in our own country if only we could utilize our resources in the most efficient way. Japan furnished me with an actual demonstration.

I have returned from Japan firmly convinced about the necessity of adopting the language of the provinces as the medium of instruction in the University. Perhaps in Bihar, the movement for adoption of Hindi as the medium of instruction in all stages in schools and colleges has already begun.

It has often been urged that the absence of suitable technical terms in Indian languages may prove to be a hindrance for the study of the sciences. I am not a purist, and would welcome the continued use of technical and scientific terms in English. If our boys can understand them easily, they will continue as loan words, and I hope they will ultimately be absorbed and enrich our national vocabulary. Many words which

were originally of foreign origin are now current in all Indian languages, and are easily understood by our common people.

I hope with the progress of scientific education, many more words could be pressed into service, and ultimately suitably transformed to suit the genius of our people.

Very often the labour of translation of scientific terms would prove to be a vain effort — railways, restaurant, kilogram, centimetre, wheel, lathe, thermometer, pressure gauges, boiler, cutters, electrons, atoms, bacteria, fungus, differential co-efficient integration, are understood by almost everybody; paper, chair, tables, are now in daily use. I need not multiply instances, and I hope the teacher and his pupils will easily come to a mutual understanding when they discuss their subject, and it will be really the duty of the teacher to keep himself abreast with the recent developments in this task of adopting the language for the popularization of the scientific outlook.

A scientist is often forced to join issues with his opponent, and carry on wordy contest — but he is happy when he can set up a suitable experiment whose results would help him to demonstrate his propositions. I feel this very desirable that somewhere in our country, the experiment of running a modern university with a strong bias towards sciences and sociology, where the medium will be an Indian language, be launched. With an eminent scientist as Chairman of the U G C, I have every hope that the scheme will find favour and good support.

In a residential university of the type I have in mind, free intercourse among students and teachers would be easy. If we are all united in our common effort to raise high our national flag in all the spheres of life, the petty misunderstandings and senseless jealousy should easily disappear. The ideal of the modern university should not be only learning and advancement of knowledge. It should preach the essential unity of men, irrespective of caste, creed and religion, of status, wealth or origin.

Man is the ultimate and the supreme truth which we should realize and I would hope that in one of the universities of Bihar, this supreme realistic view preached by the ancient poet of the land should be accepted as a symbol and as an ideal. 'Man is the truth above everything else' (*Sabār uparey mānush satya*) is something to which all western countries are also waking up.

A modern Indian university will I hope be a centre of an age-long sacrifice (*yajna*) where the teacher-priest and the student acolytes would sit together, and try to learn and realize for themselves what everywhere men have realized after centuries of strife, sacrifice and bloodshed. May there be a place found for such penances, and when the final libation to the supreme truth 'man as the ultimate reality' be poured, let there be a reawakening throughout the length and breadth of our land. Let us thus realize one of the fervent prayers of our great poet Rabindranath :

Where the mind is without fear and
the head is held high;
Where knowledge is free;
Where the world has not been broken up into

Fragments by narrow domestic walls;
Where words come out from the depth of truth;
Where tireless striving stretches its arm
towards perfection;
Where the clear stream of reason has not lost
its way into the dreary desert sand of dead habit;
Where the mind is led forward by thee into
ever-widening thought and action—
Into that heaven of freedom, my
Father, let my country awake.

Convocation address delivered at Ranchi University on 11 May 1963.

9

On his Seventieth Birthday

Respected Chief Minister Prafullachandra, my dear friend Humayun Kabir, Dr Deben Bose who has been like a teacher to me, and all those who have loved me and others who are present here — my greetings to you all.

Seventy years of my life have gone by. One could say every day now is a gift of God. The allotted time span of a man's life is over; the time span over which a man can depend on his own strength to carry on scientific research and social work is also part of the past. But I still have time, perhaps to repent a little for the wasteful activities — of which one or two will find mention here — a little less of which might have allowed for more valuable work to be done. But even the Omniscient cannot tell now if I could have really done something useful. For although psychology deals with how a person's mind works, one cannot always definitely perceive what one's mind is inclined to or what one ought to devote oneself to. In the old days in such cases it was the guru who enlightened one — even that depends now on the mercy of God.

As I was saying yesterday, we started our life at a time when there was a great movement raging through the country. Today, I recall that the few of us who were then the first to feel inclined towards science and were able to achieve one thing or another for the country, all of us went to college at the same time. Many of these friends are no more. Their names have been recorded in our country's history. Not only did they make a name for themselves in science, they also went on to be successful in several constructive activities; Bengalis will never forget them. Particular mention must be made of Dr Saha. On the one hand he changed the character of the Indian Association for the Cultivation of Science founded by Mahendralal Sircar, and on the other hand, added to the University of Calcutta as an integral part a faculty for the teaching and the research

of a special subject — the faculty that now bears his name. It was through Dr Meghnad Saha's initiative that the study of nuclear science began in the university, and then in a different way all over India at different centres.

The First World War started soon after we had learnt a few things as students. We heard of many new discoveries after the war ended. We found that both sides had made immense strides in science through competition. We were not aware of them for a long time because in those days not many scientific journals came to India, and our prescribed textbooks also did not carry such information. I remember how eagerly we looked forward to the return of Dr Deben Bose who was interned in Germany for many years and was gathering much information, in the hope that he would bring us news of the latest progress in various areas of science in Germany during the war. I was not yet familiar with the German language, but Dr Saha had studied a little German and acquired his Intermediate Certificate. There was no end to our aspirations. We went to Dr Brühl's house, searched through his library and collected many useful books which Dr Brühl himself might not have read. We were completely enthralled by the information we gleaned from the writings of Maxwell, Boltzmann, Planck and many other scientists and were totally engrossed in them. At this point of time Dr Deben Bose arrived. He brought with him the reports published in Germany to mark the sixtieth birthday of the great Planck. They contained brief accounts of many new aspects of the new science of which we had no idea. We pleaded with him to read them out to us and we tried to translate (particularly me since Dr Saha was not always present) every word of those accounts into our own English. This yielded good results. Along with the new scientific information that I gathered, I also came to be acquainted with the German language at first hand. This gave me a lot of confidence. Some time after the 1919 solar eclipse a startling news appeared in newspapers in our country as well as elsewhere in the world. Einstein's theory had been completely vindicated. We were all amazed. Everybody was eager to know what new things Einstein had to say after Newton. We worked hard to translate those papers — partly Dr Saha and partly me. It took the form of a book. The preface was written by our friend Prasanta Mahalanobis who was then teaching relativity at Presidency College, Calcutta. The book was published by the Calcutta University, a fact which they must have forgotten by now. The book was used in the East for a long time. Then some time before the Second World War German racism assumed such monstrous proportions that all non-Aryan races were driven out of that country, resulting in riots in many cities in which knives were frequently used in secret. My guru, Einstein, who was then not in Germany, protested. As a result, he too had to leave Germany. Our book also went out of circulation thereafter.

When man begins to regard nationalism as the highest treasure of life, his sense of duty and his norms get distorted. The same thing happened in this case as well. I was then in Dhaka, teaching at the University. I had a few German friends there. They said, 'What has your friend done?' I considered it my greatest honour to have Einstein referred to as my friend. For the paper for which I am known might have been triflingly short but Einstein himself had translated it — very few people would be able to boast of such an achievement. In fact, he wrote two papers within three months of reading my paper, using

it for all elaborations and extensions possible at the time. Curiously enough, in one of the papers he refers to me as Dr D Bose. I found that quite often the surname Bose meant one family to foreigners. Shortly after this I had the opportunity to go abroad. His words of praise became my passport there. As a result, I could with little effort meet and discuss science with such stalwarts as very few people have had the good fortune to. Among them were Madame Curie, Professor Langevin, even Professor Gehrcke and also others who have become immortal in the world of science. I could not only speak to them about their work but also had access into their well guarded fortress-like laboratories and familiarized myself with secrets that were not usually accessible to foreigners. What is more, I did not have to pay any deposit to borrow books from the State Library of Berlin, the Staat Bibliotheki. On Einstein's letter of recommendation I could borrow three or four books at a time, just like any university professor of that country.

Of course, those were the days before Hitler. My sojourn in Europe lasted for two years. Naturally I remember those days particularly well. We project ourselves as people who honour learning, but I have seen for myself the extent of the respect really learned people in Europe used to command. Things have changed since then. Those who are aware of the facts of history know very well how scientists and professors were harassed by the Hitlerite racists. Some of us had aspired for a level of scientific research that would establish a tradition of science in India, so that she would be recognized and respected everywhere not only as the land of the Vedanta, destined to remain for ever a poor relation of the world-famous Aryans, but as a contributor to modern civilization. The Germans also have great regard for our Upanishads and philosophy. They believe that there is a close relationship between the ancient Aryans of India and the German race of today. At the same time they seem to think that the present day Indians only carry the Aryan name but have no real connection with that great race ; like the present day Greeks, who are descendants of people who lived in the hilly tracts of Macedonia and are not the inheritors of the ancient Greek tradition. At times I had the feeling that similarly the Germans looked down on us with charity. But coming to think of it, do we also really believe in what our forefathers had said ? For instance, we claim that our feeling of equality for all living things is a part of our tradition, yet at the same time defend our caste system as the great solution to the world's problems. There are many who are of this view, including well-known philosophers who will explain the issue very well. While they generally advocate the four *varnas* they are indifferent to the various kinds of spectra that can be produced now which is a modern phenomenon. We are very proud of our Golden Age but we feel reluctant to talk about the backward peoples who lived in this very country at that time and the way we treated them. Still, sometimes these truths get exposed. Take the case of the non-Aryan Shambuk during the reign of Rama. The poor fellow had a tough time in Rāma-rājya. He wanted to acquire through rigorous prayer the same things that the Aryans had made use of to develop themselves. As a result, apparently there was a great drought in the land of the Aryans, and they decided that such a man should not be allowed to live. Therefore he was sentenced to death, even in the reign of the good king Rama. This may sound somewhat irrelevant but I am using it to give you an idea of what we had in mind as students — it is not entirely true to say that we wanted to bring back

the same good old days in our country. Actually, most of us wanted to see the country free, and ideas were afloat as to how to get rid of the British.

Like the more recent 'Down with English' movement with which I got involved, we were taken up with the slogan 'Down with the British' in those days and got carried away in various directions by various currents of thought. Some of us were convinced of the need to educate the masses of our country, to create in them a sense of belonging. But this feeling had to grow to be a part of them through thorough education and not merely because somebody in authority like a guru or a zamindar said so. With this in view we had organized some mass education centres in Calcutta — but those night schools were short-lived because the British thought they were training centres for bomb-making. So they had to be closed. That chapter is likely to have been forgotten by now. Then there were those who believed in the power of the bomb, who said — look, don't indulge in such childish things. Let's drive the British out first and then you will see everything will be fine. Now the British have left. Many of my extremist friends have also died. Those who are still around seem quite confused. After all, educating the masses was not as easy as they thought. Sometimes I wonder if some of our old values like our feeling that we had to have the same feeling for all had really persisted, then would we not have been able to nip in the bud many of the newer dangers that are looming in today's troubled times ? One cannot be sure. Even today there are astrologers in our country who believe that whatever we do or whatever happens is pre-destined — our horoscopes tell us not only about this life but about many previous and future lives as well. This is a belief deeply ingrained in many of us. Even a Muslim friend of mine had once gone to Benaras to get his horoscope prepared and found that even the first letter of his wife's name got revealed. He came back very upset. He asked me : How did they do this ? An Indian scientist is often faced with such ticklish problems. For instance, is it possible to reconcile the idea of many births with scientific evolution? The scientist turns to the philosopher who merely smiles. The scientist feels the urge to proclaim that scriptures, philosophy, even religion are not of much consequence. They would kill all initiative in man to work. If you say, no matter what you do, what is going to happen eventually is predestined, nobody would have any initiative. Nobody would even attempt to mitigate the painful tribulations we encounter today. It would be enough to offer prayers at the Kalighat temple or Tarakeswar. The theory of repeated births is a great riddle to scientists. Many of them are breaking their heads over it. I am reminded of an incident. I had once put the same question to a holy man. I happened to have great respect for him. My friend, Bidhubhushan Roy, considered him to be a saint with supernatural powers, for he firmly believed that it was by the holy man's grace that he survived after he lost consciousness in the Science College from some dire disease. Both of us asked him on a suitable occasion, 'Baba, is rebirth a fact?' He said, 'Yes'. But we were not convinced. We kept asking him, 'You say rebirth is true, but have you really found this out for yourself ?' He said that he was told by another yogi. Neither Bidhubhushan nor I was satisfied. We persisted with our question from various angles. Finally he broke into a song, went into a trance and tears rolled down his cheeks. The song went like this — 'Where have you brought me, mother? My entire body is covered with wounds inflicted by the nettles and thorns of this

forest. Mother, when will you take me back into your arms?' We came back disappointed that day. Since then I have occasionally felt that the Bengalis today have been wounded in various ways. Perhaps the mother will take them into her arms one day, but before bothering her, it would perhaps be more fair on our part to see what we can do ourselves to alleviate the pain. The thinking and self-confidence required to do this do not necessarily imply that everyone has to become an atheist. One has only to remember that the world has changed a lot over thousands of years. The environment in which happiness once overflowed in our land from prosperity and abundance does not exist any more. People who lived then would not have understood our current problems. Naturally it is futile to look for solutions in the old modes of thinking. Every century throws up new problems to a nation. Finding a solution to these problems, these imbalances, is our responsibility.

The fairy tale prince had one day asked the Sphinx : What is the future of man and what should he do ? Man ought not to have been born, replied the Sphinx, and now that he is born, he should die as soon as possible, that would be the best thing for him. The Sphinx has disappeared but her terrible cynicism springs to our mind from time to time, specially when man is in deep trouble and cannot find a way out. But history indicates that man's attitudes keep changing. It is true that hatred and violence have not abated — man keeps on using firearms and killing — yet efforts are also made in the midst of all that to give relief to the distressed. People are spending money from their own pockets, saving from their own produce to feed the famished. In the utter darkness of despair what holds out hope for a better future for man are perhaps these small incidents.

For various reasons perhaps only a small part of our youthful dreams have turned into reality. Nevertheless, Bengalis cannot afford already to be weighed down by pessimism and think that we can do no more than just turn the dreary pages of our life, or that everything is pre-ordained, there is nothing new for us to strive for. We badly need a change of this attitude. After all we have produced Acharya Jagadischandra, Prafullachandra and Dr Saha who dedicated their entire lives in the service of science. So it is hard to believe that getting a degree or a job will turn out to be the last goal for a Bengali, or that Bengali idealism will be reduced to the demand for the reservation of a certain percentage of government jobs for them. It is not a new thing for Bengalis to be concerned about others. They have often thought nothing of sacrificing their self-interest for the sake of giving concrete shape to a higher ideal. This makes me feel that we can expect great things from them which can help in the solution of national problems. I believe that the Bengalis do have this universal outlook. The history of their heritage will bear me out. My respected friend Prafullachandra is the Chief Minister now. He had done a lot in the cause of society early in life. I think his experience will be of good use to our country and our society. Some cynics might say that the office of a king made even Lord Krishna forget about Vrindavan when he ascended the throne at Mathura. But I am an optimist. I think that a Krishna of the Kaliyuga need not act in the same manner. The humanism and love for all mankind which the Bengalis have cherished is not a quality imported from the West. I have read a story about the great Rammohun Roy, who heard that a certain country in South America had achieved independence and, delighted, he threw a

party at once. I would not read it as merely a publicity stunt because the joy that he shared with the invitees ended there — the news did not appear in the next day's papers. Nor are we aware that the Raja received any award for it. Perhaps this feeling of kinship with humankind is shared by everybody — but it is suppressed in practice by petty self-interest.

I have gathered varied experiences from my visits to different countries. I had to move around in Dhaka during communal tensions between the Hindus and Muslims. I found that affection and sympathy between the Hindus and the Muslims were not entirely wanting. Of course, we tend to make a lot of fuss about religion. So I am scared of the religious kind and prefer to give them a wide berth when they are particularly vocal about it. It seems religion does not have such a dominating influence in China. That is why they could achieve so much progress so soon, although it seems the 'great leap forward' has eventually landed them in a ditch.

Bengal has always been distinguished by the absence of religious fanaticism. It was not at all rare to find Hindus and Muslims taking food together — again, Hindu writers jostled with their Muslim colleagues to liven up the literary experience. There was a time when we had hoped to build a harmonious home for the Hindus and Muslims, children of mother Bengal. The soil of mother Bengal has blended the hopes and wishes of God knows how many countless Hindus and Muslims — who can tell now! It was our hope to create a model of communal harmony here that would be our gift to the world. But our hopes have been dashed like castles in the air. Nobody knows what the cruel countenance of time that has gradually revealed itself would lead us to.

Children who have assembled here with a lot of zest for science, I would like to tell you this : do not take up science for the sake of passing examinations. Just as on the one hand, one needs to study literature to appreciate one's own country and its problems, to look for the causes of misery of one's own countrymen and to be acquainted with what men are doing for one another in other countries, so, on the other hand, does one need to study science. It enables one to have an idea of what can be done to get rid of man's miseries. Your interest in science should be rooted in this quest. Today we are constantly being told that man is no longer interested in living on this planet. Soon he is going to fly off to Mars or somewhere far beyond. Maybe we have brought the earth to such a pass that there is no easy way to survive here. Someone from a civilized country has said, 'We are spending billions to launch a rocket'. If they had given us the money without squandering it this way, we could have found some easy means to feed hundreds of thousands of hungry people. But who is going to raise such issues? It is for you to do so. As for me, I shall stop here.

Address given in Bengali in reply to felicitations offered to S N Bose on the occasion of his seventieth birth anniversary, at the Mahajati Sadan auditorium, Calcutta, on 1 January 1964, later published in Jnan O Vijnan, *May 1964, and reprinted in* Rachanā Sankalan. *Translated from Bengali. Prafullachandra Sen, then Chief Minister of the State of West Bengal, was President of the Professor S N Bose Seventieth Birthday Celebration Committee, the body that convened the gathering, and Humayun Kabir (1906–69), then a Union Minister, was one of the Patrons of the committee.*

10

Convocation Address at the Indian Institute of Technology, Kharagpur

Mr Sen, Director of the Institute, fellow colleagues and scientists, students and ladies and gentlemen who are present, my grateful thanks for this kind invitation to deliver the Convocation Address of the year. The Kharagpur Institute has grown enormously during the last fourteen years. We have now many fine hostels and laboratories and the number of students from all parts of the country now exceeds 2,000. It grants degrees to engineers and architects, diplomas to skilled technical workers and doctorate degrees to successful research workers. The Central Government has started five similar institutes in the different states on the model of Kharagpur. Numerous technical colleges are also now functioning inside the various universities of India and the country is thus endeavouring to supply the enormous demands for technical personnel which our plans of industrialization have created.

I hope graduates, masters and doctors of Kharagpur, will play useful and important roles and be completely absorbed in the different projects set up countrywide in this tremendous endeavour to usher in the era of industrialization. In an underdeveloped country, it is perhaps premature to try to take stock of the progress achieved so far, as we are still struggling hard to obtain a breakthrough against all our obstacles heaped up by centuries of inaction and slavery and our plans have also not matured and brought in the expected results on all counts. However, on the eve of the fourth five-year plan, it may be necessary to be more wide awake to our realistic needs. Unforeseen difficulties have arisen during the last two decades. In 1963, during the convocation

address, Dr A N Khosla, the Governor of Orissa, who had taken a very important share in the drawing up of national plans, spoke on the deterioration of our relation with our neighbours, specially with China, which has necessitated heavy defence preparations diverting our financial resources in unexpected and unproductive channels. While we record worsening friendly relations and negotiations and talks continue and leaders try to arrive at a peaceful solution of our difficulties, we must at the same time try to increase our fighting strength, come what difficulties may. We must protect our country against all invasions and in all future plans and programmes this decision is bound to play a preponderant role, though it may cut across our plan of economic regeneration. To increase our distress, the population in India is on the increase at an alarming rate. In 1951 just after the partition we were barely thirty crores. According to the results of the last 1961 census the population is over forty-four crores and will in all probability continue to grow at this alarming rate. While we sing of fertility, of benediction, of Heaven for this lovely land, historians have recorded recurrence of frequent famines in ancient times. That, in spite of bumper harvests, famine may be produced by our thoughtlessness and mismanagement has been demonstrated during the last war. We still have recollection of the man-made famine of 1942. Yet our plans have not taken account sufficiently of the need of agricultural development to make our country self-sufficient in the matter of food. The difficulties cannot be denied and whether we will ever be able to solve our problems without bringing about a large scale socialistic reconstruction remains as yet unproved. In our race for industrialization we have spent our valuable reserves not only in importing machineries and defence appliances but also in importing food materials. We have experimented with deficit financing, and now the scheme also has to be called to a halt. We have to pay heavy interest on loans and we cannot depend on our friends to continue their aid unless we can prove that we have been trying to utilize the aid given in the best way possible. Thus on the eve of the fourth five-year plan the difficulties have increased and the target seems yet far away. In 1956 our late beloved leader, Sri Jawaharlal Nehru, while addressing this convocation, had deplored the short-sightedness of the petty squabbles which wasted the efforts and resources in the adjustment of the provincial boundaries on linguistic basis. He felt that such internal dissensions do forbode a real danger and he exhorted our countrymen to postpone the settlement of minor differences and to go forth with one united purpose in the exciting adventure of raising the status of our country to the level of a modern progressive state according to the western ideals. However, the plans seem to have neglected one important task, that is the removal of illiteracy which in the ultimate analysis seems to spell a real danger against the attainment of true democratic ideals. If the majority of our countrymen are not able to read, to understand what is best for them, or are not trained sufficiently to judge the supreme necessity of avoiding all senseless turmoils, the task of keeping the country together and on the right path devolves on the minority and the danger of their succumbing temporarily to a partisan spirit may well become very real indeed. Jawaharlal Nehru had warned us against such demonstration; but even ten years after he had spoken, riots have broken out all over the country on the language question. Valuable public properties

worth crores of rupees have been wantonly destroyed and whatever may be the reason we may put forward to exonerate mass demonstration, our leaders cannot be completely absolved from all blame for having created this unfortunate turmoil at a critical moment when our enemies seem more than ever bent on forcing issues and this crisis requires all our countrymen to unite. During the first Chinese invasion there was a spontaneous outburst of patriotism which made all of us think that we have firmly realized that India for better or for worse is moulded into one common whole; but that we had not wisely judged the situation seems to be apparent now. I have mentioned only some of the difficulties and they are very real and may dishearten even the most optimistic among us. But it is better to be frank about the tasks that lie before us all, specially before the young men and women who have attained the status of graduates and doctors today. In whatever station they may have to work in future, they will have to remember the ideal of one country for all and they at least should try to steer clear of petty provincialism. The students of Kharagpur have the advantage of having been in residence together for years, and thus must have realized, by their daily cooperative efforts, the essential unity of India in the midst of apparent diversity. They have to keep this conviction not only strong in themselves but I hope they try to act as missionaries preaching and practising the duties which a common citizenship of united India connote. Scientists of the country have to realize that we cannot forever continue to depend on our friends for furnishing us with all our machineries in exchange of export of raw materials. The research workers at Kharagpur as well as those that are engaged in industrial pursuits should feel the necessity of making our country self-sufficient in all respects. When we try to build up factories and big centres of industrial production our countrymen should be trained to make plans for this purpose and captains of industry should be ready to utilize them and with indigenous resources, talent and capital try to usher in the industrial era. Though our production of skilled workers has now increased than what it was before independence, yet in comparison with many other countries they do not appear either very brilliant or spectacular. I have often thought that we might have gone very much further if we had tried to utilize our manpower by making them at first literate amd educated in the quickest way possible and by utilizing our mother tongue for achieving the enormous task of bringing about the growth of a mentality suited to a modern age in this country of medieval superstition and unproductive economy. Japan and even our neighbour China have, I think, demonstrated that to achieve large-scale industrial progress the utilization of a foreign language as the medium of instruction in schools and colleges is not a sine-qua-non. I have spoken about that often enough and I have been always misunderstood. Once in a Science Congress meeting a well-meaning countryman of ours, who had been forty years abroad during our political struggle, came back and having heard all our difficulties caused by groups having different mother tongues advised us to adopt English as our language and the medium of instruction for all. That this was not the right solution of our difficulties will be obvious if one ponders on the relatively slender achievement in literacy by earnest efforts during the hundred years of British rule. Also our neighbours have demonstrated that the mother language utilized for

education can give quick and enduring results. If our temporary madness ends and sanity comes to prevail, while we make the fourth Five Year Plan we should have a more realistic approach to problems. That we must bring about a hundred per cent literacy has been often reiterated from election platforms. It is time that we now work out a workable plan which will co-ordinate all efforts and produce this desired result and thus bring about a complete harmony regarding our aims and purpose.

In conclusion, I shall quote a few lines from Sri Jawaharlal Nehru's address of 1956, which so admirably sets forth our duties, and our ideals :

> We wish to solve our problem in India, . . . we want to advance in the technological sphere and the scientific sphere rapidly; and yet we want to adhere to certain methods which normally speaking do not help rapid advance. . . .
>
> Well, only the future in history will show how far we have succeeded — we have done well, I think, in spite of any number of difficulties and obstructions, . . .
>
> Difficulties and obstruction arise out of our own failings. It is because of our own failings and weaknesses that we stumble and fall and sometimes are pulled back. .
>
> If we function rightly, if we, in India, function with unity and co-operate with each other and go in the right direction,. . . there is nothing in the wide world that can come in our way.

The graduates of the present year may profitably also ponder over the noble words of our departed and beloved leader, and adopt these as their motto of life :

> A big drama is being enacted on the stage of India today. It is a part of the vast drama of this wide world which so often verges on the tragic, and which may, if we are not wide awake and careful, become absolutely tragic and end in disaster.
>
> I want you to see this, the dramatic aspect of what we are doing in India today, because there is a great deal of drama in it, the building up of the ancient country, and then see yourself as partners in a vast undertaking. Individuals are something more than mere individuals; in doing your little bit you are sharing in the great work.
>
> I should like you to feel that wherever you may work you are partners in a tremendous undertaking. So rely on yourselves, think of this your work in this big way, and grow big with it.

May God give you all strength and will to do your task in whatever situation you are placed in life, in the manner and the spirit of the above words.

Jai Hind.

Convocation address delivered at the Indian Institute of Technology, Kharagpur, on 7 April 1965.

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Agriculture in India

Director of the Institute, Hon'ble Ministers and illustrious guests of the Convocation, Students and Friends.

Allow me to express my gratitude in giving me the high honour and privilege of addressing the annual convocation where medals, certificates of merit have been distributed to successful and brilliant students of this institution. I am by profession a man of pure science, who has completed the normal span of three score years and ten usually allotted to a human being, and I rejoice greatly to be with you at the end of my life and admire the magnificent achievements brought out by years of careful planning and tireless work. Our great subcontinent has been the favourite home of man for many centuries, where people of many races and various faiths have mingled and struggled. Gradually they have been fused into a single nation evolving a standard of values and habits of life which have a distinctive place in the history of civilization.

When I was young I took to science hoping thereby to contribute in my own humble way to develop our country by helping dissemination of knowledge among our teeming millions. It was in an atmosphere of struggle when the future did not yet appear clear, that we began our life's work. Towards the end of my career as a teacher, I hailed the dawn of Independence which has brought new hopes and has called for greater endeavours. This old country is in many ways still in the transitional stage. Though our poets have sung of fertile lands and mighty rivers, we have not yet been able to utilize the natural resources of our country to make us self-sufficient in the matter of food and raiment and the usual necessities of life. Our economy is still mostly agricultural. About 75 per cent of our people depend on agriculture or its industrial subsidi-

aries for their livelihood. We try to grow our own food, maize, wheat, barley and rice, sugarcane, potato, tobacco nd cotton. We have developed also growing of jute and tea, which we export to foreign countries to earn valuable foreign exchange. However, it is common knowledge that our country is not yet self-sufficient in the matter of food and our experts tell us that if we can manure our fields and take precaution against insects and pests, we will be able to diminish the margin of deficit considerably between production and consumption. I am told India's average production of foodgrains is about 67 million tons, but only 40 per cent of it comes out in the market for sale. This reduction is mostly due to insects and rodents. Also we are to take note of the fact that compared with other agricultural countries, our yields per acre have remained disquietingly low. This calls for careful survey, collection of statistics, indroduction of new manures and educating our farmers to adapt themselves to new habits and ways of life.

The common man in India is an agriculturist, content with a very low standard of life in their village hovels, where they face stoically privations, misery and disease. However, all these are now changing rapidly. People have come in contact with foreigners having different standards of comfort and have learnt that by ceaseless effort and careful planning it is also possible to improve their lot and gradually produce an atmosphere of hope and prosperity as seen now in the advanced countries of the West.

In early times, every Indian village was almost self-contained. It grew not only its own food but provided all its simple wants by local industry. Its cloth was spun in its hamlets, its oils and dyes were mostly extracted from local herbs and seeds. The native physician catered to the needs of the sufferer by administrating suitable remedies to their manifold woes. This state of affairs still continues in some parts of our country, though we are rapidly opening up our hinterland by magnificent roads which bring townsmen and villagers together. Modern vehicular traffic is now gradually replacing the animal-driven carts. These transport facilities have helped our agricultural farmers to market their commodities and to introduce in their lives the urban facilities. I am told our farmer is quick to learn and if we send out among them teachers who will tell them about the new conquest in the agricultural field, preachers who will demonstrate the actual capabilities of production, they will quickly adapt themselves to these innovations.

The Indian Agricultural Research Institute has been famous for its various activities. The Department of Agriculture had been established in 1901, and the Agricultural Advisor to the Government of India had under him, along with this one, various other Institutes for conducting fundamental research, postgraduate training in general agricultural practice, plant breeding, agricultural chemistry, mycology, etc. The Institute here has a long record of useful work. They have developed rust-resisting varieties of wheat and disease-resistant varieties of rice, millet and sugarcane. They have worked on the diseases of plants due to fungus, and other pests of crops — cotton, sugarcane and fruit trees. Their work on plant breeding and genetics, the reclamation of soil, improvement of fertility and res arch on soil microbiology are of a very high

order and their value has been universally recognized.

With the advent of the new age after Independence, our Research Institutes have come in closer collaboration with similar institutions in other countries and the movement has started for the introduction of high-yielding varieties of cereals. I am told that there have been striking and noticeable improvement in the average yield in wheat and paddy, achieved mainly by the introduction of new varieties. This year we are blessed with a fairly good monsoon and the results have raised high hopes that we may even be self-sufficient in our food within a few years and that have been talked about in our parlours and Parliament and flashed in our local newspapers.

However, we have not yet been able to get a complete picture in all its details. I am told that the new varieties are not so resistant to local disease. Their demand of chemical manure has to be met with erection of various factories which will produce the newer varieties of manures. Fields are to be irrigated properly to produce the best result. All these raise a host of problems calling for fertility surveys about the nature of the soil, water levels underground, irrigation canals and utilization of river water by suitable erection of dams and barrages.

We are warned that we may not rest on our oars on the first flush of success. We are warned that India's population is rapidly increasing at a greater rate than in other countries. We have thus to gird up our loins to wage war on both fronts; we have to wrest from nature a bountiful harvest and we have to practise self-control and believe in family planning. Though our poor farmer is a fatalist, the new generation of researchers, workers and graduates of this institute should learn the lessons of the modern age. No difficulty is insurmountable to persistent and consistent endeavour. Modern man, by means of science, proposes to unlock all the secrets of nature and utilize them for the benefit and betterment of mankind. In other countries, there have been similar years of trial. They have persisted in their endeavours, they have surmounted their obstacles and they are now in the vanguard of human progress. We should raise high hopes and ambition in the younger generation of Indian scientists. They should continue their investigations and preach their new ideals among the masses. There will be smiling villages, contented agriculturists, flourishing cities connected by a network of good roads, and there will be the laughter of happiness and cheer in every Indian home. Graduates of the Institute, this is an ideal worth your lives' sacrifice.

The other Indian scientists will envy you, but they will have this consolation that they have struggled against odds and have lived to see the dawn of a new era and to hear the clamours of a young generation for a more vigorous life and better opportunities and freedom.

Address delivered at the Eighth Convocation of the Indian Agricultural Research Institute, New Delhi, on 1 March 1969.

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The Krishnan Memorial Lecture, 1971

Dear Director Verma, I see many familiar faces before me. I won't name them separately but to all of my friends I say, I feel highly honoured for being asked to come among you and to say something on this very memorable day. I had a great sensation of pride being associated with the National Physical Laboratory where, as Dr Verma has said, by dint of merit and hard work, bands of young people are grouped together with the main purpose of solving many problems — either the manufacture of highly sophisticated instruments which may ultimately be utilized for purposes of research, or finding out all about certain commercial processes to enable the manufacturers of our country to produce and thus help our industry, our people and, incidentally, our countrymen also by saving a lot of foreign exchange. We had been discussing these problems together and I remember, years ago when we began our life as students of science, that there were no such facilities offered to the young and meritorious aspirants those days. There had remained, besides the government colleges where alone to some extent facilities were provided for, the solitary Indian Association for the Cultivation of Science, founded by Dr Mahendralal Sircar, a doctor, a patriotic doctor who had thought of the future of the country, of the possibilities that are provided by native talent, and who had collected money and founded the Indian Association where people could be given news about the recent developments of science, and where people may be given facilities for work. Then there was no question of utilizing the resources of the laboratory for research purposes to solve problems which defied the industrialists. It was more or less about a century ago that people were thinking of how to make our countrymen modern in the sense that they would be able to look at science with

eyes not half as troubled with superstitions, so that they would able to look at major and natural phenomena and reason as to how things are brought about, how things develop, how things are made and so on.

Dr Sircar had himself a history. He had begun as a doctor trained in western allopathic methods, but very soon he convinced himself that it was possible to effect cures by homeopathic methods. And he was brave enough to adopt this method after he had carefully gone about and tested for himself the efficacies of these drugs. As a result, he ran the risk of being thrown out of the cadre of physicians, because he was a graduate himself of medical science of Calcutta University. The doctors were all up against him for having gone in favour of a method for which they didn't vouchsafe any reasonable efficacy. However, he continued. He was the Dean of Faculty of Medicine. He continued to be in that position and he valiantly collected private subscriptions and erected a modest building and laboratory. In this laboratory, originally, there were many short-course lectures.

Sir Asutosh himself, when he was a young man just out of the university , delivered a course of lectures in mathematics on his research work. When we were students, we found his name in a textbook as a person who had given a very ingenious and brilliant explanation of the equation of mass. He had ambitions. He wanted to do research work but then ultimately, since our country didn't then offer any facilities and no avenue of progress or advancement was available to any scientist, he turned to law. The legal profession was then highly honoured, and somehow or the other, science had curiously got wedded to law in our country, in the sense that not only had Sir Asutosh, a great lawyer as well as a great mathematician, brought about the revolution in the teaching of modern science, there were also two great lawyers, whose statues stand in the foreground of the College of Science, who by their magnificent donations made science possible in those days. Formerly, before our days, the students had mostly to rely on the government facilities being offered, with the solitary exception of two great names which have gone down in the history of science in our country — Sir Jagadischandra Bose and Prafullachandra Ray. They had gone abroad, studied science elsewhere and then returned to our country and continued to work with the native equipment — sharpened by western teaching — and find new directions in their work and in their jurisdiction.

Sir Jagadis's name had remained connected with the applications of science for unravelling the mysteries of plant life. Sir P C Ray's name had remained associated with the first efforts by Indians to manufacture drugs in our country and the first ingredients which go towards basic industry — manufacture of sulphuric acid, etc. But most of the scientists usually had their training and turned to law. However, there was a great change in the history of our country — there came the partition and the Swadeshi Movement. And then people at once thought how futile it was to go about learning things and at the same time remaining thoroughly dependent upon others, even for our own immediate necessities. There was a movement which wanted people to become completely independent so far as the daily necessities were concerned. There were students who had gone abroad to learn the technique of modern industry — to

study how things were spun, how mills were erected, how chemical industries were developed, etc. There were also others who had gone abroad to study the sciences. They had hopes and ambitions that they would return and set up for themselves some industry. However, this did not find favour with the rulers of the country then, and there were repressions, persecutions and revolt. There was started the University at Jadavpur where people wanted to give complete education through the mother tongue — where people came, endowed their wherewithals for the furtherance of education in our country. Well, there was suddenly at the same time a sort of revision of the curricula of the Calcutta University, and the training of science was given a more modern look. Students had to work in the laboratories to justify the award of their degrees. Even for the first degree in science you had to do two years' practical work. At that time the Indian Association laboratories offered facilities for students who were anxious to learn science but could not find enough facilities in the modest colleges that then existed in Calcutta. The Indian Association offered facilities for doing intermediate practical courses in physics, chemistry and botany, and students in large numbers thronged to the Association to do work. Well, I tell you all this simply as a sort of introduction, and to remind you that the facilities of the Indian Association were very modest indeed. And we had simply thought of it as a place where beginnings had been made but could not be very much improved because of the failing resources.

Mahendralal Sircar had died, and the laboratories still had very modest resources. When we were students for the Master's degree in Calcutta about 1913, there were new things, new endowments and there were new persons — eminent scientists who had been invited to come to Calcutta. Dr Ganesh Prasad had been appointed Rashbehari Ghose Professor. There were endowments given by the two eminent laywers, Sir Taraknath Palit and Sir Rashbehari Ghose — their names will remain written in letters of gold in our history. The first selections were made of the professors. Of course, curiously enough, when the appointments with the endowments were made and money had been given for the establishment of these professorships, there was a clause which produced a great deal of ire among our rulers, because it said that the person who would be appointed should be an Indian. According to the law, as Indians were defined, it was just barring for ever appointment of anybody other than Indians to these posts. Actually, then most of our science posts had gone to Europeans with the solitary exception of the two illustrious names which I just mentioned — Sir Jagadis Bose and Sir Prafullachandra Ray. All the other professors were mostly Europeans and Scotsmen who did not have very brilliant academic careers, but it was considered sufficient to be educated to teach the Indians a smattering of science. However, some money had already been spent for the teaching of science, and when we were students, we used to look through the almirahs in which were packed things at which we wondered — for example, an Echelon Grating or some other thing — and which were occasionally reverently put on the lecture table and again put back inside the showcase. The professors thought that if they were touched by people, they would be spoiled for ever and so much money wasted. Now, that was science! Well, you had to respect and have reverent superstition and look at these things — and at the same time remember that

these apparatus were manufactured not in India but elsewhere — to be shown for our edification, to be treasured and to be always objects of perpetual adulation. At that time, about 1913, when we were students and Sir P C Ray was doing some research on the amine nitrides and Sir J C Bose was doing his magnetic chrescograph, endowments had come, and Sir Asutosh had to seek out certain probable recipients of these endowment chairs.

He had noted a brilliant young man who had come down to Calcutta, not as a scientist but as an officer of the finance department, but who had chosen to pass all the time that he could spare from his official work in the laboratory. I am mentioning Sir C V Raman's early debut in the scientific world as a person, as a volunteer, who came to the Indian Association and utilized all the facilities which the Association had to offer. He chose his dwelling house very near so that he could come and enter the Association by the back door and continue his work in the long hours after sunset.

Sir Asutosh had observed this because he was then the President of the Indian Association. And when Raman was still comparatively unknown among the professors of the various colleges, he had predicted that this young man would go very far. I remember the day when I had gone to see Sir Asutosh and we were wondering about the new appointments, and he said — I know that he will go very far.

So we were all wondering. We occasionally went to the Indian Association, and he was already a name among the aspirants who wanted to do work. He was working like any other student of science — mornings and evenings — making his observations. He was studying what you call asymmetric diffraction and publishing his papers in the *Philosophical Magazine*. And then to the young aspirants who had just passed out of the Calcutta University, he had given problems to solve. He was a very keen observer, Sir C V Raman. For example, I remember there was a very old apparatus in the Indian Association which was set up to show the effects of elasticity. There were two balls which were pulled apart and then allowed to come down and to have a collision and then rebound and jump up to a certain height, depending on how far it could retrace its position. Well, you could see that this one was more elastic than the other, and so on. That was a very old apparatus. Sir C V Raman had taken two of these elastic balls together, knocked one against the other and curiously enough, had observed that it sounded very different if you heard it from one end rather than the broad-side position. It went very far on one side and sounded more metallic than the others, and he was curious. Well, things were all set and balls knocked about in that apparatus, but nobody had ever thought that there was this asymmetry in the production and propagation of sound when two balls collided. But he was curious, and he had observed this thing. And, of course, he was not a master mathematician like Ganesh Prasad, so he asked Ganesh Prasad. They were friends then. Usually before people become great, they remain friendly, and it is only afterwards that they quarrel. So C V Raman was still friendly with Ganesh Prasad and consulted him . He shook his head very wisely and then gave it as a problem to one of the young men. Result — Sudhanshu Banerjee got his doctorate.

Well, he solved the problem. He explained how it had to be explained by regarding these balls as sources of short waves, and he analysed the short waves being reflected one from the other, and then by curious combination getting together, you see, and producing a sort of double source, just like a magnet — you have these magnetic lines proceeding more rapidly, having greater force, in one direction than on the other. He had deduced by means of analysis, of course, as to why these short waves were very much stronger in one direction than the other. I just tell you this to illustrate the keen obvservation power of Sir C V Raman— how out of very modest things he could find certain things which needed explanation, and how he could lead on to very abstruse calculations and thus furnish a sort of enduring result for science. There were such things which I remember as, for example, the Wolfe note he used to show from the violin; how the maintenance of vibration should be very difficult when certain critical vibrations occurred; the colour of thin films, and so on. And for every such little thing which one could observe, he found something out of the argument and gave them as problems to be solved to young men and they were very glad. They were grateful because at last something was given to them to whet their own innate intelligence and to apply the newly acquired scientific methods which they had learnt at the feet of their teachers. But the teachers had not been able to offer them any problems, and now problems were found for them. And that was how people whom we had admired in our young days — like Mitra or Phani Ghosh — were given problems, and they began their life as investigators and got their inspiration from Sir C V Raman. They didn't remain tagged on to him but developed their own capacity for work. Sisir Mitra became one of the pioneers of the study of the ionosphere. And many students who will remember his name with respect and had their first lessons in wireless, in microwaves, were given prizes by the National Physical Laboratory. He had begun the studies of these waves, and he was finally made a Fellow of the Royal Society. Phani Ghosh had developed his powers of organization — he had started the Department of Applied Physics which has grown to be of respectable dimensions now. Every year they are training students who are absorbed by the industries, by government departments, etc. Sir C V Raman continued and observed certain things about which he spoke himself — about the blue colour of the seas which he thought was not reflection of the blue of the heavens — and he went on and ultimately discovered the Raman Effect. And with this discovery was associated a comparatively young man who was a very quiet sort of person. We found out that he was very much younger than us and was an ardent student. When Professor Sommerfeld came and lectured on the new developments of physics, he took down notes which the professor found suitable and valuable for the publication of a brochure containing these lectures. He continued his work in the modest Indian Association and Sir C V Raman continued to work on the problem of the mysterious colour and had guessed that there was not only scattering of light, there was also a change of wavelengths.

He had put people on that problem. I suppose there were many illustrious people who had worked on it, but the problem could not be solved. Ultimately he had found that there was actually a change of wavelength. But somehow or the other, he was not

clear as to what was happening. The first paper which he published had talked vaguely about the change and something like the Compton effect — the Compton effect had recently been discovered and Compton had reported about the change of wavelength of X-rays. It was only by dint of very careful work at the Indian Association — with very modest apparatus and with an ordinary spectrograph in which exposures took over 100 hours — that the actual discovery of the Raman Effect could be clinched to a successful issue. Krishnan had worked assiduously and without sparing himself, and as a result he had almost blinded himself by this continuous work and the exposure to the ultraviolet rays. However, that brought fame to India. They were so energetic, and organizationally so able was Sir C V Raman that as soon as he recognized that there was something very new, he put into the problem all the resources that could possibly be utilized and pressed into service and had the problem searched out. And Krishnan had remained associated with him with this work.

Of course, some people thought that it should be called the Raman-Krishnan Effect. But Raman himself would have been very angry if anybody had said it was the Raman-Krishnan Effect. The Germans had thought that Smekal had already thought of the possibility of a similar phenomenon and had talked about it, though, of course, he had done no experiments himself. The first work of Raman was not with atoms but with molecules, and yet the Germans continued to talk about it as the Raman-Smekal effect, making Sir C V Raman very angry. So anybody could talk about it other than as the Raman Effect, though there were the Russians who talked about Landsberg and Mandelstam as the first to notice this effect. They put forward, I suppose, some sort of claim that it was observed in the solids. But, however, so energetic were Raman and his pupils that they convinced the world that here was a novel effect — an effect which neither Stokes nor Rayleigh had thought of — and there was proof positive. Hundreds of experiments were done, all with very modest apparatus at the Indian Association — either the old superannuated apparatus which were treasured there or with some of the new apparatus which were bought for postgraduate work at the College of Science. Sir C V Raman had proved another thing — that it was possible by modest means — if only one really had a sense of the new and the mysterious and were not daunted and went on solving problems — to solve them with even ordinary apparatus. One need not wait for all the apparatus to come from abroad for research purposes; if one was anxious, one could do it. So Raman and Krishnan worked together. Then after some time, when there was the necessity at the newly instituted University at Dhaka to make appointment for a reader of physics, Krishnan was taken in and Raman came and recommended Krishnan as one of the brainiest of his pupils.

Krishnan came. He thought for a few months about continuing the Raman Effect experiments there, but then gave up. And then he took to a new problem where he made his mark, where his pupils have developed methods — the method of making asymmetric magnetic measurements and crystals. One of his pupils talked about it yesterday, and you should remember that though they have now developed and got new things — imported liquefiers etc. from abroad with which they intend to go down to the temperature of liquid helium — the first measurements were made with apparatus with the

coarse fibre which were drawn in the laboratory, measurements done with ordinary instruments and so on. That demonstrated once more that if you have problems and if you really want to solve them, there will be methods by means of which you could really make a sort of brilliant beginning. I don't mean to, I don't want to discourage the government from giving grants to science saying that it is not necessary. But what I mean to say is, even before such grants are given, ample facilities already exist in our country to prove the capacity of the various people, and there is no immediate necessity of getting new things imported from abroad. Nowadays the question of getting apparatus from abroad has become rather of second grade importance because of the development of our industries; because of a large number of trained technicians who are now available in our country. We remember when we were students, the professors used to hold on to their mechanics as some of the most precious possessions. For example, there was a mechanic with Sir J C Bose and he did very fine work — indeed to his amazement. Because Sir C V Raman had cast his jealous eyes on this mechanic or another, he grew furious. He was very angry and Sir C V Raman had to content himself with getting somebody who had voluntarily given up service because of the higher emoluments at the Indian Association which he had procured for him .But this Sir J C Bose wouldn't forget and wouldn't forgive.

I tell you these simple, curious little stories and anecdotes to remind you that about seventy/eighty years ago there was a paucity of even technicians in our country, and we used to quarrel about possessing a particularly gifted person or the other to be kept inside the laboratory rather than giving him away to a competitive person.Now there are a lot of technicians in our market and we are now finding it difficult to give suitable employment to these technicians. And if, therefore, there were persons in our country who had problems which they wished to solve — which would require facilities and gifts of a peculiar nature and ability — well, I think among the many thousands or many hundreds of thousands of people who have now been trained during all these years, there would not be any dearth of people who, if suitably used, organized and put to work, will solve problems which the country has to face.

I am telling you all this simply because while reviewing the early history of the introduction of science in our country, I remember great names who began their life very humbly, who had very humble resources at their disposal but who had indomitable patience, who could solve problems for themselves, who would enthuse young men by their example, and thereby place a goal for others and our young men — and place India, as you say, on the map of science. Well, gentlemen, I have talked a great deal. I talked only to remind you of the picture of K S Krishnan as he was then, a modest investigator in the laboratory of Dhaka, who loved his pupils, who lived like an Indian guru surrounded by his students and thereby produced enthusiasm — thereby not only had Krishnan progressed, solved problems, created fresh problems, he became one of the founders of this National Physical Laboratory where we have all assembled. Throughout the country there are now students — his own pupils — who had worked, who had found work, found problems, who had been at it, who had more or less been inspired by his example and thereby had produced for India a rich resource with men

and money — men in the sense that laboratories have sprung up all over the country where young men are trained and given suitable opportunities to solve problems. Only somehow or the other, there is something — I suppose it is a sort of evil star in our country — so that though we make very good beginnings, we finish rather a sort of a third or something.

Though we had produced these brilliant young men, at the same time we have not been able to produce or to make as much progress as we could really hope for ultimately in the field of research and industries. Why? I remember that when we were young, people used to point a finger towards Japan, and we had a song that though they were half savages, they could boast of independence — we had no independence, and so on. There was that celebrated song by Hemchandra. However, they had thought of western methods and used them resolutely by virtue of the few very far-seeing statesmen who had chosen upon themselves the task of getting scientific knowledge from abroad. The result — within a hundred years or so, Japan had become a name to be respected in the history of science. And even now, though Japan was humiliated in the last war and devastated and has been under the heels of the aggressor, Japan has been able to maintain its supremacy and its place in the first ranks of the scientists by means of its own efforts. Well, I sometimes wonder why it is that Indians who had very often been able to see very much farther than others — who had seen the sun rise very much earlier than many other people — had not been able to take advantage of the early seers and have not been able to continue this advantage and to maintain their pre-eminent position in the world.

Well, I leave these questions to history and to the new generation of young people who somehow or other feel that they have got nothing in common with the old fogeys. Especially in Bengal they talk about the generation gap. The gap is there. One cannot gainsay it, and it is a challenge. It is a perpetual challenge to the Indian genius as to how, even though the country is endowed with such natural resources and had such a brilliant history, it continues to remain third-rate in spite of so many resources and so much manpower. Well, gentlemen, I stop here with these questions to our young men.

Krishnan Memorial Lecture delivered at the National Physical Laboratory on 12 March 1971.

13

Education and the New Age

First of all, let me acknowledge with all respect that I feel honoured to be with you today. In this old age I feel deeply touched by your invitation. Times have changed. When we were young, we used to view our motherland in a particular way, and we had also developed certain ideals. We staked all that we had in life and dedicated ourselves completely to the fulfilment of those ideals. Those thoughts and ideas have undergone so much change today, that I cannot help feeling quite overwhelmed.

Respected Madam Vice-Chancellor has spoken of many things. She remarked that initially everything sprang from bliss — all these things. Indeed.

But we have different ways of looking for this bliss. Rabindranath could not perhaps find 'bliss' in these social and domestic surroundings, and so, leaving everything behind, he went to the open fields of Santiniketan. There he tried out a radical reform of the education system in an entirely new way, so that we can develop our future in harmony with the people of the world.

Think of our ancient sages who sacrificed everything for the sake of this bliss. They used to say that unless we could free ourselves from the bonds of attachment, happiness would not find its total fulfilment.

Hence, it is hard to say how much of that happiness can be realized in this city.

Rabindranath had once walked these very grounds. There was a time when he sat in these very rooms in the old world style of his forefathers. Again, through these very rooms came drifting new idioms — the first page wafted into the hands of Maharshi Devendranath — then emerged the great poet with whom we are trying to attune

ourselves today.

However, the notes of the new times will not quite ring familiar to old ears. For us the focus of Visva-Bharati or Rabindra Bharati is the culture of mankind. What has sustained us all these years has been the belief that if we could keep faith in people and move forward, then the daily problem of survival would cease to loom so large. It is not possible to say what lies at the root of human culture. The stomach has been placed at the root of modern culture. Yet, even to this old age we have retained the faith that maybe ultimately our inspiration flows from a little above, from where also will flow the spirit of culture — out of which we can hope to build the world anew.

A little above the stomach lies the heart — a lot is being said about it these days. There is a lot of talk about brotherhood of man and of a summons from the heart; yet if we look at the world, what we find is, to quote the poet, a 'delirium of hatred'.

Our students are trying today to discover their Bengali identity — but what constitutes this national identity ? Where is the authentic identity of the Bengali? So many historians, bards and musicians have described the Bengali identity in so many ways.

As children, it made us mad to read about the Bengali character in our English texts! A lot of what was written in history those days made us angry. We used to feel we must build something and prove our worth to these people. This was sheer miscalculation! But this is exactly the way we were led to think those days in our universities where despite all the lack of resources and the grumblings, we thought we would still be able to achieve something significant.

I feel hopeful after listening to the Vice-Chancellor. Perhaps the message of a new era will emerge from the kind of education that is being imparted here at Rabindra Bharati. And we must truly understand and ponder over words that will echo through and stir the minds of the youth as a result. Consequently, I too am rather curious to know how our students feel, what they think, how they are trying to shape themselves in the light of the new dawn.

I live close to a school. I am quite disappointed to see and live in the regular confusion that prevails there. It seems we have not been able to achieve much in the field of education, which is why there is so much unrest.

Anyway, before anybody says anything, I would admit that we have not been able to achieve much after independence, and hence there is so much confusion today. But there should have been confusion long ago. Since this anarchy has surfaced so soon after independence, we must then welcome this independence with the words : Long live Independence!

Even 700 years of subjugation have failed to open our eyes. Today, through independence we have arrived at a new arena, and in its light we can perceive a new direction. Many of the older generation will recall how they dedicated themselves fully to this cause, this independence. Yet, they had of course had a most conventional education. A new ideal has to be set for education, a new system introduced for those who are coming forward to be educated but find the present system irrelevant for a

livelihood. The system of education should be so designed today that on completion of studies, they will all be called to serve the nation. Of course, those who have already been called to serve will not want to move away from their special jobs once they settle down. And so there is the apprehension that perhaps a new movement will be needed to remove them. Anyway, one hears a lot of talk today about this education, and this culture and our country's misfortune — it all sounds like some distant music whose meaning is blurred. Today all the universities have been called upon to provide livelihood to the students. Swearing by the old ideals will now sound like a hoax and will no longer work among the students.

Ideals! People are no longer prepared to believe that ideals alone can open up the vistas of a real world beyond. Under the circumstances it is rather embarrassing for you to have invited someone like me to tell you what you ought to do, and consequently, I too am equally embarrassed. Yet, I can at least tell you that the old culture that has survived to this day as well as those you have invited to this special convocation, belong, like me, to the same age.

It is often said that the present generation cannot get along with people from an earlier generation because a generation gap has yawned between them. We can see today all the hopelessness and hunger and we know what to do. Only God knows what this is all about. I don't know what joy there is in it. Who can tell whether the dance-rhythms of Nataraja are audible in the destructive game of Maharudra?

I am an optimist. I hope we will be able to proceed despite all this. My appeal to those who have got their degrees today is — have something novel to say. If there is anything new to say, mere words will not suffice; you must also do something about it, and in so doing, also demonstrate that the effort cannot be merely destructive. So, to those of you who have become inspired today in Rabindra Bharati, those who have become eager to express yourselves, to you my only request is — merely glorifying the revolution will not do, you have to try and train yourselves up too. Hatred and violence alone cannot be the last word of revolution.

The world has seen many revolutions. Everywhere violence and hatred have been dominant. I would not like to believe that this hatred and violence are the last words of our civilization. All the time a nagging thought continues to trouble man — whatever I have done has not been right. Watching the present, people of my generation will say — our ideals allowed us to love each other. That undoubtedly showed us the right direction. Kill the person you cannot love — is that the new order of the day? If this kind of liquidation continues, our minds will become such that there will only be hatred in them and no room for liberality.

Anyway, I will not say more. I am glad I have been able to share my thoughts with you today. I hope what so many workers are trying to build up single-mindedly is successful, that your mutual affection grows, and may God bless you. May all other ventures in Bengal like this one also be successful.

Today my mind goes back specially to those days long ago when many like me had rushed to Dhaka University. The trumpet of destruction that blares there today bears

to us also a few tidings of the realization of some of the dreams we had dreamt then. Those of us who had gone then wanted men, irrespective of race or religion, to stand side by side with one another and declare their love for one another as brothers. This wretched person is your brother — you too must be moved by his sorrow — we had tried to accomplish this a lot and for a long time. It all comes back to me today. We have been fighting for one Bengal — the ideal we hear of everywhere today was the same that inspired us then. But those who were groping for it faced a lot of harassment — they had to run here and there.

I have also been to places where today's freedom fighters are locked in combat — of course in the special conditions that prevailed then. But enough, there is no point in talking about times past. All I want to say is that the seeds we sowed a long time ago are today bearing harvest — therein lies the pleasure of sowing.

Therefore it would perhaps be correct to say that one day the seeds of creation we keep sowing will bear fruit in spite of an apparent phase of destruction.

Bearing all these in mind, I appeal to you workers — there will be many obstacles in your way, but cling to your ideals. The students will say many things — they will write much on the walls, but everything is not visible — all is being recorded in the annals of time, and the seeds properly sown will certainly yield harvest in the fitness of time.

With this hope in my mind I await, until the end, the final day.

Address delivered in Bengali at the Convocation of the Rabindra Bharati University, on 10 May 1971, and published in the Rabindra Bharati Patrika, *vol. 9, no.4. Translated from the Bengali. Dr Rama Chaudhuri was then the Vice-Chancellor of the University. At a point in his speach, Bose refers to the two aspects of the god Shiva — as Nataraja or Creator, and Rudra as destroyer.*

14

At the Calcutta University Convocation 1973

I consider it a great privilege to be present as the Chief Guest at the Convocation of the University. I have reached my eighties, I am physically not very fit; the mind, too, has begun losing its agility. Under the circumstances I feel a little hesitant as to what I could possibly say. Among the learned audience I see lively faces sparkling with intelligence. They have finished their studies in science, history, philosophy, earned fame in research, and have thrown new light on several problems related to the living and non-living. They take my mind back to the past. I, too, received my passport to life at a similar Convocation of this university. Later, along the long course of life, I have had to face various adverse situations; I have gathered various experiences including the humiliation of defeat, the happiness of victory, the company of great men, and especially, the inexpressible satisfaction of being able to wipe off the ignominy of subjugation and sit in the rank of free nations. Perhaps the new travellers will draw some guidance from a veteran predecessor. With this expectation let me begin. . .

We were hardly out of school when the tide of nationalism swept over the country. In our adolescence we used to roam the streets singing songs of the *Rakhi* festival, trying to feel that we were all like brothers, all children of Mother India irrespective of caste and religion. We would have to wipe away the sorrows of our poor Mother India, break the shackles of foreign domination. We would have to save a great nation with an ancient heritage from the ruthless rule and exploitation of foreigners, to instil a feeling of the modern in their old-fashioned descendants, and eradicate illiteracy. That was how the youth of those days conceived the challenge of destiny. We heard great men, eloquent patriots, speak at street meetings, trying to inspire the youth of the

country. I feel an urge to tell you those stories.

After passing my Entrance Examination I got admitted to Presidency College. I had the good fortune of studying under eminent and memorable teachers. We heard Acharya Jagadischandra speak about his discovery of radio waves. My first initiation into chemistry was under Acharya Prafullachandra. I had the rare privilege of studying English with Professor Percival. I still have a sense of pride when I recount all this.

The first World War started before we had finished our six years of college. The national movement had brought upon itself all over the country ruthless persecution unleashed by the foreign exploiters, leading to open politics giving way to an era of revolutionary politics all over India, in Bengal, Bihar, Uttar Pradesh, Punjab, Maharashtra. Many noble souls sacrificed their lives, many of their colleagues spread to different countries. Along the long journey of my life I have lost many a friend. Many had staked everything to hold on to their ideals — and sought to achieve the impossible, to liberate the country from the foreign yoke. Now, after independence, it is only natural to remember them.

Sir Asutosh wanted to build a new temple of science within the University where the priests and the masters of the lore would all be Indians. In spite of the antagonism of the foreign rulers there was no dearth of money. The donors have become immortal. The grateful countrymen have erected their marble statues at the portals of the College. A postgraduate section was opened in the newly established College. Chemistry was given the first priority. After retiring from government service Prafullachandra took charge as the Palit Professor. A bachelor totally devoted to his work, he stayed in one of the rooms in the College building even in his leisure hours. Soon a group of young students gathered around him; they were his constant companions, and they looked after him all the time.

Despite the problems posed by the war, Sir Asutosh agreed to begin postgraduate classes in physics with a handful of young enthusiasts like us. Later, of course, C V Raman joined, quitting his government post. In the flush of fresh inspiration, the students plunged into research. Several achievements of the Science College entered the pages of history. From this city of Calcutta Professor Raman won his Nobel Prize. Meanwhile, his colleagues, too, won fame — their researches have been documented in books.

Under the leadership of Sir Asutosh an active centre of research grew up in the Calcutta University in chemistry, biology, history, philosophy and other disciplines. The rule of the country still rested with the foreigners. Yet, in response to the call of the future, young men started competing in international fields and despite adverse conditions could progress steadily, winning honours for the nation. Sir Asutosh had faith in their idealism, self-confidence, intelligence and competence. They also tried to live up to his faith in them. All these may sound irrelevant when the bright picture of the University seems overcast under the burden of several unfortunate circumstances. Yet, as an optimist, I am trying to suggest through these effusive references to the past that even today if we could instil idealism into our students through proper guidance,

then they will again be able to stake all and retrieve the lost glory of their Alma Mater.

That was half a century ago. Then destiny drove me to Dhaka. A commission was set up to carry out reforms at the Calcutta University. It consisted of many learned and experienced men from home and abroad. After reviewing many evidences a plan was chalked out for extensive dissemination of higher education in the country. Based on it, new universities came up in many provinces. But Calcutta was not ready to comply with it. A new residential university was set up in Dhaka in accordance with the new recommendations. Some of us from the University of Calcutta received invitations to implement the new system there.

Meanwhile the political atmosphere within the country had undergone considerable change. Gandhiji had appeared on the scene and called for non-cooperation. Some of the talented students left research to take to the spinning wheel and reorganize the country. The wave was felt within the universities too, but it did not lead to any permanent change.

In the meantime seeds of division had been sown by the foreign ruler, dividing the Hindus and Muslims. This had an impact on the political leaders of the country. In the days of the Swadeshi, Hindus, Muslims, Christians were all children of Mother India. The basic principle of the struggle was to strengthen the bond of unity and stand up against the foreign ruler. Later came Khilafat, the dream of a religious state for the Muslims. The rulers cleverly used it as the main instrument of their divisive policy.

The twenty-five years I spent in Dhaka were varied and eventful. That the charisma of a sincere teacher was no match for that of political heroes was proved time and again. Yet I had to keep performing my duty as a teacher in that difficult situation, without paying any heed to caste or creed. But watching the developments in Bangladesh now I feel that perhaps my twenty-five years of teaching have not been in vain. Bangladesh has emerged from the dead East Pakistan. I have heard the hopeful news that they will have a secular administration. But riots and acts of violence are also going on. It is difficult to guess before some time goes by what the future of conflict-ridden Bangladesh is going to be.

I returned to Calcutta after twenty-five years. The Second World War was about to end. The Cripps Mission had gone back. Threats of direct action were in the air. Within a year violence broke out. At last came the independence the Bengalis had dreamt of. In the process their state was also split into two. Millions of families were uprooted in Bengal and Punjab, millions lost all their belongings.

The Bengalis did not hesitate to sacrifice a great deal for freedom. Perhaps some among my audience here, hearing of the losses, will remember their homes in East Bengal with silent grief. Yet I feel it would be good to take stock after twenty-five long years. It might even help in future planning. Some might say that when even nationalism has to be sung on a low key against love for universal man, taking stock of the Bengali's fortunes sounds rather old-fashioned. Perhaps some of my friends would remember a thesis submitted by a scholar who was awarded a doctoral degree and was much appreciated by a university for upholding the view that 'in the old days there was

no concept of India as a single nation in the minds of the natives — it grew only with the coming of the British.' At the age of eighty I admit that I am old-fashioned enough to hold the Bengalis so dear and have my mind filled with their joys and sorrows.

Twenty-five years have passed since independence, and Bengalis have lost a lot. They were known for their scholarship abroad. Today there is a feeling of antagonism against them in other provinces. What are we going to do about it? It makes us think again of the greater losses we have suffered compared to other nations. In our time we were drawn towards science in the hope that it would bring prosperity to the country, and increase business. We used to think of such things. We thought P C Ray had done the right thing by establishing Bengal Chemicals. But the industries set up during the Swadeshi era have either closed down or are shifting to other states. The number of truly Bengali organizations of which we can be proud, has dwindled. We are lagging behind other states in the eradication of illiteracy and in industrialization.

How Bengalis can prosper in the changed circumstances should be a matter of concern and responsibility for all the youth of the country. I will now end my talk with an appeal to the youth to try and restore the Bengalis to their old position of eminence.

Convocation address delivered at Calcutta University in Bengali on 16 June 1973. Published in S N Bose, Rachanā Sankalan. *Translated from Bengali.*

15

Address at Seminar on Scientific Contributions of S N Bose

Professor Kapur, I should be grateful if you allow me to speak seated.

Friends, fifty years ago when we were young, we had the great idea that we would have to prove to the world that Indian intelligence was no less than anybody else's, and therefore we were anxious to show our own intelligence, our originality and to say something which people would accept — that was a great thing in those days, because as you know, this institution was started in the teeth of the objection of our rulers. The first clause that the Professors should be only Indians had set their backs up and they did not really help the Calcutta University to get on as otherwise they would have done. However, we, the Indians, felt that we had our own things to do in this particular matter — we had to work ourselves, we had to create enthusiasm among our people. And fifty years ago we had started the ball rolling which caught people's imagination — I feel really grateful. At the same time our problems have become more complex with the getting of our independence — that is to say, we are not only to show ourselves on paper that we are intelligent people but we are to show in our work, in our hearth and home, in our laboratories and workshops, that we too could rise to the occasion, utilize our own natural resources and build up a country strong, vast and progressive. Of course, a twenty-five years' span of time is really very little compared with hundreds of years of history during which we have left things moving of themselves and allowed ourselves to roll back into ignorance and so on. These things are to be removed. We are very curious people — as somebody has said, we live in this torrid zone and we get easily excited but very easily depressed at the same time. However, I think the cold waves

generally coming across the frontier will enable us to remain to be true to ourselves. So, in spite of occasional difficulties, these will help keep our spirit, and at the same time among our young people keep alive the hope that we would be able to remove all our past shackles and difficulties. For example, one thing in this caste-ridden country people quarrel about is how they were born and so on. These things still persist even though Gandhiji had preached the removal of these things. But still we hear of people being persecuted, being condemned, being burnt to death etc. All these things still persist among our people. I think it is not simply the question of solving problems but the question of really changing the way of looking at things that really faces us. And therefore, it is not simply our intelligentsia but also our politicians we have to look forward to. They should be able to take proper guidance from the lessons of history — they should see that it is only by the removal of restrictions that we would be able really to bring out from the Indian spirit the best that can be produced. At one time when there was such a call, people were not wanting who despite difficulties came forward to regain self-respect. When I remember my young friends who had all grown up together — like Saha, Dr Sen and other young people — Mukherjees and Ghoshes — it was really something very surprising and at the same time very very stimulating for us. If such things could be really possible in a small span of time, why could it not be repeated? There is no reason why now that there are greater facilities in our own country, they will not only remember something which was done but do something which people will remember ever afterwards. I hope this occasion when you have started the ball rolling will bring out not only the innermost spirit among our people but also the spirit of cooperation which we feel we are really losing at the present moment. Because, after all in order that we could make the best use of our manpower and our vast country and our large sea line, so to say, we must have to feel like one nation. And everywhere in the country, in every village we should have to feel that we are now going a step forward. Now that there are people who are thinking not only of solving our problems of calculus but also problems of agriculture and other factors, now that the fifth plan is being mooted, I hope in this fifth plan all the difficulties which we had experienced in the last twenty-five years will be remembered and removed so that in another five years we will be looking forward to a country which will make vast progress, which will recognize its own difficulties and which will find strength to overcome them within a short period. Gentlemen, I feel at the age of eighty the weight of the years but I feel very much enthusiastic when I see young faces — when I see Bhaduri, Bose, Dasgupta, Bagchi — people who had been with me when I was really much older than they. We could help ourselves at a time when things were difficult. But now since things are going on getting improved, since now there are various institutions and various universities, there are new factories and new collaborations etc. all over the country, we hope that in the next five years — or perhaps before the end of this Bose year function — you will be able to look forward — to say we have really done something which exceeds very much more the old stage in statistics; that here is something really the country will have got in agriculture, in machineries and in chemistry, in our production. I see here Dr Mukherjee who is now in the Agriculture

Commission — he is now away from the University; I see Mr Basu who is in the Indian Association — the Indian Association has gone a great way further since the time of Sir C V Raman; now the laboratories may be built — and everybody else.

And, of course, Dr Dutta who is still at the helm of things — he is doing what is to be done for the old Calcutta Mathematical Society. I hope that the Calcutta Mathematical Society will still continue not only to be really a sole property of Calcutta mathematicians but really something which the whole of India can regard as their society and the place where they can publish their original contributions. In a similar manner, all our different institutions which are connected with our universities of Bengal should really continue not only to guide our young men but also show a sort of beacon light to all the other universities of India. In India what we really need is really a sort of total abolition of all the racial spirits — the spirits of dissension that still trouble us occasionally to such an extent that we old people really get lost, and we cannot guess as to where our young people are being led on. But I hope, these very few people who are still at the helm, who in spite of all difficulties still really believe in the high ideals in India, still think that India has something to give to the World — the spirit of cooperation and love of humanity — we still believe that in spite of all these dissensions, with all these difficulties, all our fights, perhaps we will able to solve our problems in a sort of bilateral fashion instead of asking people to take to arms — well, to solve our problems ourselves and at the same time to solve the problem of oil which really faces us. At one time we thought that perhaps we could from our splendid remains of coal get something done. But then it looked too expensive as compared with what we could get easily from West Asia, and we left it at that particular stage. But I think now people should really think how to get together, how to utilize, as some people have already done in other countries, the enormous power that is developed by the waves. They tried to see that the sea waves could be utilized for generation of power. I think there should be things done not only to solve our problems for the time to come but also, at any rate, to show that to the World so that science grows. I think it will be possible for us — keeping the right spirit of cooperation among humanity — to keep this humanity living in this civilization progressively in spite of various difficulties that loom forward, in spite of the vanishing reserves in resources. And gentlemen, I think, it is now time that we stop.

I thank you very much from the bottom of my heart for having been all here together so as to give an old man at this age a sort of great satisfaction that life has not really been in vain.

Speech delivered at inaugural session of Seminar on the Scientific Contributions of Professor S N Bose, held at the Calcutta Mathematical Society on 29 December 1973. Published in the Proceedings *of the Seminar, pp. 15-19.*

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Contributions to Parliamentary Debates

I DEBATE ON THE UNIVERSITY GRANTS COMMISSION BILL IN THE RAJYA SABHA
21 March 1955

Mr Deputy Chairman, I have listened with great interest to the speeches made on this Bill and I rise to welcome this measure, the first of its kind which the Central Government had taken to help the Universities. I think at this stage it is necessary to very carefully consider the question in all its aspects, because we are now going to take a new step, that is to say, help the universities in carrying out their duties. Much has been said about the present low standard attained by our students in the universities and some examples have been adduced to support this contention. I venture to say that one has to separate carefully the task of primary and secondary education from the needs of the universities. The criticisms that have been directed here against the universities' standards may more properly be urged against the system that prevails in our country of imparting primary and secondary education. At the present moment the primary and secondary schools are directly under the control of the State, of the various States, and I think the States should take necessary and proper steps to improve the standard of primary and secondary education in our country. We have to remember that the universities everywhere and also in this country serve a two-fold function. Of course, the way in which they may fulfil it depends to a large extent upon the material that they are supplied with, in the form of entrants to these universities. The students come to the universities after they have gone through a course of study, at a certain age, and the universities take them afterwards under their care for a period

of about six years or perhaps a little more. Therefore, the result that we achieve in these universities will depend not only upon the standard of teaching that is imparted in the universities, but also upon the standard that has been attained by these entrants in the pre-universities stage. This is a point which we have to remember. I welcome this Bill mainly for this reason that in clause 12, there are the two sub-clauses (*a*) and (*b*) which say that this Commission will inquire into the financial needs of universities; and allocate and disburse funds for any general or specified purpose. I think it is practically the most important thing that has to be considered at the present moment; that our universities have not at their disposal funds sufficient to provide for a healthy growth in all directions.

So far as maintaining the standards of universities is concerned, or about the reform and the improvement of university education, I would venture to say this much, that this country has now got about thirty-one universities and in each of these universities there are efficient groups of teachers who take care of education. There are also different faculties and Boards of Studies in these universities and I think they are the best persons in whose care we can entrust the cause of education. In all the universities at the present moment there exist not only Boards of Studies but also Academic Councils and Senates where we have not only the officers of the university but also distinguished persons who are educationists or who have got high distinctions in various spheres. There are also at the same time representative leaders of public opinion in many of the controlling authorities of these universities. Therefore, the tasks of maintaining the ideals of University Education, I think, are in safe hands. I think that the Central Ministry of Education also is largely convinced that this particular task is being done by the universities properly. The universities have to serve a two-fold purpose. First, they will have to train men who will take up the affairs of the State. They will also be training persons who will be training the future generation of students. They have also to be at the same time centres of creative effort in our country. Now, all these attempted things are in all universities. In our country the ancient practice had been that the torch of learning was kept lit by the teacher. I think, even when we have this modern paraphernalia, we will have ultimately to go to the teacher to have the inspiration and the necessary guidance in all matters concerning education. I do not think that the problem of education is a problem of simply setting up of standards. It is not merely by saying that the boys should attain such and such a standard that you will be able to achieve your end, and solve the problem. The most important agents here are the people who serve you in the colleges, in the universities, in the laboratories, and in the different research institutes. Ultimately, they are the persons — who have been working there all the time — who have got the experience and who will be able to tell you exactly what is needed, if you want to improve the standards of education in our land. I feel that it will be a terrible task for any Commission, if you simply say that they will have to recommend to the universities the measures necessary for the reform and the improvement of university education. I do not think, Sir, that you will be able to find in this land persons who will be ready to take upon themselves this terrible responsibility of guiding the course of education

through the various mazes of subjects which the different universities now teach. Therefore, Sir, I think that the aim of this Bill should be a modest one. It is only necessary to co-ordinate the standards of teaching and examinations. I think, at the present moment, there is an inter-university Board, where we have discussions about these subjects, but these discussions have been mostly ineffective. The setting up of this Commission is thus very welcome. Very often, the universities themselves have felt the need for reforms, for introducing modern methods to attain a higher standard of efficiency, but they have not been able to do much in this direction for want of funds. I am not going to repeat, Sir, the various figures that have been quoted by the various speakers who have pointed out the paucity of the grants that we have been allotting up to the present moment to the universities, and to the cause of education generally. I think, therefore, that we should ask the Select Committee, which will go into the provisions of this Bill, to see that the task of this Commission is circumscribed and limited to mainly solving the question of co-ordination and determination of standards of teaching in the different universities, and also to the question of studying carefully and sympathetically the demands of the various universities about grants, giving due consideration to the developments in those particular universities. I, therefore, feel, Sir, that sub-clause (*c*) of clause 12 of this Bill should be omitted, or at any rate, should be re-worded as follows :

> recommend to any university the measures necessary for the maintenance of standards of teaching and examination.

That is all that is necessary. I feel, Sir, that any step that you are going to take about improvement of education will bear fruit only when you are able to inspire a spirit of cooperation among all the teachers in the different universities of India.

It has, of course, been suggested that this particular Commission would be able to solve the problem of education in this country. I may, however, point out that in this task we will ultimately have to depend upon the responsible people that are in charge of education inside the universities. I also know, Sir, that every teacher in our country is alive to his responsibility, and the corporate bodies that we have set up in our land are also serious about their tasks. I do not feel, Sir, that the standard of our universities has gone down to a very ludicrous level. I do not feel so, because I remember the long list of persons that have been turned out by these universities who had served the national cause, and who have been and who still continue to be the leaders of thought in our country. We also remember, Sir, that in many of these universities, there are young people and workers who have achieved fame and distinction, who have contributed substantially to the cause of learning, and who have made the name of India respected in different lands. Therefore, Sir, you need not have any misgivings about the way in which work is being done in our universities. All that is wanted is that you should be generous, and you should set up an efficient machinery which should be able to get in touch with the people who work at different centres, and should find out what they need. And you should be able to supply those needs in the form of grants, which may either be given out of the Consolidated Fund of the Government or out of the fund

which is set apart for this purpose, and which would be administered by the Commission.

Secondly, Sir, about the composition of the Commission, I feel you may have some representatives from the Government services in order just to tell the Commission about the particular needs — of the public services — or of your army, or of your defence etc. But still, Sir, the number should not be such as to overweigh other considerations in this particular field. I think the best advisers can be only those persons who have been working for the cause of education, or even our Vice-Chancellors. I am not sure, Sir, whether a very large number of Members of this Commission is really needed for that particular purpose — to lay down the rules in a non-interfering spirit, etc. I am afraid, Sir, that some of the clauses that have been incorporated in this Bill may be read in a way which may suggest that the Central Government will be interfering with the normal working of these universities. I think this question should be gone into by the Select Committee and the provisions amended to remove this impression. About the terms and conditions of appointment I suggest that all appointments should be at least for a period of six years. You cannot really hope for any results in a lesser number of years, and any member of the Commission cannot have that particular sense of responsibility, if you say that he may have to go out after a year or two. From the way things move in our country we may presume that at least a year will elapse before the members of the Commission are able to understand the problems and the needs of the different universities in India. And if you ask them to go out after two or three years, I do not think that they will feel themselves responsible enough, and the whole burden of working out the aims of this Bill will shift upon the perpetual members, who will be Government servants, or perhaps only the Chairman.

Therefore, I suggest this as an alternative : Let the number of the members be limited to about five as originally suggested by the University Commission, but let them be responsible people and let them be entrusted with the task of looking to the needs of the different universities and let this work be left with them for a certain number of years. We have been talking about planning and it has become customary to talk about a term of five years; why not think of the University Commission enquiring into the needs of the universities in the same way and allot a period of five or six years for its work? Choose your commissioners and entrust them with this particular task. Let them be sympathetic persons who understand these problems, who will go to and consult the different universities, find out their needs and see what particular subjects can be fostered or nourished in any particular place. India is a vast country and it need not be that all the universities should grow up according to the same pattern. What is really wanted is that the standard should be kept more or less at a certain degree of efficiency so that the persons that we want, the workers that the State may require, have got the requisite amount of general education. At the same time you should allow the different universities to specialize in particular subjects for which the teachers in that particular corner of the country may be famous. Formerly, it was customary for the student to go to the teacher. Now also there should be some way found by means of which students from one part of the country may travel easily

to another part where they can find teachers who are famous for the particular subject in which they are interested. Such a custom prevails in Germany, and I think it would not be difficult for this country also to bring about a similar arrangement by which students from one part of our country may go over to another part and finish their education at a third university if necessary. I suggest this from a certain point of view. I am not only thinking of the efficient teachers who will be working at a certain corner of our country; but this custom if allowed to grow will bring about a healthy interchange of students among different parts of the country. We all want India to grow as one indivisible country. We want all parochial ideas to disappear. We want that the future Indian should feel that he is an Indian and nothing else, and such a thing can be brought about quickly in our country by suitable interchange of students. The future generation of Indians who will be able to travel about and see the country for themselves will be able to understand the aspirations of the different sections in our country. I think the University Commission should also consider the feasibility of introducing this practice in our universities and I request the Select Committee to take into their consideration this suggestion that I have made.

From Parliamentary Debates, Rajya Sabha, vol. 9, 1955, pp. 2598-2605

II DEBATE ON COMMITTEE TO ENQUIRE INTO CONDITIONS OF REFUGEES

24 February 1956

Sir, we have often heard it said that we Bengalis are parochial, are not able to solve our problems, whereas people from other parts of the country are able to tackle their problems successfully. I would simply remind this House that there was a time when Bengalis did not remain within Bengal but spread over the whole of India, at least over the whole of North India, but now, of course, the people who are outside Bengal, who are Bengalis, feel they are not wanted in those States. I simply remark that to say that Bengalis are parochial will not perhaps explain the difficulties in which the Bengalis find themselves at the present moment. What we really want is not a solution of our problems by asking Pakistan to mend its ways. I think it is not India's policy. In such an event, in other countries there will be rattling of arms, but here we believe and we are firm in principles of peace. What we really desire is that we should really think of this problem. I have heard for example on many occasions people in power and responsibility admitting frankly that most of the money that has been given for relief and rehabilitation has been wasted. That has been admitted but the point that I want the House really to consider is this : It is a problem of such complexity and difficulty that to say that a few Members of this House should be associated with this problem is not really asking for something which should not be granted. Suppose I ask : has the Government studied the question as to how these people could be employed, or what were their occupations when they were in Pakistan. I don't think the Government has at their disposal any statistics as regards the way in which these people were employed

when they were in Pakistan. You have not thought of that at all. When these people came, you were not prepared. In the beginning you were not prepared. You had not thought of tackling the problem in that manner but you have left things to drift themselves. After a long period of seven years of inaction when the original initiative that might have been inside these people themselves must have atrophied, now you have the picture of people who rely mainly on doles, who think that after all it is the Government's duty to maintain them at any cost. But it is not always the fault of these people who have been reduced to such a state because where formerly they were in their own countries, they were industrious people, working out their existence and having their own corners where they could work and busy themselves in gainful employment. We had never studied as to how these people could be employed and where they could be settled and how they could be fitted into the various problems, into the various things which are of national importance. For example, Eastern Bengal people are supposed to love water and I suppose, they would like to be settled in such a place where it will be more or less congenial with their surroundings. Now you think of rehabilitating them in the sub-Himalayan regions. These people are not noted to be climbers. They are good people for certain walks of life. I would request these points to be taken into consideration and we should not only think of spending money but we should also see that the manpower — the additional men that we have got — are also utilized for the best interests of this State. There is no question of being able to answer as to whether this continuous drifting across our frontier will ever cease or not. Well, at least, we must know that if the economic conditions deteriorate in the other part, there will always be this little drift and the drift will continue for some time. People on the other side — the Hindus — have got no future in the sense that though they may be now suffering to stay there, their future generation — the Hindus that may be living in East Pakistan — will have no possibility of any gainful employment. That is really obvious to every Bengali who had been in East Bengal. Even though there are people who continue to carry on in East Pakistan, they try to send their families across the border. While we ask you to solve the problem, while we ask you to let all right-thinking people to think of this problem because we don't think that this problem has been successfully tackled — the problem is of such magnitude that we have not really given thought to this problem at all while we ask you to not only to think on the ministerial level or the secretariat level or at the bureaucratic level but when we ask you to see that there should be some Members of Parliament who should be associated also, who should be allowed a chance of studying the problems, it really means this that we are just as anxious as anybody else to help the Government and try to sit down and see that there may be other possible ways of tackling this question. We don't say that this problem will continue to remain insoluble but we say that perhaps if it is rightly done, if the problem is tackled at the right moment, perhaps we would be able to valiantly find our way out of the insuperable difficulties which face us at the present moment.

Parliamentary Debates, Rajya Sabha, vol. 12, 1956, pp. 743-746

III DEBATE ON WORKING OF PREVENTIVE DETENTION ACT, 1950
31 March 1956

Mr Deputy Chairman, I do not belong to any political party, but unfortunately I had been a teacher in Bengal and my memory is a distressing one. The story of the Preventive Detention Act and the speeches that I have heard in support remind me of the old regime. I remember first when only eight leaders of this country were arrested—only eight, not 117—and that was to keep the law and order going and let the ship of State roll on smoothly. Well, I have heard that the ancient regime is at an end, but I suppose that the evil that it has done lives after it. I have heard people speaking in this House of Parliament—I do not know how many of them were under the spell of this law of detention. . .

[SHRI H P SAKSENA : Should we rise up to show you how many we are?]

No, Sir. I do not want it. I think it will depress me still further. What I wanted to say is this that I have heard the partisans of law and order before, during the British regime. I have heard of the eminent necessity which kept people without trial for months. I have seen how it vitiates life, how it destroys families, how it disturbs the peace by driving underground the whole movement, how one man kept in detention maintains people who feel the inequity of it all and how it grows and grows. On the other hand I saw also other things happen in this country. I have been in Calcutta on the day when we heard with great jubilation the declaration that we shall have the maintenance of law and order in our own hands. Calcutta was then passing through a spell of internecine war. People had gone through the streets, people had gone through what may be called grave risks : there were no incidents because this simple recognition of the rights of the people had changed the mentality of the people. There have been people of different races and different religions embracing one another. This may happen in India. This will happen in India if India does not forget the philosophy that it had stood for. I have felt that we very often repeat our adherence to our ancient philosophy. But is it really for export? Is it really for earning the foreign exchange that we have shown our adherence to the great teachings of Lord Buddha ? I do not know. But to say that simply because one of the great officials of the State has been detained for some hours not being able to reach a more salubrious climate than Delhi, he would be able to spill the blood of eight men and not feel it, is it so simple ? Do you feel that such things had happened in India ? And you still maintain that it is only in the name of law and order that you have done it ? Is it not your laziness—as one revered speaker has said—that it is normal laziness and short cut to the maintenance of law and order that has kept this iniquitous law on the Statute Book ? Is it not possible, armed with all the powers at your command and all the goodwill of the majority of the people, to maintain law and order ? Is it not possible for this great country and for the persons in whom the majority of the people have confidence to maintain law and order, to keep in check a few people when the majority is behind you ? It is really sad you have put in places persons who do not believe in the great task that India has taken upon herself. I believe that whenever we are playing a game which we have learnt from the Western

States, we are not really respecting ourselves, respecting our great traditions. Why should it not be possible to use this Parliament as a clearing house of the different kinds of opinion when things are discussed in great amity ? Why should it not be possible without throwing slanderous remarks against one another to discuss really grave things which affect the State ? I feel it is a very grave thing that you have within the brief span of seven years of your Republic shot down as many people as perhaps in the last fifty years of British rule.

[HON. MEMBERS : Shame.]

Sir, it is really a shame. I think not only inside Parliament but outside India every Indian hangs down his head in shame when he feels that in spite of the best traditions of our fathers, in spite of centuries of civilization, in spite of all the teachings we had from our people, we are playing like ordinary brats.

[AN HON. MEMBER : No.]

Well, none would be more glad if it were really proved, but I despair. I see before me streams of blood ; I see before me desolate homes ; I see before me people who have been detained without trial and I see before me people who are praising that such a method of detention is, however, the best thing that we can take recourse to under the circumstances. Sir, I feel that this great Parliament, this great country, the people who have great traditions, should have also belief in peace. It will not do merely to say that we are fortunate in having with us men of peace who have proclaimed peace, who have proclaimed the five laws of peace. That certainly becomes great and certainly becomes capable of carrying on the great traditions of this country. Let us, however, have belief in our traditions and let us have belief in our own capacity. I do not think in India at the present moment there exists any situation which cannot be tackled if the man is resolute enough, if he has got belief in the traditions of his forefathers, if he has got belief in the recent teaching that the Father of the Nation has given to this country.

I believe in non-violence and I do not believe that, if the person is non-violent, the aggressors will not melt down and will not be disarmed if their look is threatening. I have seen such things happening. I think you also have perhaps seen—perhaps it is only after you have come into Parliament that these things have vanished. These things have not happened only once here. But I believe in India they may happen often and often.

Therefore, I say : Have belief in yourself ; have belief in your own powers ; have belief in your aggressors also ; have belief in your Opposition who are also Indians and let there not be this blot on the name of India which has long over-run its due.

Parliamentary Debates, Rajya Sabha, vol. 13, 1956, pp. 4267-4270.

IV DEBATE ON MOTION OF THANKS ON THE PRESIDENT'S ADDRESS
19 March 1957

Sir, I wanted to say something connected with what my honourable friend Professor Kabir said about improving our foreign exchange position by exporting ores. He said that we still have tremendous reserves of iron ore and we could very easily afford to export some and then, of course, buy iron from outside; ore is thus really worked somewhere else and is given back to us as iron, increasing the price, I do not know how many fold. But what I would like to say is that there is just this aspect of the question which the economists and legislators should think about. Even supposing we have enormous reserves of ores, all these ores are not of the same uniform quality. Their qualities differ and naturally if you want to export your ores, it will be really those ores that are of the very best quality that will be required by others. Therefore, it will mean that if we want to get foreign exchange, we must agree to deplete ourselves of the best quality ores that are now available, and the best quality means that it is easily worked, without any very disagreeable properties and so on. So, if we at once increase our export of ore and we do not sit down at once to smelt our iron ore in our own country, and thus utilize the resources which nature has given us, it will mean that we will be left without much good quality ores at the end. Ultimately, when the workers are properly educated and when we have erected the steel plants, by that time, considerable amounts of good quality ores would have vanished from our country and we would be short of the good quality ores. Thus, export of ores is not a very good suggestion.

Similar is the case of coal. We hear that we have got tremendous reserves of coal. But all this coal is unfortunately of a very low quality. We had been merrily burning away all our good reserves with the result that when the need appears of getting good quality coal for the principal metallurgical processes, we know we are hard put to it to find such coal in the quantities that we want. So we have to utilize various expensive processes in order to conserve our slender reserves.

Let us not dispose of the good ores because, once we export them, we would not get them back. By these ores we will not only be able to improve the quality of our iron produced but we will also be able to progress towards increasing the prosperity of our country. In regard to this point, I am very happy to hear that the social dynamics are now such that we have just begun to move. For heaven's sake, don't put a spoke in the wheel of progress by exporting the valuable materials which an ordinary country, which is just beginning to industrialize, can easily use instead of going for difficult processes and use of poor materials at the end.

Parliamentary Debates, Rajya Sabha, 1957, pp. 72-73.

S N Bose : Miscellaneous Pieces : Science and Culture in India

1

A Scientist's Apologia

A few days ago the Science College celebrated the centenary of Rabindranath Tagore. Chief guest for the occasion was Shri Surajit Lahiri, then Vice-Chancellor of the University. What emerged from his analysis of Tagore's poetic temper and philosophical outlook would amount approximately to this — that the current emphasis in this country and elsewhere on science teaching and scientific research was inordinate, the ultimate outcome of which would not perhaps be beneficial to human society, that the impression we get of Tagore's ideals from his writings has no essential connection with the scientists' vision. Science was perhaps misdirecting us, displacing the desired ideal in favour of the immediately attractive, etc. As a result, the discussions that day were deflected to a different course altogether; and the scientists were called upon to answer the questions raised by the philosopher to the best of their abilities. As Chairman of the session and representative of the scientific community, I had to say a lot of things that day. One cannot remember all of it at this distance of time. Recently my students discovered that a taping machine had been duly installed that day to record the proceedings accurately. So many months later they made an attempt to transcribe the discussions from the magnetic tape in order to satisfy their curiosity. But they were not entirely successful. Regrettably, the Vice-Chancellor's speech could not be deciphered. My own speech too was only partially recovered. Whatever they could make out they recorded and sent to me with a request — if I could spruce up with the help of the incomplete note the plea I had tried to make on behalf of the scientific community. This is what resulted from my attempt.

Our philosopher Vice-Chancellor had raised a very complex question indeed. We of

course had not anticipated that such issues could at all rise from the discussion we had envisaged. Nevertheless I feel that scientists ought to say something about this because the question that has occurred in the Vice-Chancellor's mind could very well occur to many of our leaders, especially since many in this country are born philosophers. Their attitude to scientists is somewhat like this : scientists started the railways, so we can travel great distances with ease; they've blackened and soiled themselves to break coal and wrest some energy from Nature; this may have changed the external features of our civilization. But through his absorption in such matters the scientist has lost the right philosophical outlook. He has perhaps never been concerned with the creator's mind that lies behind this creation, about the soul of man, about God, and the many schools of philosophy that have arisen here and abroad from such contemplation do not interest him in the least.

We scientists might admit that we do not understand such things, and therefore evade such questions, or we might think that the creator alone would know the significance of his creation and its essential nature, maybe we find no voice of reassurance from all those philosophical schools that have emerged. Nor do we know the philosophical beliefs of our philosopher-guest. But we cannot help being curious. Does he for instance believe in God alone? Or does the Devil too sneak into his thoughts?

Man has been trying for long to fathom the mysteries of creation; scientists too have been trying. But can he with his limited capacity truly grasp its full power? The scientist cannot conceive of it, maybe the philosopher thinks even that to be possible. But to attain such a level, he must think that this very physical frame itself is Maya and that truly he himself is an inseparable part of that Great Power. Of course in our country this line of thought has been propagated in many forms ; perhaps even now some may do believe in it heart and soul. Once one comes to this realization we say about him, 'Now he is liberated'. The scientist wonders where has he gone then ! For he no longer has any links with our world!

We hear of the lives of many of our sages, who are supposed to have sacrificed themselves totally, even their very mind and soul. Is that how God would have it? To forward this question to Him however, we must enlist the philosopher's help since scientists are blind here, they cannot meet God. Humankind is gradually ascending upwards through a process of evolution. Civilization is on its way up. This the scientist does believe. What he does not believe is that man's vehicle of progress can advance on the mere fancy. Civilization must be founded on the knowledge acquired from Nature's store of power. That is why he is not prepared to cling on to philosophy alone. Otherwise, alternatively we might fall back on philosophy to imagine that we are all human, all alike, and we could live out our lives without envy and animus, but so far this has resulted in a futile mentality. This would be quite apparent if we survey thousands of years of history. The kind of transcendental philosophy our philosopher guest has referred to is not new to this country. It is very much native to India, or so we are proud to believe. The poet D L Roy once wrote, 'You can lambast us if you please, but do try and read the Gita if you can'. But here one is tempted to rejoin that even when this particular philosophical outlook was supposed fully prevalent, even then was there no

respite from enmity and strife. Of course those who were pure of heart managed to get around this fact quite charmingly. There is a story I would like to recall here. The Rishi Vashishtha always maintained that the Brahma was Truth, and that his world was an illusion. So the king set an elephant to chase him, and Vashishtha of course promptly took to his heels, his hair lock still unknotted. So his disciple the king asked him, 'How now, O Great Sage, where are you off to? This is nothing after all, just illusion.' However, even if one were to argue here today on similar lines on the essential nature of Truth and Falsehood, we could not dismiss as Maya (or illusion) the widespread poverty and ignorance that prevails so glaringly in the world today. At least as long as man lives on this earth, this ambivalence in his mind must remain. As long as man lives, he will continue to try to wipe these features out of human existence, and seek to build a society in which these accidental dangers do not loom large. For that we need knowledge, we need a large vision.

Today scientists can say that if the world were guided by philosophers alone (as India is supposed to have been guided once) perhaps even then animosities would not have been eliminated. Without scientific research, there may not have been the atom bomb, but the older weapons of death and destruction would still be in use, with effects no less cruel and lethal.

I don't think that through the ages men have showed any special concern for their fellow men as human beings. It is only in this twentieth century that we have started working towards that ideal. But of course what the Vice-Chancellor had to say is very true indeed. There is no future for us unless this concern for one's fellow men is felt by everyone. Scientists believe this too; but unfortunately, the men at the helm in every country are not men of science. Many may be religious, many nationalists. Many a leader believes that his own race was born into this world to fulfil a certain destiny; and his own responsibility is to pulverize the earth and ride it over with his particular notion of civilization. I know (but I am not sure if any of you do) that the man we all wished to put in the dock as the prime accused responsible for the recently concluded World War II (of course his suicide prevented us), that same dictator, Hitler, was in fact a deeply religious man. If we go into his life, we will find that he was a strict vegetarian, and so on and so forth. From all those outward signs that we take into account it would have seemed that whatever we received from this individual would truly benefit mankind. When the German people were in acute peril and sinking to the direst depths, he came forward as their saviour. And those who listened to him on the radio as I did during the war, could realize his wariness. 'If we lost this war, it will be a thousand years before the German nation can rise again,' he warned. Well, so much else fell to Germany's lot. And today one would merely like to say this much, that if this nation which had sunk so low has risen once again it has been in the strength of science. It is the scientists who have pulled her up again. What the German nation have to give us is not philosophy alone; the knowledge the people had acquired through every moment of daily work, was pressed into service for human good so that within a short time they could rebuild their war-shattered economy. The scientist firmly believes that religious practice alone can achieve very little. What man is according to

religious doctrine, what his relationship with his anima is, such philosophical questions and their study should be taken up in private. From such discourse man may draw inspiration for work, but once he begins to work, he must work with his whole mind in action ; he has to know the rope for a rope, and never mistake it for a snake. If we from a variety of notions choose to negate everything, there will still remain hundreds and thousands of people suffering from disease, depressed with sorrow, terrified forever of so many dangers, who do not know how to make the land yield four handfuls of rice instead of two. A little knowledge may be of help to them — knowledge which may not give them a complete notion of the Universal Creator, but this little knowledge has helped avert much trauma. This fact our Vice-Chancellor will possibly admit.

We did not wish to launch on this theme today. What we had intended was to show what place Rabindranath himself had given to science — in his writing, his life — not restricted to poetry alone — he had given many lectures at many places, written many books, where his attitude to science is made very clear; that is what this gathering was to discuss. We were not prepared for such deep philosophical questions. But let me say this much at least, that we cannot stop at the eternal disposition of the eastern mind — because he too must be aware that at least three or four hundred years ago St Francis of Assisi would say such things as that the water he considered sister, the wind his brother. That is how, throughout his life, he tried to offer mankind all that came into being within him, and thus draw them into kinship with himself. Just as this sense of universal brotherhood existed in this country, so in fact everywhere in the east as well as in the west. But nowhere do we see men prepared to embrace this philosophy as the truth and hold the whole nation in kinship. But one only hopes that the truth of this will dawn on people when the opposing camps both possess equally powerful weapons.

At times it seems that people are so interested in India only because of the availability of an oil line. But if today we were to discover ways and means to construct lethal weapons from plain water, then this brutal competition for forcible occupation of territories might diminish considerably. When the hydrogen bomb was constructed it had an explosive power two hundred thousand times greater than that of the atomic bomb. When a single atom bomb devastated Hiroshima, a scientist friend of mine said to me: 'God, at least now one could no longer say that people were trying to acquire some rare material from a particular place.' If we are clever enough we can develop the ultimate lethal weapons from any common material. Of course this is not exactly how the scientist's mind works. It would be the job of those who reign over people, all those leaders of men who are engaged through daily propaganda in turning truth to lies, and lies to truth, to build up to this idea, with hand on heart. For then no one nation would have a monopoly over lethal weaponry. If war breaks out both sides would have the power to break havoc on earth using such weaponry. Once an eminent astronomer in Calcutta had said that if you turned your eyes to the sky you would notice how a new bright star emerged all of a sudden, and disappeared again after a few days. He thought that this phenomenon ought to make men cautious. If we are not cautious, if we do not change our attitude, this earth may cease to exist. We may ourselves become such a

passing star. The creator alone knows if this star magic will happen. Who knows which way mankind is thinking these days?

But the only cause for hope lies in the fact that many small countries which had made nationalism their sole concern have now learnt to think in terms of all humanity. They are getting together time and again to discuss what duties they owe towards the future of mankind. If man is to survive and develop further we must all change our attitudes accordingly, the way that it has now become possible for America on the one side, and the Soviet Union on the other to think along the same lines. South Africa has not yet come round to this. The southern peoples have usually preferred to stand apart, yet many other countries would like to think together. In a way, none of this would have been possible without science. Which is why I still believe that when the scientist still proceeds with his work through all manner of peril and danger, he shows a certain strength of mind — he is truly a humanist. He will not think that his only duty is to his own group, he loves mankind. He places it at the centre of his daily concern.

Every true scientist who engages in research not merely for his self-satisfaction or egotism dreams all the time that he might discover some fundamental principle that will help us to build a monument to human progress. And again, the scientist who, test tube in hand, pursues the trail of a little known disease, he too tries to discover some means by which he may eradicate a deadly epidemic. That is the mark of a true scientist. Probably all scientists will agree with my evaluation.

Had Rabindranath been concerned solely with his own poetry, he would not have built Sriniketan, nor would he have bothered to go out to the countryside to encourage development of agriculture and cottage industries. He aspired to dedicate all of human knowledge first to the Creator of Life with all reverence and then to put this knowledge into the hands of mankind for them to put to use, with good sense.

Bose's own reworked text of his talk at the Tagore Centenary Celebrations at the Calcutta University Science College in 1961. Published first in Bengali in Shraddhānjali / Homage, Professor Satyendranath Bose Seventieth Birthday volume, *Professor S N Bose Seventieth Birthday Celebration Committee, Calcutta, 1 January 1964. Translated from Bengali by Madhuchhanda Karlekar.*

2

The Progress of Science in Ancient India

There is now a rush all over India to go to the West to study science. Factories are sprouting everywhere and our boys are going abroad so that they can return equipped to contribute to the country's progress. The government too is keen on hiring scholars from abroad to reform our education system so that we can acquire in every field an approach adequate to the present time. Under the circumstances it is only natural to feel curious about the progress of science in our country in the past.

Soon after the Battle of Plassey, the British gained supremacy in Bengal. The Portuguese, British, French, Dutch and Danish merchants had started coming to India by sea a century earlier. They used to buy fine muslin, silk and other things here at a cheap rate to sell them at a high price in Europe. Indian handicrafts were highly valued abroad. Obviously various technological applications were known in many parts of the country.

Unfortunately all that died out gradually after the coming of the British. At the time of the Sepoy Mutiny in 1857, the British had established their hold all over India. The princes of the Native States who had not yet been ousted accepted the overlordship of the British. They were mere puppets in the hands of the British envoys. However, in spite of this utter humiliation, there were streaks of a silver lining. Contact with the Western civilization shook the nation awake from centuries of deep slumber. The country's intellectuals gathered information from all over the world, and came to know about the remarkable gains in science, wealth, technology and business made by Europe. Anyone with a progressive mind turned towards the West for knowledge. There still persisted in this country a system of education rooted in Sanskrit and confined to the upper caste Hindus. The Muslims went to the madrasas. And, since Persian was the state language

during the Muslim period, it was taught and used by Hindus and Muslims alike.

People began to think that with all the time and energy put into it, this system failed to provide knowledge more suited to the times. Instead of grammar, religion, statistics, *nyaya* and astrology, they sought the dissemination of modern science — higher mathematics, chemistry, physics, linguistics and architecture. There was no choice in those days but to study these subjects in English. However, the predominant attitude remained conservative. So, after a lot of debate the suggestions of Raja Rammohun Roy and the forward-looking sections were accepted. The new universities that came up provided for Western education. Most people of the country had by then lost faith in the old traditions. The convention of recording history chronologically was unknown in this country. Besides, many old texts and manuscripts had been lost during periods of political unrest. The common people did not understand Sanskrit. The Brahmans alone used Sanskrit. The priests were the society's lawmakers. All their discussion centred on religion. Most people were illiterate — whatever moral lessons or general knowledge they acquired was through the oral tradition — the songs that constituted part of the religious rituals and ceremonies, the retelling of myths and folk theatre.

At this difficult hour, when we had descended to the very nadir of our culture, fortunately for us there came with the foreigners scholars who wanted to delve into the past of the human race. They learnt Sanskrit, Arabic and Persian from our scholars and rediscovered many new facts, even as they endeavoured to dig out India's lost history. And it was through their efforts that the Asiatic Society was established in Calcutta. They took information about the Sanskrit language and its literature to Europe. They unearthed old coins, inscriptions and copper plaques and other historical evidence in many places. A new vista of knowledge was opened up to Europe. Linguists discovered a close connection between Sanskrit and ancient Greek, Latin and other European languages. They discovered links among the early civilizations of Greece, Rome, Egypt, the Arab world and India. They set out to write a new history of ancient times on the basis of their findings. The information came to circulate in this country. Through western education the information reached India that our old literature, *puranas* and scriptures contain a wealth of facts, enough to excite the curiosity, respect and reverence of western scholars. The Indian mind turned once again to the old heritage. Some of the educated Indians followed the foreign perceptions in their entirety. They went through translations done by foreigners to rediscover their own culture and history. By the time our universities were set up, there had emerged nationalist consciousness among our thinkers. Those who had acquired higher education felt it their duty to write the history of their own civilization. The country's learned had to take this responsibility upon themselves. For foreigners are not always able to grasp the reality of our civilization. They have a different language, a different religion. It is hard for everyone to absorb the culture of an alien community. Hence there was considerable fear that the foreigner may not do justice to Indian culture.

There is deep reverence for the Greco-Roman civilization in the European mind. When they came to study the culture of this country, the Europeans thought that India's advances in several fields were a direct result of coming in contact with the Greeks. They

felt that the seeds planted by Alexander, and the relations that he had initiated may have profoundly influenced India's ancient culture.

Nationalism in the modern sense has entrenched itself quite securely in our minds from Bankim's time. In his *Krishna Charitra* and other essays Bankim tried to counter the foreigners' theories. We find Romesh Chandra Dutt coming up with an assessment of our literature. His history of civilization written in English continues to amaze us. His Bengali translation of the Rig Veda stands out as evidence of his profound regard for our ancient heritage. We are also reminded of Radhakanta Deb's *Shabdakalpadrum* and Shourindramohan Tagore's history of Indian music. Balgangadhar Tilak, writing about the first habitation of the ancient Hindus tried to establish on the basis of his studies of the Vedas and Jyotisha that the Aryans were originally settled in the polar regions of the North; some of their experiences from those times could be traced even now in Vedic literature.

All this however concerns literature, religion and history. But a search for our scientific heritage goes further back. Those days astrology and Hindu Mathematics used to be studied in Sanskrit. Ayurvedic medicine was kept alive by the Vaidya community. Doctors following the Charak-Sushruta line served to compensate for a major lack in the country by their diagnosis of and prescription for diseases. The achievements of the ancient Hindus in the field of mathematics were first revealed by Colebrooke. In 1817 he published an English version of Brahmagupta and Bhaskar's texts on algebra and arithmetic with an erudite introduction. Colebrooke sought to assess the level of excellence reached by the Hindus in the ancient period in mathematics and astrology. He had studied the original texts and annotations of Bhaskaracharya's *Līlāvatī* and *Vīja-gaṇita* and Brahmagupta's *Gaṇitādhyay* and *Kuṭṭakādhyay*. He proved that Bhaskar had written *Līlāvati* and *Siddhānta Shiromaṇi* around AD 1150 and it was from these texts that he gathered information about the earlier mathematicians.

Before Colebrooke, Dr William Hunter had spent some time with astrologers in Ujjain and concluded that Brahmagupta lived around the year AD 628. From the elaborate annotations that grew around the ancient texts he came to know of a famous mathematician before Brahmagupta by the name of Aryabhata. None of Aryabhata's original texts have been recovered, but later mathematicians have quoted him extensively in their works, in agreement or in disagreement; providing an impression of the theories of this great mathematician. Aryabhata seems to have been teaching that the earth goes round its axis every day. He had discovered the correct explanations of the lunar and solar eclipses and said that neither the moon nor the planets have any light of their own, actually they are lit up by the sun. Aryabhata calculated the diameter of the earth to be 1050 yojanas. On that basis, holding the ratio of circumference and diameter to be 22/7, he calculated the circumference of the earth to be 3300 yojanas. This comes pretty close to the actual figure, because if one yojana is measured as four crosh, then one yojana will be 1.9 mile. Accordingly, the earth's circumference will stand at 25080 miles.

Colebrooke has given a lot of information about Hindu algebra and geometry. He has shown that the Hindus knew the properties of surds. They even used negative numbers in their calculations. They had derived formulae of quadratic equation, sometimes they

solved even more complex equations. They had worked out some extremely complex solutions of problems, which were rediscovered by the Europeans in the middle of the eighteenth century. Later Indian scientists have studied these topics in detail. More than a hundred years after Colebrooke, Dr Bibhuti Dutta and Ayudhesh Narayan wrote a scholarly history of Indian mathematics, which contains thoroughly researched findings on many of the above-mentioned subjects.

After this it were the Indians who took up this study in the history of science. Acharya Prafullachandra Ray and Acharya Brajendranath Seal must be mentioned as pioneers in the field. Acharya Ray went abroad to acquire expertise in chemistry and returned home to dedicate himself to research and teaching. He worked for the rejuvenation of the chemical industry in the country. At the same time he studied ancient texts and manuscripts for information about the past.

Whatever spare time he got after teaching and research he spent on these ancient texts. He took the help of Sanskrit pundits to retrieve a lot of information from a close study of the early works of the Vedic corpus. He learned that the ancient Hindus knew the method of preparation of acid and alkali. They could purify iron, lead, copper, tin, mercury etc. They employed various purification techniques for the purpose. They were aware of highly advanced methods of preparing metal-ash. They had designed and produced the necessary apparatus for all these processes. The chemical inferences they drew from sublimation, distillation and partial distillation astonish us. Several texts describe the many changes that can occur in mercury as well as the preparation of various chemical compounds from the reaction of different metals with sulphur.

The History of Hindu Chemistry, vol.I, was published in 1902. The second volume was completed in 1908. After many years of labour Acharya Prafullachandra won for himself a place of honour among scholars of the world by providing information on an unknown chapter of Hindu scientific studies and practice. That many mathematical formulae reached the West from India via the Arab world had already been shown by Colebrooke. Acharya Ray established that many principles of chemical processes were discovered in India and carried to different parts of the world by the Muslims. With the translation of Charak and Sushruta, many Indian formulae of chemistry reached far and wide. Recently Professor Priyadaranjan Ray, one of P C Ray's able students, has published a revised and recast version of the *History of Hindu Chemistry*.

From a whole range of findings made in the past forty years, we now have a reliable notion of the extent of knowledge of the ancient Aryans. The age of Mohenjodaro dates back to 4,000 years ago. Excavations have revealed many artifacts belonging to that civilization. We have learnt that coloured glass was first produced in this country. We have also found many bronze and copper alloy articles which were used by other people, in other places. Later India used to produce pure iron and export it to other countries. The secret of making steel orginated here. Indians ought to feel proud that Indian steel was used for making the famous swords of Damascus and Toledo.

Now we come to the age of Swadeshi. I will conclude my article with a reference to Acharya Seal's research. He was a world-famous philosopher, born on 3 September, 1864. He was a versatile genius, who had mastered mathematics, science, several languages,

poetry and philosophy while still quite young. He helped Acharya Ray when he was writing his *History of Hindu Chemistry,* the last chapter of which contains an account of how the ancient Hindus produced different objects and their theories about the properties of things. The theory of evolution according to the Sankhya school, the speculations on the chemical and physical qualities of matter according to Vedanta, Mimamsa, Buddhism and Charak have been critically presented by Acharya Seal in this chapter. He depended mainly on *Vyasbhāshya*, *Charak Samhitā*, Yuddotkar's *Bartikā*, the Commentaries of Prashastapada and Varahamihir's *Vrihatsamhitā* in his account of these theories. How the world was created from void as a result of differences in the qualites of *satta*, *tamah* and *rajah*; how creation reached the macro-molecular state from the five elements; how material objects came to hold beauty, taste and smell owing to the fusion and break-up of macro-molecules; how the Hindu and Buddhist philosophers have sought to describe the location of sky-air-water-earth — Acharya Seal has addressed himself to all these things. We might see the first principles of modern atomic theory in his account. From the way Acharya Seal has described the attraction and repulsion of atoms, he has given us an early glimpse of chemical affinity. Later, after further research, Dr Seal published his important work, *The Positive Sciences of the Ancient Hindus,* in 1915. In the preface he said that he had gathered together in his work many facts for the use of historians of the different sciences. The Greeks and Hindus had laid the foundation of the laws of natural sciences now in use. The basic facts they discovered regarding the properties of matter are being applied in modern industrial technology. The scientific outlook of the Hindus and the logic they used for drawing conclusions deeply influenced both the West and the East, as will be understood if we think of China, Japan or the Saracen Empire. So it is important to make a comparative study of the Hindu and Greek systems and Dr Seal has been quite successful in doing this.

Acharya Seal sought to make a comparative study of different philosophies, in terms of all those fields of knowledge reflected in philosophy, viz. linguistics, physiology, geometry and mathematics. Just as we now have an idea of the Greek pursuit of knowledge, so it was Dr Seal's objective to introduce readers to the Hindus' mastery in these fundamental sciences. To present his arguments convincingly, he quotes from the original Sanskrit texts proving to his readers that there was nothing in his account of his own invention. This book will thus prove immensely useful for a comparative study of the Hindu shastras.

The book contains a lot of information — in chemistry, the chemical composition of different elements and compounds like mercury, bronze, tin and mica; and their analyses — and calculations of time and distance; information about the transitory motion of planetary bodies. Dr Seal says that the Hindus had developed some principles of differential calculus, such as $d(\text{Sin } \Theta)=\cos \Theta d\Theta$, long before Newton. Spottiswoode was persuaded to accept this theory.

He has also dealt with dye technology in this book, describing how the Hindus had invented the science of permanently dyeing silk and other materials with herbal products and alum and how highly valued Indian goods were abroad.

Then again he tells us how the Hindus had classified the plant and animal worlds.

Finally he draws on the medical texts to describe their conceptions of the human anatomy and the nervous system. From the tantras he draws their notions of the Ida, Pingala and Sushumna. He has also tried to explain their concepts of life and spirit, taking from Charbak, Sankhya and the Vedantas. Acharya Seal, in the course of his analysis of the Hindu conventions of argument and proof, refers to the Buddhists and the Vedantists and their approaches. The methods the Hindus adopted to reach the truth, how scholars with different views argued with each other, Dr Seal has analysed in his lucid style. In his autobiography he has singled out this chapter as especially important.

That is all there is in Acharya Seal's history of science. But he always enjoyed being surrounded by inquisitive listeners with whom he could exchange thoughts. He inspired many with valuable advice and led them along the path of investigation. The late Dr Radhakamal Mukhopadhyay is supposed to have been greatly inspired by Acharya Seal when he was writing his famous *History of Ancient Shipping*. So was Dr Prasantachandra Mahalanobis.

This study of the science practised by the ancient Hindus has now at last come to the present. I have discussed the past, I am thinking about the future now. There is a nationwide movement going on to transform India into a mainly industrialized country on the model of other nations. Does a country, whose economy is based on agriculture, have to follow other countries and go the industrial way? I would like to end this article with a few lines from Acharya Seal's unpublished autobiography:

> Our usual notion of a rivalry between industry and agriculture in the national life is basically wrong. Both are essential and fundamental. The latest slogan in England — i.e. 'Back to the soil' — seems to me a healthy trend. In India farming will remain a basic and chief occupation as in China. The ratio between agriculture and industry should be 3:1 in our country. Agricultural methods should be improved with scientific use of machinery. I consider better utilization of solar power and electricity desirable. Thus rate of production would increase — and that would be a fitting reply to the economic argument that continued farming would reduce production and profit. This is cited as the main objection against agriculture *vis-à-vis* industry. Increased use of machinery in Indian agriculture is a good way to make use of our enormous resources — manpower and land — and to take us to the forefront of nations.

Chamberlain had once said that everyone should have three acres of land and a cow. We must raise the same slogan here. At the same time industrial growth is also necessary. Till today industrial projects constitute only four per cent of our economy, against agriculture. This is unnatural and that is why we are so deeply immersed in poverty. At this hour of food crisis we need to pay heed to Acharya Seal's advice.

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3

The Problem of Education in Bengal and Asutosh

Bengal has always been a source of worry for the rulers. Even if we leave out ancient history, during British rule itself there was no end to conflicts between the ruler and the ruled. History is full of accounts of the exploitation and humiliation of our countrymen. The preference for indigenous products evolved here long before it emerged in other provinces. Our great men were in favour of introducing an education and administrative system suited to the needs of the time. But unfortunately they could never reach agreement — group rivalry and conflicts were rampant. The foreign rulers have now left. We are living in an era of democracy, free from the oppression of the rulers. But good days are yet to come to Bengal. Bengalis remain burdened with much suffering. Our worries still persist, our in-fightings have not ceased. Bengali intellectuals right from the beginning of British rule have been concerned with religion, society and ideals of education and its medium. Some are engaged in quoting from the scriptures and theorizing, while others who are more innovative are keen to introduce a new system of social norms and education according to their own individual perceptions. Religion is also not being ignored. With everybody pulling the ropes, one should from time to time reckon how far Juggernaut's chariot has moved in our country. Let me offer the few thoughts on education that spring to my mind to the memory of Sir Asutosh on the occasion of the centenary of his birth. Elasticity is possibly a characteristic of the society of an ancient race. There is a tendency to slide back while moving in a direction for a while with fervour. It is difficult to sustain a permanent change within these pulls and counter-pulls. How the Bengali race will appear on the canvas of human civilization is something Destiny alone will determine.

Let us go back to a time long before the birth of Sri Asutosh. Controversies over education had already assumed serious dimensions several times. The Christian missionaries, Rammohun and Radhakanta Deb wanted to introduce western education to end the slumber of centuries with the help of the cold draught from the West. A lot of opposition came from the traditionalists. There was even an attempt to propagate western knowledge and science through the use of Bengali. Of course, all these happened before the establishment of the university. But although the mixed blessings, that were produced when the intoxicating wine of western education and culture was served in Indian bottles, threatened to inundate society, it could not infiltrate deep enough into society. Of course, some were converted to Christianity. The students of Hindu College cut off the holy tufts from the heads of Brahmans and tried to scandalize society by taking forbidden food. Eventually when these rebels returned to the fold with the waning of enthusiasm , the conservatives did not allow them any place in society. They ended up hurting themselves with the thorns of their misdemeanours. But occasionally the thorns turned into roses and enriched Bengali literature. One remembers Madhusudan and his last hopes and wishes: 'Rekho mā dāserey money, ey minoti kori padey' [Please remember your servant, mother — that is my only appeal].

Again, many may not remember the scientist Radhanath Sikdar, a bright student of Hindu College. He was extraordinarily gifted in mathematics. It was apparently he who determined, for the first time, the unequalled height of Mount Everest. The survey departments of the government used to follow his methods of calculation. Radhanath used to wonder how in spite of their smaller numbers, people from western countries were able to dominate the whole world. Eventually he concluded that the source of their unfailing strength lay in the stronger values of their food. He experimented on himself by changing over to western food and trying to feel the difference. We find him on his return from Dehradun to Bengal, devoting himself to eradicating illiteracy. He wrote books and essays in Bengali so that children could learn to adopt a modern attitude towards life.

After a while the enthusiasm of the Bengalis cooled off. Meanwhile the British rulers were coming round to the conclusion that it would be their obligation to keep the teaching of English confined to the middle classes. After all they were the people who would be looking for jobs in government departments; let people from the other strata of society find out their own solutions, let them move the way they liked. Hence, there was a mad scramble for opportunities for western education along the banks of the Hugli. The Brahmans, the Kayasthas and the Vaidyas came forward, while the Muslims by and large stuck to their madrasas and makhtabs. The Brahmans wanted to retain the Sanskrit *tols* as well. They used to learn English and at the same time took all care to retain their caste identity (Bhudev Mukhopadhyay was an example), and if you were to count heads, such persons would have perhaps outnumbered others. This policy of eating the cake and having it too became popular. Thus in most Bengali homes the inner and outer chambers remained separate.

By this time the educated upper class Bengali had managed to establish himself as the junior Sahib. He accompanied his English masters on their trips abroad, but

usually kept to himself and did not wish to mix with the traditionally minded local people.

Then came the Sepoy Mutiny. Unrest and violence spread throughout the land. Initially there was some trouble in Bengal, but the north-west bore the brunt of the turmoil. The British of course won in the end. The old Badshah of Delhi was imprisoned. His son and grandson were killed before the Khuni Darwaza. The state of Jhansi fell, Nana Saheb went underground. The winners went on a course of inhuman revenge; in village after village young men were hanged on trees so that Indians would remember the might and merciless revenge of the British for years to come.

But the British were clever enough to find out that it was quite easy to dupe the people of this country. Therefore, the name of the East India Company was soon replaced by the name of the Queen as ruler. Thus began the reign of the Great Queen Victoria. Proclamations made in her name were full of sweet assurances. Lawyers and other educated people took this to be their Magna Carta. They knew the history of England by heart. Everything comes to those who wait silently, they thought. There was a time when a class of Englishmen felt that it was no use educating those who were not going to work for them. But in 1861, the representative of the Queen (Canning) said after establishing the University that this was not the right policy, for it could lead to extremely dangerous and harmful consequences. (Perhaps the betrayal of several zamindars and bargadars [landholders and big tenants] during the Sepoy Mutiny was fresh in his memory.) From the platform of the university the official representative went on to say, 'I hope the English have learnt that they have something to offer in return to India. If they want the friendship of the Indian people, then it is their duty to share a part of their western heritage and their kind of education with the Indians. That was why universities were established. I do not expect any magical result from the University of Calcutta. But I think that many years from now the gentlemen of this country will eagerly seek the titles I am distributing today. I call them titles because later, earning them through their own effort will be the ambition of the rich who do not merely look for jobs.' [From the 1917 Convocation address of Devaprasad Sarbadhikary]

The University of Calcutta thus came into being. It happened closely following the suppression of the Sepoy Mutiny. In the words of Sir Asutosh, 'This will remain as an eternal monument of the liberality, mental equanimity, courage to do what they will and tenacity of the British. They have succeeded in spreading European education among our people. The British have not only worked to protect the law and order of our country, but they have simultaneously been continuing the task of upgrading the culture of this country as their responsibility. That it is possible to civilize others without spreading education is an extremely disgraceful idea.' The government however was reluctant to spend money on education. The schools and colleges continued as before. The University did not wish to take up the responsibility of teaching. According to the instructions of Lord Canning, degrees continued to be distributed at convocations. Neither the vernacular nor oriental studies had much importance there; to get a degree it was not even necessary to know much science. It was just a competition

in how far one could master English idioms. Some time later Sir Henry Maine said, 'We are senior judges of two high courts of UP. Sometimes we are embarrassed by our young Indian assistants' mastery of the language. It is no exaggeration to say that they are coming up fast, pretty soon they will reach European standards.' Even now the young judges are still fond of showing off their knowledge of English. But that is a different issue altogether. Even today Indian professors often believe that if they can write English well, they will be able to provide strong support to their fragile theories. In other words, once the modern Anglicized judge hears good English, he will immediately accept the substance of the statement. So English came to stay and the country overflowed with lawyers and magistrates. All of them learnt how to cite documents and precedents and argue on their basis. The Congress came into being. Educated people from the various provinces came together and with the Queen's Proclamation for their argument, demanded 'Home Rule'. Thus the educated class followed the path shown by the English and drifted away from the common people. It was their firm conviction that this was the right path to achieve the highest good. When a section of nationalists expressed their doubts about the system and wished to bring a radical change in the method of education, Asutosh, speaking as a representative of the political moderates, told the students in one of his convocation speeches, 'You have acquired the ability to interpret western culture to the people of the East — a matter of great pride. Learn everything, digest western philosophy, science — take it to your fellow countrymen, who have not been as fortunate as you to get English education. But you must not look down upon Indian thought and ideas and be unpatriotic. Make an effort to study your own vernacular [vernaculars were not taught in the universities then] because you have to share the wealth you have acquired in your fields of study with your countrymen through the medium of your mother tongue.' This is the way most of the educated people in our country thought. Then came the Viceroy, Lord Curzon. Seated on the imperial throne, the young man felt he was God's chosen prophet in India. Educated in an aristocratic college in England, he looked condescendingly upon the 'College Square' brand of education in this country. He also set up a commission to improve the standard of education. He was convinced that the province of Bengal was at the root of all trouble. Its sprawling expanse made it difficult to keep it under control, so he proceeded to split the province into two for the sake of administrative convenience. The high-handed governor did not listen to anyone and had his own way. There was a great upsurge in Bengal. It set the intelligentsia thinking why this happened. As Rabindranath wrote in his article 'The Disease and its Cure', 'For some time the Bengali mind has been greatly shaken by the partition of Bengal. Our people have for the first time clearly understood one thing — No matter how deeply we may be hurt, the government will never feel its intensity and will not be moved at all. Never before have our common people understood with such clarity how distant and alien the Government is. How the authorities could dare to ignore the will of our people with such cruel arrogance is a question that has, for quite a while, troubled our minds. . . Can Government policy continue to ignore the entire people of a country? When we see it can, we not only feel the pain of humiliation, but our minds are filled with terror.' Hence people all over the

country were initially drawn to the ideal of Swadeshi — like Surendranath, Rabindranath, Ashu Choudhury, Hiren Datta, Brahmabandhab, Bipin Pal and others. All of them had lost faith in the rulers. So meetings were held everywhere; foreign clothes were set on fire as part of a ceremonial offering to the great cause of Swadeshi. Young Bengalis, men and women, took a new pledge of brotherhood and tied the 'rakhi' on one another's wrists. Only a short while before this (1904) a new Act for Indian Universities had been passed. A meeting was held with selected members to find out ways of implementing the new regulation. This was when Asutosh made his first appearance on the education scene in Bengal. Thinker and scholar in his own right, he had earned fame quite early for his original papers in mathematics. Paying a tribute to him in 1924 Professor Ganesh Prasad wrote, 'Reading the essays of Asutosh, one feels that he could have made important new contributions to pure mathematics, but there was nobody in this country to guide him. Even the English mathematicians learnt of the principal findings in this area (that Asutosh thought about and was known in Europe from 1859) much later.' Ganesh Prasad added that Asutosh must be acknowledged as the first Indian after Bhaskaracharya to have done mathematical research and made a fundamental contribution to the field. However, eventually Asutosh turned to law. He got his doctorate in Law in 1894 and in 1904 became a High Court judge. He had enjoyed doing fundamental research for a while.

I have heard him talk about his own findings at the Indian Association established by Mahendralal. In addition, he wrote a book on the theory of the ellipse, parabola and hyperbola (conic section) following the Greek method. It was used as a textbook in this country for a long time. But the narrow confines of mathematical research could not hold him for long. He entered the Legislative Assembly quite early in 1899 after he had become a Fellow of the University in 1889. Through his intelligence, eloquence and cooperative and judicious participation in the assembly he earned the trust of the government. Lord Curzon praised his sincerity in serving the country. Finally, the responsibility of implementing the new regulation was entrusted to him in 1906. The working rules had not yet been clearly framed. Curzon was succeeded by Lord Minto. He too had great faith in Asutosh. Appointed Vice-Chancellor of Calcutta University in 1906, Asutosh assumed responsibility for the University's future direction. The Swadeshi movement was in full swing at the time. Many eminent and thoughtful national leaders sought to bring about a radical transformation of education, and felt that the responsibility for education in the country must rest with the natives. The curriculum would include issues of life, philosophy and ideals on the one hand, and on the other hand, the history, literature and philosophy of the West and an account of the synthesis of East and West in such areas. Western science and technology needed to be introduced and developed to increase the country's material wealth and eradicate poverty, and the spread of education through the mother tongue ought to be made possible. It must be admitted that at that time, before the implementation of new rules, none of these could be achieved in the University of Calcutta. The leaders said, 'The University is nothing but a house of slaves, where only a slavish mentality is sedulously inculcated in the minds of the students. We have lost all faith in the English rulers.

We will build our cherished National University and National Council of Education on our own. We would send our students abroad — to Germany, Japan or the USA. They will come back qualified to devote themselves to the nation's industrial development and the growth of a robust and self-sufficient nation.'

To maintain the nation's self-respect, an open war was thus declared against the established ruling power. But Asutosh did not join the nationalist movement; rather, he boldly turned his back on this extremely hostile upheaval and steadfastly continued at the helm of the university. His convocation addresses provide a clear picture of his stance and his thoughts. For instance, in his address in the year 1914, he sets out his reasons for his assuming this formidable and demanding responsibility. 'Not only did I have to continue as the Vice-Chancellor for an inordinately long period of time,' he said, 'but had also to bear a burden of responsibility which no one ever had to in the past. When I first joined, the new working plan was not yet decided upon. When after months of discussion with many a circumspect colleague a plan of action was chalked out, a greater burden of responsibility descended upon me in full measure : to implement the new working plan in revamping the proper functioning of the university. Many a brave and ambitious man would have recoiled from this task in horror and would have thought twice before taking up the challenge. There is no need to go into details. However, I reposed complete trust in my own ability. In that hour of need, I took up the challenge, considering it as an honour to have been asked to do so. It had always been my wish to do something of lasting importance for my Alma Mater, the very thought of which used to fire my imagination and inspire my awe. In fact the more I faced obstacles the more determined I became in my mission. I have been fortunate to receive encouragement and sympathy from many friends. I have earned the praise and enjoyed the trust of both the Chancellors and the Rectors. There was of course another factor — the joy of fighting against danger in the teeth of severe competition and adversity.' This was how the British succeeded in winning over the idealist man of action that Asutosh was.

In the first phase Asutosh was at the helm of the university from 1906 to 1914. His ideals and goals in education are to be found in his convocation addresses during this period. He felt that reform and subsequent rapid development of the university in this way would usher in a new era. In the process the University would regain the faith and respect of the nation. In the first year the nationalist movement was in full swing. He said in his 1907 convocation address, 'I admit that getting good professors will take time. Right now we may get only a few readers and lecturers — but the significance and ideals of the new Act are of a high order. From now on the University is not just an institution issuing certificates, nor is it even a conglomeration of colleges. These will certainly remain and a good many innovations will also have to be effected. This will be a centre of learning and the expansion of the frontiers of knowledge. This is precisely the true ideal of the University. There will be hurdles on the path of progress but reaching this goal is absolutely essential and to reach it is not wholly impossible. . . . We cannot, however, reach anywhere near the ideal if we do not have extensive research and a large team of talented research workers capable of doing research in

every field. This will entail a huge expenditure which must be borne either by the government or by the wealthy people in the country. A great deal needs to be done — upgrading the existing colleges, reorganizing the schools and a radical reform of the system of education. Suitable means for imparting knowledge to the students have to be devised. Along with improving their mental faculties, we must also attend to their moral and physical education. For this we require the active cooperation of the members of the Senate.' Addressing the students in particular he said, 'England has a special regard for India and it is for you as true representatives of the country to carry on a dialogue regarding India's demands on England. Do away with mistrust and depression and create in this country a spirit of confidence. In whatever you do, be cautious and never go to extremes. Take pains to gain a profound understanding of all issues. Do not let anyone make sentimental appeals to your imagination and get carried away by them merely because you find them comfortable and convenient. Never submit to the pressures of the leaders. Judge things with rigour, and accept only those things that pass the measure of logic.' Meanwhile the nationalists were beating a retreat. The national schools had to be closed down for want of students; the scarcity of capable teachers and students became the greatest stumbling block in the way of implementing the plans of the National Council of Education.

Asutosh continued his onward march. He got the Maharaja of Darbhanga to donate Rs 2.5 lakhs for building a library; Guruprasanna Ghosh came forward with another two and a half for technical training of the students in Europe, America and Japan. Lectures began with Thibaud speaking on ancient eastern astrology, Schuster on the new advances in physics, Kosambi taught Pali, Satyabrata Samashrami gave discourses on the Vedas and Ramavatar Sharma on the Vedanta. Twelve postgraduate scholarships were instituted, each worth Rs 32. The Darbhanga prize was instituted as an incentive to research in medical science. The government agreed to bear the expenses for one professorship.

Meanwhile, the work of reform was going on in fifty colleges and more than six hundred schools. None of these was equipped to teach in the new modes of instruction. They neither had good teachers nor books or scientific equipment. Even the government colleges were in bad shape. There was need for plenty of funds everywhere. Asutosh threatened Presidency College with closure if it did not improve its functioning. He preferred to leave moral education to the parents but deemed it effective to draw upon Hindu and Muslim scriptures in which the students would find exemplary instances of truthfulness, liberality, self-sacrifice, gratitude, respect for the teacher, sense of duty, marital fidelity, love for children and loyalty to the king. His opponents had advocated radical reform in the field of education and teaching through the vernacular. They were critical of the university's proposed methods. In one of his convocation addresses Asutosh countered their charges thus: 'I feel that the need to acquire of the wisdom of the West through English lies at the very base of our educational reforms; through the open gate of the West should shine forth the light of learning, not through Eastern shutters. Many decent, eminent and educated people have raised serious objections to this; this merits some consideration. After what the Viceroy has said, I need not repeat

my old arguments. Madrasas and the Sanskrit College were established in the days when the government did not show any interest in education. Rammohun Roy and David Hare had then advocated a liberal education and the use of English as the key to unlocking the vast reserves of western literature and learning. The conflict between the Anglicists and the Orientalists had ended in 1835. Macaulay and Lord Bentinck had put their seal of approval on the scheme of higher education — those who want to go in for higher education must undergo a rigorous training in English. There was also Wood's Education Despatch which was described by Lord Dalhousie as being much more informative and liberal than what the provincial governments could produce — we consider it a charter for India's higher education. Recently people have been making noise that the Despatch contains serious loopholes. I can assure you it is neither wrong nor dangerous. What the English have been doing is for our good. It will also be evident from the speeches of Ilbert, Napier and others.'

Thus the claims of the nationalists were dismissed by the High Court judge with the help of mere documents and precedences. From the beginning Asutosh wished to have the youth of the country on his side. In the year of the jubilee celebrations many eminent scientists and scholars from other universities of repute were honoured with appellations in order to infuse the graduates with enthusiasm. He advised the students, 'Train yourselves to listen to the voice of the ruling power with your heart and soul; send forth the message to the world that you are upholders of peace. [Already in 1909 the Alipore Bomb case had been filed.] Prove yourselves as loyal citizens and show the world that with the deepening of your learning your attachment to the British rulers has grown stronger.' The nationalists, meanwhile, were heading for a split. Those who had wanted to steer the wheels of the Juggernaut's chariot along a new path were disappointed and dejected. On one side stood Rabindranath whose plea for the mother tongue was dismissed by Asutosh. Tagore wrote: 'In any warfare, armed or unarmed, one has to assess one's own strength and mobilization. Mere bluster is no war. When we flexed our muscles on the platforms of political speeches and declared war, we had not taken stock of our own man-power and arms. This was because however much we love our country, we do not know it well enough. Again, when with all our whining and moaning we couldn't touch the hearts of our masters we decided to wage a battle through boycott. Did we then believe that the battle would be one-sided? To go on the offensive expecting that the other party would not take up arms is ludicrous — but we see that that is precisely what our vain hope was. We had complete faith in the English law and the Englishman's patience — not in our own powers [an analysis of the mentality of the moderates].

'We had misjudged our opponents at first, but at the same time we had also misjudged ourselves. Today we lament that the British are secretly setting the Muslims against the Hindus. If that is so, why should we blame the British? Is there any reason for us to think that the British would be such fools as not make best use of their advantages ?

'What is worth pondering is the fact that the Muslims can at all be set against the Hindus; who does it is not so important. Saturn, the god of evil, can enter through the

minutest hole so it is better to beware of holes, than of Saturn. The enemy will naturally pounce upon us where we are the weakest, sooner or later. If one enemy does not do so, some other will. Blame the weakness, not the enemy. We have a certain sense of guilt in our country regarding Hindu-Muslim relations. We are never conscious of habitual sins. It is precisely for this reason that the emergence of the Devil in a terrible form must be looked upon as a blessing in disguise.'

I cannot resist the temptation of quoting from the poet's warning — 'Those who have kept themselves divided, and have a greater penchant for dissension than for unity, are never likely to be freed from misery, disgrace and subordination. The country will not be our own even after the departure of the foreign ruler. The rebuilding of a nation should be done by the efforts of its own people. When the English-educated urban people call the illiterate villagers their brothers, then the poor fellows do not understand the meaning of the word 'brother'. So what is to be done? Well, make it clear to all that they have a responsibility towards their own country. They must do something worthwhile — this applies to all groups, old and new — to prove that they can take up the responsibility of serving their nation. If we do not have a co-ordinated action plan, if we cannot mobilize our resources, if none of the parties can make proper arrangements for food, clothing, health and education for all, then all our present bragging will only end in futile weariness. If we cannot unite and launch into action following our plan, let us concentrate quietly on private and individual efforts.'

Nationalism was shipwrecked; there was a split in the Congress. When the nation fell apart in factions and dissensions, the poet, having spoken his mind in these words, turned to his own scheme of constructive work in Santiniketan. Even idealist students were divided into two camps. One camp wanted freedom first: the British must be driven out at any cost. Everything else would follow automatically. The other wished to concentrate on western education first, particularly science and technology. This, they thought, would pave the way for service to the nation, if not outright freedom. In this way, the initial obstacles to Asutosh's work disappeared. He had the majority of the students on his side. He wanted to transform the university into a veritable centre for higher studies. Some scholars engaged in historical research, linguistics etc. joined the university. One professorship of mathematics was instituted with the help of a government grant. The colleges procured some equipment for science experiments. Many educational institutions in the country owed their origin to individual efforts. That is why most of these institutions were not entirely free from autocratic control and vested interests. The university authorities were determined to put a stop to such an antiquated system. Managing committees were set up at all these institutions, including representatives of parents and teachers. There was a sharp increase in the number of students for higher education. All university examinations came to be conducted primarily on fees collected from the examinees. There came to be a surplus in the university treasury after fees and so on had been paid to the examiners.

Fuller was then the Governor of East Bengal. There was student unrest all around: consequently the schools incurred the wrath of the Governor. One way to punish these institutions was to ask the university to withdraw permission to send their students

up for the examination. The university did not always act according to the recommendation of the inspector. The middle grade of the English administrative hierarchy was thus hostile to Asutosh. Asutosh had to face stiff opposition in his attempt to convert the university from being merely an examining authority to an institute of higher education and to open postgraduate classes in several disciplines in the humanities. When efforts were being made to reform the system of education in the country at the university level, some had remonstrated that the students would be unable to cope with higher education since the standard of education in most schools had gone down from lack of competent teachers. Yet, when the number of students increased greatly and the percentage of successful students increased manifold compared to the past, there were many who remarked that to obtain a university degree one had no longer to work as hard as before and that in the rush to increase the number of highly educated people, the quality of examination was being compromised in many ways.

In his 1912 convocation address Asutosh spelt out his ideas of reforming the university in the following words : 'The people want the University to turn its attention to the imparting of higher education, rather than merely conduct examinations. I admit that this is a very justified demand, but if we wish to satisfy the needs of the country then we must state our objectives clearly and confine our efforts within well-planned limits. If we look to the West, we find a two-fold activity in their universities — the professors impart to the students knowledge acquired down the ages, and at the same time lead their students in research, and make remarkable discoveries which contribute greatly to the sum total of human knowledge. True, our educational institutions are yet to reach the standards of the West. But it would not be correct to say that western standards will always be beyond our reach, for so far our interests had never been turned in that direction. In ancient times India was a great centre for creative ideas which is a matter of pride for us and also a source of inspiration for future progress. But those on whom the country so long depended for its progress and revival of learning were not fully aware of their responsibility. This was a serious shortcoming of which we must now rid ourselves. I think I am merely expressing the thoughts of many of my countrymen. It is a matter of pride and happiness that our kind and wise Emperor has expressed similar feelings in his reply to our felicitation. We have resolved to inscribe his speech in letters of gold. The Emperor has declared that at present no university can be called complete without the teaching of the arts, literature and science and proper facilities for research in such fields . . . I have prepared a draft for postgraduate teaching in arts and science subjects. This is being criticized by many. Some are of the opinion that priority should be given to primary education; otherwise it would be useless to spend money on higher education and research. My answer to such criticism is that if we have to wait for that long, then we will never be able to make any progress in higher education. These two are completely different issues. If somebody says that it would be a mistake to introduce postgraduate studies before schools and colleges are improved, my answer would be the same. For the last fifty years we have been teaching undergraduate and postgraduate courses in colleges. Has it not produced any positive result at all? If anything there are now many talented

students in the colleges who, given proper guidance, are ready to endeavour to produce work of the highest order. Every year we receive a number of articles and theses for Research Prize Scholarships and doctoral degrees, which show that the ability of many of our students to do good research work is praiseworthy by any standards. I consider it the prime duty of our University to employ suitable professors to guide and motivate such students.'

Meanwhile almost fourteen lakhs of rupees were donated in two instalments to the University treasury by Sir Taraknath Palit. He wanted to have two professorships in chemistry and physics instituted with the help of the interest returns on the deposits. Earlier money had been obtained from the government for the Hardinge chair in mathematics and the George V chair in philosophy. Thibaud was appointed to the Ancient Indian History and Culture chair from the University's own resources. Teaching started in dead earnest in various subjects in the arts. There was no dearth of students applying for admission; in fact, in some subjects the number exceeded one hundred. Some of the critics hinted that the University will now compete with the colleges. But Asutosh said, 'The charge is unfair; the colleges simply had no place to accommodate the increasing number of students.' He added, 'We want to extend the sphere of knowledge and turn our ideals into reality. In addition, many more classrooms would be needed; the Law College needs a hostel.' Thus new buildings came up, and a Law College hostel was also opened.

The critics grumbled that Sir Asutosh was trying to bring all educational institutions under his dictatorial sway. Teaching M A courses and guiding research students were not one and the same thing; the young men teaching M A courses were entirely without experience. They were determined to oppose him in the Senate. According to the regulation those days the introduction of anything new required the assent of the Chancellor. The office of the Viceroy had shifted from Calcutta to Delhi and Shimla. The detractors had enough opportunity for adverse instigation. Meanwhile Sir Rashbehari Ghose came forward after Sir Taraknath. He donated ten lakhs of rupees for research and teaching in pure and applied sciences. Both Sir Taraknath and Sir Rashbehari were involved in the nationalist movement; when the National Council for Education failed to go far in constructive work, they were a little disappointed. It was in Taraknath's residence that the Bengal Technical Institute was opened. Later, when there was a difference of opinion, the National Council shifted elsewhere, and the entire property was handed over to Asutosh. Initially, Asutosh spent all his energy in making arrangements for the teaching of literature, philosophy and law. The introduction of science teaching was more expensive. With the money donated by Taraknath Palit in hand he wanted the government to supplement it for this task. The government, however, was not in favour of science education, perhaps out of an apprehension that the Bengali students would misuse scientific knowledge. Moreover, the Palit endowment had the stipulation that the professors would have to be Indian and the University had agreed to this. The Viceroy's office had already shifted to Delhi. In spite of many unpleasant confrontations, the trusted advisers of the government had seen to it that science education did not receive the government's approval. Finally, Rashbehari

Ghose lent Asutosh a helping hand. With the help of two great donors, a substantial amount of twenty-five lakhs of rupees was accumulated for construction of the Science College. The foundation stone was laid on 27 March 1914. On this occasion Asutosh recalled that he had received donations from Sir Taŕaknath on 15 June and 8 October amounting to fourteen lakhs of rupees. On 30 December 1912 the Syndicate had asked for roughly an equal amount from the government hoping that the claims of science and technology would not be turned down. But unfortunately the government did not respond. Disappointed, Asutosh was about to reduce the scope of his project when a memorable incident took place. It was evident that the time was right, there would not be any further delay. On 8 August 1913 ten lakhs of rupees came from Rashbehari Ghose. The University decided to disburse rupees three lakhs from its own fund and at the same time an appeal for financial assistance was sent to the Indian government on 4 October. The response came after a long time. Later they were told that their demand would be considered in due course along with other demands when the government had funds to spare.

The founding fathers of the College of Science realized that the University would have to be self-supporting and depend on liberal donors like Sir Rashbehari and Sir Taraknath whose names will be remembered by posterity long after many men whom we call 'great' to appease them, fade from public memory. The foundation of the College was laid. Acharya Prafullachandra and Dr Prafulla Mitter joined the chemistry department, Chandrasekhar Venkata Raman and Debendramohan Bose were selected for physics, Ganesh Prasad for applied mathematics and Shankar Purushottam Agharkar for botany. Asutosh was convinced that this institution will never face closure from lack of resources and assistance. He said that perhaps it would even be possible to invite eminent scientists from other countries from time to time. It had a modest beginning but it was hoped that it would always be active and continue to prosper. This is how work began in the science departments, and Prafullachandra on his retirement from Presidency College joined the Science College. He lived in one of the rooms in the three-storeyed building that came up right on the main road. The southern part housed the research wing. Dr Prafulla Mitter started work with his students. Raman who had not yet given up his government job carried on his research in the Indian Association at 210 Bowbazar Street every morning and evening at his convenience.

The Chancellor, Lord Hardinge, did not attend the 1914 convocation. That year Asutosh retired after eight years of hard labour from the Vice-Chancellor's post. By this time the area under the jurisdiction of Calcutta University had been greatly reduced; the Punjab and Allahabad zones had separated long back. Now other universities were set up one by one, at Patna, Dhaka, Burma, Guwahati. Now we have Jadavpur, Bardhaman and Kalyani within the State. Deben Bose and Agharkar went to Germany to acquire experience in research. But the First World War broke out and they were interned in Germany for four years. In 1915 after receiving their M Sc degrees a group of students appealed to Sir Asutosh to start postgraduate classes in mathematics and physics as well. They promised him that they would do their utmost to help him. Asutosh could assess people rightly, and had faith in the young students

of Bengal. After tackling many obstacles, he finally persuaded the University authorities to open M Sc classes in physics and mathematics. Of the first batch of lecturers in physics, eventually two could not join for political reasons. Sailen Ghosh had to flee to America and Jatindramohan Seth was interned. Nevertheless teaching continued. Were the young teachers able to maintain a high standard of teaching and research? The opposition claimed that some of the papers that were published as original contributions were not really so. Letters were published with claims and counter-claims. Some Englishmen believed that the University lacked a correct policy. Besides, plans were afoot to open new universities in the country. A new Commission was set up seeking the dependable opinion of Western scholars. Michael Sadler came as the President along with distinguished professors of many British universities. Sir Asutosh, too, was a member. At the end, the Commission supported the setting up of postgraduate classes in the University. Their report, running into a few hundred pages, was published; it was recognized as an authoritative set of instructions relating to education in India. The rationale of Sir Asutosh's actions was fully endorsed. As President of the postgraduate council Asutosh regulated the progress made by the postgraduate classes for some time. Another sixteen lakhs of rupees were donated by Rashbehari Ghose for the improvement of studies in the applied sciences. The young lecturers tried their best to enhance the prestige of Calcutta as a centre of learning. Meanwhile Asutosh remained attached, as he was in the past, to various other scientific research projects in Calcutta.

He became the President of the Asiatic Society four times. It was through his efforts that the Calcutta Mathematical Society was established in 1908. He was the President of the first session of the Indian Science Congress in 1914. In one of his convocation addresses in Calcutta he said with pride that other universities were coming up in our country. Calcutta will no longer be the seat of the Viceroy's office; perhaps Bengalis will no longer be in the vanguard of every major venture in the country, but it is likely that in the field of education they will continue to dominate. This was his desire and his dream; to achieve this he directed all his efforts and devoted his whole life to making Calcutta University the leading centre for education and research.

The non-cooperation movement started in 1920. The leaders once again wanted the students to leave the University and join the movement. In the intervening years, other Vice-Chancellors had come and gone. Once the authorities understood how much faith the student community had in Asutosh, the Governor as well as the Viceroy requested him to take on the mantle of the Vice-Chancellorship once again. So he was back in 1921 and stayed on for the next two years.

Then in 1920 more endowments came — five and a half lakhs of rupees from the Khaira family. This was for science and the fine arts. In 1923 after retiring from the High Court Asutosh had hoped to devote all his energies to the cause of education. It was unfortunate for the country that this was not to be. Suddenly on 25 May 1924 he passed away. Forty years have gone by. India has attained independence. There were only three universities when western education was introduced in India — now they possibly number more than forty. Calcutta University has been trying to follow the

ideals set by Sir Asutosh. The University has progressed more or less as Asutosh had envisaged. Perhaps its sphere of activity has shrunk. There have been some minor changes. The nationalists had wanted the vernaçular to be the medium of education. Although Asutosh had admitted the vernacular into a corner of the vast curriculum of the University, he wanted English to be the medium of instruction in higher education. I personally feel a change is overdue; for this has hindered the progress of education in our country. Though we have Bengali up to a certain level now, English is still used in the higher classes. Who knows what the future has in store for us?

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4

The Early Phase of My Explorations in Science

From my childhood I was keen to go abroad. I passed my M Sc at the age of twenty-one. Professor Mullick asked me to see him and said affectionately, 'You have secured embarrassingly high marks, my dear boy'. Perhaps this meant a government scholarship for going abroad, I thought hopefully. But I was out of luck. They all wanted boys to be trained in chemical engineering. That would lead to the growth of indigenous factories, increase of wealth — and we would become self-reliant. My skills were limited to mathematics. That year chemistry won, though the victorious student eventually entered the grooves of academia, but that would not be relevant here.

Meanwhile Acharya Ray had joined the University College of Science after retiring from Presidency College. Europe was at war. Sir Asutosh had already selected Professors of science with the financial backing of Palit and Ghose. But two of his chosen men — Debendramohan Bose and Agharkar — were interned in Germany where they had gone for higher studies. Raman was still working as an efficient government officer in the Accounts Department. In his leisure he was carrying on research in the Association founded by Mahendralal at Bowbazar. His papers were being published in the foreign journal, *Philosophical Magazine*. Asutosh had great faith in him. 'One day he is going to be a great scientist', he told us once. Besides Acharya Ray, Dr Prafulla Mitter had also joined Science College. He had worked for some time in the Bengal Technical in the early days of Swadeshi. The Swadeshi bosses did not see eye to eye with Palit, so the Palits left the Parsibagan house and moved to Muraripukur. Taraknath had so much faith in Sir Asutosh that he donated all his property to the Calcutta University. Sir Rashbehari had already donated one million rupees as a first instalment.

It was Dr Prafulla Mitter who supervised the construction of the Science College in Calcutta. That is why the three-storey building has drains in every room. It had to be, because all of them were to be used as chemistry laboratories. However, worktables had been put in only in a few of the ground floor rooms in the south; and gas, water and electricity connections were being installed. Both Acharya Ray and Dr Prafulla Mitter had joined and started their research. Ray lived in a room on the first floor of the same part of the building. He had left the Bengal Chemical office. He was a bachelor and led a very simple life. His students looked after him. He had free accommodation — quite spacious too. Dr Mitter would often bring food for him in a tiffin carrier. His admirers also brought sweets and other delicacies. After the day's work Acharya Ray would take a ride in a horse- drawn 'compass' carriage to the monument [the Ochterlony Monument, now known as Saheed Minar] in the maidan [open green area adjoining Chowringhee]. His old friends, students and admirers assembled there — it was an exclusive group; others also went along. Our good friend Devaprasad Ghosh was the best student of our time; he was extremely bright and versatile. He was teaching at Ripon College and also attending the High Court to try his luck in the legal profession. He understood the war strategy of the Germans very well. He could visualize what was coming and explained every move to his admiring listeners.

Our batchmates Jnan, Meghnad and others stayed at 110 College Street. Sureshchandra Bandyopadhyay, Nilratan Dhar and their students were all 'Swadeshi' sympathizers and followers of Dr Ray. They were all members of his evening club. Acharya Ray used to return to his room in the college late in the evening. He never switched on the light. His health was ruined by overwork. Some believed it was the result of handling mercurous nitrite. His disciples studied on their own. At times they had heated political discussions. A lot of impostors were around, including, as some claimed, police informers. Prafullachandra Ray had turned Gandhian — he wore a short lungi, and worked at the spinning wheel to develop 'soul-force'. But every morning he went down to the laboratory and spent a few hours doing his research. The students who assisted him were mostly Palit scholars; some were honorary volunteers. There was a poor student Dr Ray had picked and brought up. He helped him in his research. Dr Ray had great confidence in his analysis. He was concerned then with a lot of weird metal compounds — trying to increase the valency of sulphur, dabbling with compounds of gold and platinum; and nearly all his ideas found corroboration in the student's analysis. He ridiculed the students who had degrees. His boy was the best as far as analysis was concerned. Even though Dr Mitter did not always see eye to eye with him, he was too polite to say so. He was the model of a perfect, well-mannered Bengali gentleman. However too much praise went to the student's head. The alcohol in Dr Ray's store started to evaporate a little too quickly. The gold and platinum compounds also started disappearing. Eventually Dr Ray had to change his opinion, but he was kind and fond of his students. The boy merely lost his job; he was not put to any other difficulty.

These things happened a little later. To come back to the beginning, those of us who had not studied chemistry began to ponder about the next course of action.

* * *

In 1912/13 (?) Dr Ganesh Prasad joined as Professor of Applied Mathematics in the Rashbehari Chair. He had a very good reputation. Apparently he had worked with Dr Cline in Germany. He came from Benaras — a man with a simple life style modelled on Dr Ray's. He lived in a bare flat in Samabaya Mansion with hardly any furniture in it, but in the midst of German books and encyclopedias. We had all heard about him. His question papers bristled with out of the world problems, difficult enough to floor the most intelligent students. He had joined the newly built Science College at the bidding of Sir Asutosh. Until then Presidency College had monopolized higher studies in the sciences.

In the contentions in the Senate the group opposed to Sir Asutosh belonged to Presidency College; most of them government officers including a number of Englishmen. The government did not approve of Sir Asutosh's efforts to teach science in the University. It had taken umbrage at the stipulations of the Palit and Ghose chairs, that none but Indians were eligible. What do Indians know about the teaching of science? Till then it had been the age of Cunningham, Harrison, Peake, Vredenburg and the rest. Acharya Ray too had worked in the Provincial Service for a long time. Of course Dr Jagadis Bose was an exception. He had turned to botany from physics. But he continued to teach physics, though most of us were under the care of Dwijen-Babu, Charuchandra Bhattacharya and Barada-Babu. But when radio waves were discussed and experiments shown, we felt elated. Puffed with pride we used to disparage Marconi, saying he hadn't done anything that we couldn't have done. Eventually Dr Bose was to search for a scientific basis for the Upanishads — *Sarvam Khalvidam Brahma* [Brahma lives in everything]. However, his mathematical arguments were not particularly strong; he had to fall back on intuitive knowledge. Later he moved to the Bose Institute and plunged headlong into studying the life sciences.

Dr Ganesh Prasad was then a newcomer to the University. The students flocked to him for training in research. They were the best science students of Calcutta, though several of them had not secured high marks in Ganesh's paper in their M Sc. But the fault lay with the teachers of Presidency College, at least Ganesh Prasad thought so. The young students had to stomach adverse comments about their former teachers, too scared to answer back. After my M Sc I too presented myself to Ganesh Prasad who had been one of my examiners, though I had not fared as badly as the others. Dr Prasad was kind to me at first but I was notorious for plain speaking. I found it difficult to bear his tirade against my teachers. I had dared to counter his adverse criticism. This infuriated him. He said, 'You may have done well in the examination but that does not mean you are cut out for research'. Disappointed, I came back. I decided to work on my own. I was trying to do whatever I could when the Bihar government advertised for young teachers in the newspapers. Since I had not studied law, I decided to stick to mathematics and try my luck. I collected letters of recommendation from Principal James and Dr Mullick [Debendranath Mullick taught at Hooghly, Patna and Presidency Colleges. He was the President of the Physics and Mathematics section of the Indian Science Congress in 1919] and others and applied for the job. But my luck was

still against me. I was summoned by Dr Mullick one day. The DPI of Bihar had informed him that I happened to be too good a candidate for their requirement. So I was obliged to look around in Calcutta. One of our classmates, Sailen Ghosh, popularly known as 'Mama' [lit. uncle], had done well in physics. He too was without a job, but he was very resourceful. He went up to Sir Asutosh with a suggestion : why not start teaching subjects in the University that are not taught at Presidency? The logicians had in the meantime thought of various plans for the physics department in the University. But who was going to teach? One day we were called up by Sir Asutosh.

Meghnad, Sailen and I went up the steep stairs to the library, to the special chamber where Sir Asutosh sat. We were naturally meek and submissive and overawed by his august presence. He had heard that the younger generation wanted more modern subjects to be introduced in the University curriculum. He asked, 'What subjects are you competent to teach, boys?'

'Sir, we will try our best to teach whatever you want us to.' He smiled. We had only heard of the many new discoveries in physics, most of them made in Germany — new developments and new discoveries. Planck, Einstein, Bohr — we Bengalis had only heard of them. To know more about them one had to read books in German or research journals in other languages. During the war most of these journals did not come to India.

At long last, as the first step to a new career, we were given a special allowance of Rs 125 per month. Meghnad was assigned to study quantum theory and I had to learn Einstein's relativity theory. We came away committing ourselves to being prepared to teach within a year. But where were we to get the books from? There were some books in English on relativity; we got hold of them. But where could we get hold of the writings of Boltzmann, Kirchhoff, Planck? Suddenly, I thought of an idea. Dr Brühl was the answer.

Dr Brühl was then teaching physics at Sibpur [Bengal Engineering] College. He had set up many delicate instruments which even the Presidency College did not have. All the M Sc students had to do practical work in Brühl's laboratory — a very tough test, a difficult hurdle for most students. But some of our predecessors had acquired a great deal of skill from him. We had heard that Brühl was originally a brilliant student of botany. He fell sick, with a problem of the lungs. Doctors advised him to spend some time in a warm climate. So he left his country on foot with the idea of collecting specimens of plants from various countries. He travelled along the Danube and reached the city of the Turks — Constantinople. Crossing the Sea of Marmara, he entered Asia. From the Turkish empire he crossed Persia and finally reached Karachi. Then he took a boat to Calcutta. Many foreign scientists used to visit the Botanical Garden then.

Brühl however got married when he came here. Since this was not done with prior permission of his emperor, his scholarship was terminated. Dr Brühl stayed on in India. He was an expert in all areas of science. From the post of a mere assistant he rose to a high position. He was fond of reading and had an excellent collection of science books in his library, where we discovered many rare books. We borrowed Planck, Boltzmann, Wien; we could not have asked for more. Meghnad had taken great pains to learn

German, and even passed the Intermediate Examination in the language. I had just started. But I read French. We got hold of a few books on relativity in English. Meanwhile, Sailen had become a great favourite of Sir Asutosh and provided him with a lot of information — if new subjects were to be introduced in the Science College, where to find the fine instruments, where to set up the lecture theatres, the electric points, water taps, what the approximate expenditure would be from the University's own budget, and so on. No import was possible during the War. It was to Sailen's credit that he had gathered a lot of information.

The National Council had once attempted to introduce higher education in the sciences. The apparatus they had bought were lying at Muraripukur. There were also some useful tools at St Xaviers College and at Sibpur — all this information was at Sailen's finger-tips. Emboldened by the idea that nobody would refuse Sir Asutosh, he went as far as Krishnanath College in Baharampur to search for apparatus. But the chemists did not approve of these schemes hatched by the younger generation. They apprehended that the three-storeyed building would soon come to accommodate the other sciences too. In their opinion, Asutosh was being too hasty. Prompted by some immature youngsters he was going in before the right time. He should wait for Deben Bose and Raman to come. These youngsters were hardly capable of carrying the heavy burden.

Asutosh felt that if he could win over Acharya Ray to his side, he would not have any difficulty in carrying through his plans in the Senate. Not that Dr Ray had much faith in us, but fortunately our classmate Jnan [Jnanchandra Ghosh] had already earned the Acharya's appreciation by taking the first step towards his famous theory. The old man had faith in the capability of the young. So Asutosh had little problem in winning him over. The new system was introduced. From 1917 postgraduate courses in applied mathematics, physics, chemistry etc. were to commence in Science College as well. Of course, Presidency College would continue with its old syllabus. Students of all subjects who had done well were approached to take up teaching positions at Science College. Ganesh Prasad was the professor of mathematics; he however preferred to spend most of his time on research. We were asked to teach other subjects. It was indeed a peculiar situation. Apart from Meghnad, Sailen and myself who were chosen to teach physics, others who came were Sushil Acharya and Sisir Mitra, both bright students of Jagadis Bose. From Bangabasi College came Joges Mukhopadhyay and Phanindra Ghosh. Jatin Seth had gone to America after completing his studies at the National Council. He had come back qualified, and he too was asked to join.

There was trouble right at the outset. One night the Police Chief, Basanta Chatterjee, was shot at by unknown assailants. The spies suspected the Seths of Beadon Street; their house was ransacked and searched and all members of the family were taken into custody. Of course nothing incriminating was found. But Jatin Seth was interned. Meanwhile Sailen Ghosh was getting ready to sail abroad for higher studies. He was all set to leave when Mr Tegart summoned him. Sailen was not at home when the messenger came. When he heard of it on his way home, he promptly went into hiding. Eventually after crossing many hurdles he reached America through war-torn Europe.

We had thus lost two colleagues. But we were undeterred. We tried our best to honour our commitment to Asutosh. Meanwhile Jnan Ghosh's articles on the Theory of Electrolytic Conduction were being serialized in the *JCS* [*Journal of the Chemical Society*]. They were being appreciated in the scientific circles. Acharya Ray was much impressed by the achievements of the youngsters. Jnan Ghosh left for London on a Palit travelling fellowship. Mukherjee [Jnanendranath Mukherjee] also got a fellowship at the same time.

The moment I went to Asutosh to plead my own case, he smiled and said, 'But you have got married, you see'. The Palit fellowships were meant for bachelors only. Taraknath Palit believed that a married student would try to save money and send it back to please the newly married wife and not spend the entire money on his studies and science. That is why he had stipulated the condition. Once again I had to come back disappointed.

Meghnad and I taught in both the departments of physics and applied mathematics. However, we did not get any additional allowance for that. Things were different in history and other cultural subjects; such were Asutosh's whims. Dr Deben Bose returned in 1919 and joined Science College. More or less at the same time Raman made up his mind to resign from his accountant's job and come to Science College. He summoned the assistants at once. Until then we had been doing all the teaching. He put more emphasis on research. Poor Meghnad was not very good with his hands; he got into the bad books of the Palit Professor. The other teachers were more inclined to work with Raman to get their doctorates. Raman was involved with the diffraction of light. Phani Ghosh and Sisir Mitra got their doctoral degrees. Phani Ghosh had investigated the reasons behind the appearance of various colours in mica. Mitra studied how light waves were scattered by obstacles and how in suitable conditions the scattered waves combined to form strange lines. As for me and Meghnad, mathematics was our forte. We published a few joint papers. But Meghnad had a profound knowledge of astronomy. His articles on the brightness of stars and the related variations in temperature were highly praised. He got his doctorate from the Calcutta University and set out for Europe on a Guru Prasanna Ghosh fellowship.

In 1921, I was at Calcutta. The Dhaka University had come into being. The first Vice-Chancellor, Mr Hartog, was recruiting new teachers. He called me up one day in Calcutta. Someone in a high position had apparently told him about me. I met him and eventually got the offer of a Readership at Dhaka in 1921.

I had been teaching in Science College since its inception, for four years. Now an invitation came to go to Dhaka.

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5

Jagadischandra

Many years ago when we were still in school a complete overhauling of science teaching took place in the University of Calcutta. Intermediate courses in science commenced from 1908. So also commenced the initiation into Swadeshi — it was the era of Swadeshi. We used to roam the streets on the day of the Rakhi festival singing songs calling on people to raise aloft and use with pride the rough fabric given to us by our mothers. The wheels of the first cloth-mill set up by Bengalis had started rolling. The new plan for science education conceived by the National Council of Education was circulating among the students. Determined to go in for industry, a few Bengalis had already been to Japan and come back.

After my Matriculation in 1909 I made up my mind to study science. Bengali periodicals then often published accounts of Jagadis Bose's discoveries — that he had experimentally proved the existence of life in the non-living. This was well publicized not only in our country but also among the elite in other countries where the professor had presented his results himself and had been listened to with reverence.

As one enters Presidency College one immediately notices a glass-enclosed room. Acharya Jagadischandra used to do his research here with mysterious looking gadgets. To be able to take lessons in science from this world-famous scientist seemed to my adolescent mind to be the ultimate thing in life.

On the other side on the ground floor sat Acharya Prafullachandra Ray engrossed in his chemical research. Like me, many students flocked to Presidency College with the hope of getting their first science lessons at the feet of these two Professors. I had the good fortune of learning chemistry from Acharya Prafullachandra in the first year.

But it was only after two years that I had the opportunity to study under Jagadischandra. Meanwhile like others who had extra-curricular interests, I also went to the library from time to time and flipped through the pages of the book *Response in Living and Non-Living*. It was the era of the Magnetic Crescograph in Jagadischandra's research career. Delicate instruments were being made in the Presidency College workshops under Jagadischandra's directions. We used to look forward to his lectures. But the Professor was so involved with his research that he did not have much time to spare. Nevertheless, I will always remember those few days that I had the privilege of studying under him. Most memorable were the days when he talked about electrical waves giving demonstrations with his own instruments.

The instruments that Hertz had used for his research when he discovered radio waves were not suited for classrooms. Jagadischandra put his mind to it and developed a new kind of Coherer. His instrument was so fine that it could produce minute ether waves with wavelengths of one-sixth of an inch, the tiniest electrical waves in ether. Very soon he demonstrated in the classroom that electrical waves had all the properties of ordinary light waves.

The accounts of this apparatus and his fascinating experiments with it are well documented. His skills of invention and experimentation have been recognized in all countries. The instruments which he used to teach us with are presumably still preserved in the Bose Institute.

Since I was more inclined towards theory, I joined the applied mathematics class [should be read as 'Mixed Mathematics'—Eds.] after my B Sc. Meanwhile Jagadischandra's laboratory had shifted to the building which housed the Baker Laboratory. Our seniors were given the opportunity to do their fundamental research there. We were curious and now and then peeped inside to see what was happening in that mysterious laboratory. Very soon Jagadischandra retired from government service and used all his savings to set up the Bose Institute. From then on he devoted all his energies to the investigation of plant life. Having done the country proud with his limitless skills that brought him fame, he passed away in 1937.

His unique contributions to science have been recognized. He was elected Fellow of the Royal Society of England in 1920. His students continue to follow his methods and investigate the questions that engaged all his mind in his later years. In addition, the Bose Institute is also now a centre for research in pure physics. Dr Debendramohan Bose has now taken over the responsibility of directing research there.

Jagadischandra has set a glorious example by investing all his savings in scientific research. The scientists of Bengal have been inspired by his example to make new discoveries in science.

30 November 1958 happens to be his birth centenary. I remember with reverence that I had the good fortune once of being his student. As one of his students, I offer my tributes to him on this occasion.

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6

Remembering Acharya Prafullachandra

I came in contact with Acharya Ray when I joined Presidency College 52 years ago. Ours was the last batch to pass the old Entrance Examination. The University had introduced a number of new regulations regarding science teaching in 1909. For example, the students had to get acquainted with scientific methods from the very first year. Everyone had to carry out a set of clearly laid out practical projects in every subject in the laboratories.

Those were the days of the Swadeshi movement. Most of the good students wanted to take up science but all the colleges were not adequately equipped. So the students flocked to three or four colleges known for their science faculty. The more ambitious ones who wanted to be scientists joined the Presidency College which had the attraction of Jagadis Bose and Prafulla Ray — stalwarts the students would be able to meet every day.

Dr Ray used to teach chemistry right from the first Intermediate classes. Classes were held in the first floor gallery in the North-east corner of the old building. Very often students from other colleges also came to attend Dr Ray's lectures. He spoke in simple English and made no attempt at oratory; rather his style was informal and he spoke light-heartedly so that the basic points of chemistry would register in the minds of the students.

Those who had the opportunity of attending his chemistry classes can never forget his gestures and mannerisms. To give one example, he would embrace Sitaram, the bearer, standing beside him, in order to explain molecular attraction. He seemed to enjoy the uproarious bursts of laughter that followed such demonstrations.

He used to tell us stories about eminent scientists like Lavoisier, Dalton, Berzelius, Pasteur. He would also at times talk of Nagarjuna and the alchemists of ancient India. He would at times recite a couple of *ślokas* from some old text from pre-printing times or a long passage from English literature. Altogether the memory of his speeches those two years has grown to a wonderful impression in my mind.

I never had the reputation of being obedient and docile. Talking back to my elders has always been my problem. Something happened in Dr Ray's class one day, the details of which I do not recall, and for some reason he got the impression that I was not attentive and was distracting my friends nearby. I was summoned to the teacher's platform to take my seat on the railing at its edge, segregated from the others and next to the table with all the apparatus, where the great teacher himself used to stand and deliver his lectures every day. It turned out to be a blessing in disguise. My eyesight was poor and I could now watch every aspect of the experiments with perfect clarity. I also came to know very well the students who were engaged in important research work in the professor's personal chamber at the back. I used to listen to their discussions about many new experiments and watch them at work, but always after the classes were over.

To be quite honest I had never antagonized my teacher — in fact, I received his affectionate pats and punches all the time.

Those days we had to do our compulsory experiments in a shed in the college courtyard. For some time we shared the shed with fifth year students who used to work in one corner of the shed — the re-organization of facilities at the college was yet to be completed. Pabitra-Babu used to supervise our work. The unruly students gave him a very hard time. Sometimes it was difficult for us to resist the temptation of repeating with our own hands the interesting experiments we heard of or had seen upstairs in the classroom. Maybe one day sodium would be dropped into water without taking the necessary precautions. A loud explosion would follow and sodium would be scattered all over the room. All the bunsen flames in the room would brighten up together in a green flare. It would bring the teachers and assistants rushing fearing the worst. Those were the days of terrorist bomb explosions all over the country outside the college precincts. The teachers had to be on the alert — nothing beyond the pale of law should be allowed to happen within the college. It is no use describing those incidents in detail now. It was Chandrabhushan Bhaduri who used to investigate complicated cases. He was astute and even the most daring of the boys were scared of his grim face with thick salt-and-pepper whiskers. He dealt with the wayward boys firmly, leading them to the right path.

It was only in the first two years of college that I had the opportunity to be a student of Dr Ray's. I was never considered one of his boys, though on many occasions later I had the privilege of his company and affection. I used to visit the chemistry orbit like a comet. Perhaps that is why he would joke about me and say that I was a burning example of his pet theory of the 'Misuse of the Bengali Brain'.

Dr Ray had retired from his government job before I completed my M Sc and had become the Head of the Chemistry department in the University College of Science. He

wanted the entire building at 92 Circular Road to be used for chemistry. We were unemployed at the time and were looking for jobs. We could scuttle Dr Ray's plan in the end only by taking recourse to stratagems. Sir Asutosh himself came over to our side. He persuaded Dr Ray to change his mind. The upper floor was allotted to mathematics and the northern block to physics. All postgraduate departments came directly under the University. As a teacher of physics I continued to meet Dr Ray every day for the next four years (1918-21). For reasons of health Dr Ray used to have evening walks in the Maidan. During the First World War he and his followers met near the Monument. It was a large group comprising, besides his admiring students, researchers in mathematics, experienced astronomers, famous lawyers and educationists. They used to discuss McCensen's war strategy and niceties of diplomacy. I was never fortunate enough to be present at any of these meetings. I hope some of those who used to participate have already written about them. Around this time devastating floods in North Bengal led to a crisis that grew to national proportions and we watched Dr Ray plunge into relief work. The chemistry department almost came to a standstill. Bundles of old clothes and bags of rice filled up many rooms in the college. Volunteer boys and girls had free access everywhere. Subhas Bose came forward in the service of the nation and was welcomed by the people as their leader. Dr Ray loved his country with all his heart. He put aside his researches for a while during those difficult days. His students also worked tirelessly but politics could no longer be kept apart from the field of study.This further complicated the problems of education. The students knew of Dr Ray's sympathy for these movements. They would often get involved in these movements under false names. In the days of the 'Khaddar' Dr Ray took to wearing hand-spun lungis and kurtas. That was how he was seen working in the Science College for many years.

After I left for Dhaka our meetings were less frequent. I used to drop in to pay my respects whenever I came down to Calcutta. I found new faces crowding his laboratory. His health had deteriorated considerably before his death. Even then I used to visit him; he did not seem to recognize me.

Dr Ray had started his career in 1889 as a pioneer dedicated to the service of education and society. All his life he suffered from insomnia and ill health. His students believed that it was the terribly polluted air in the laboratory that was responsible for his health breaking down. But until the end there was no stopping him in spite of affectionate pleadings by his pupils. The laboratory was his home till finally the flickering flame went out, and he passed away.

Dr Ray first became famous at home and abroad by writing a history of Hindu Chemistry. He also discovered mercurous nitrite. His students were working with amine nitrite when we were in Presidency College. They were important pieces of research . By crystallizing them in their pure state they proved the existence of these compounds in organic chemistry. Their work was appreciated everywhere. Ever since he returned to India, Dr Ray turned his attention to the manufacture of chemicals of daily use. He was the founder of the manufacturing laboratories of Bengal Chemicals, the history of which has been recorded in his autobiography.

In his later years, he initiated several original research projects at the Science College. These led to many discoveries by his students and colleagues who earned considerable reputation later.

All his students fondly remember him as an epitome of selfless work, philanthropy and love. He devoted himself completely to ushering in a new age of research in the country. He remains a model of the spirit of sacrifice and complete unconcern about one's personal happiness in the service of the country. The future history of science will testify to what extent his efforts have been successful. He was a bachelor. His students were very close to him. Generations of students and their students all over India will remember him with admiration. As one of his insignificant pupils, I take this opportunity to remember him and offer my sincere homage to him on his birth centenary.

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7

Swami Vivekananda

Swami Vivekananda was born here in Calcutta a hundred years ago. He was then known as Narendranath Dutta. The story of his childhood, his education and how his life changed from his association with Paramhansa Deva [Ramakrishna Paramhansa (1836-86), a nineteenth century saint who had little erudition but profound wisdom. He was the priest at the famous Dakshineswar Temple near Calcutta.] is well known today. The triumphant march of his ideals which began soon after he delivered the historical speech at the Chicago Parliament of Religions continues even to this day. The movement that originated from his ideals occupies a special place among the forces active in our country today.

Indian history was passing through an important phase when Swami Vivekananda was born. The British had expanded their rule throughout India and western education had struck roots in the Indian soil. Rammohun Roy and his other great contemporaries had realized long ago that the future of India depended on the successful blending of her ancient heritage with this inflow of western ideas. They had understood that it would be equally harmful for India blindly to ape western culture or cling to its ancient manners, customs and social system. The initiative they had taken at the beginning of the nineteenth century to bring about a synthesis of the East and West found its complete fulfilment in Vivekananda.

Vivekananda was a rationalist. He believed that blind faith could never lead to any good. And it was from his innate love for work that he was drawn to certain aspects of the West that stirred him deeply. In some of his letters written from abroad he expressed his amazement at the progress the West had achieved and their deep commitment to work, and at the same time how much he was hurt by his countrymen's lethargy and

inertia. He referred to these points repeatedly in his letters to admirers and friends. These young monks constantly dreamt of building a new India.

Vivekananda believed that to deny life its basic needs and only run after otherworldly blessings was as meaningless as chasing a mirage. Very few people before him had realized so clearly that religion was not possible with an empty stomach. Hence the movement he initiated principally aimed at human welfare through service to humanity. By human welfare he meant man's overall well-being. No facet of the life of a common man could escape his attention. So he was equally interested in the spread of education and the propagation of a new religious ideal.

When he applied his mind to the spread of education, he realized that the medium of instruction must be the language in which people expressed their natural thoughts. Hence his intention to simplify the Bengali language. He could not tolerate convoluted writing in which verbosity dominated over content. He realized long ago that a foreign or artificial language could never be effective in bringing education to reach the mind of every individual.

Most of all he detested laziness. Indian lethargy used to infuriate him so much that at times he lost patience and even said that wicked action was better than laziness. He had within him a source of infinite energy which ever sought means of expression. Throughout his life this indomitable energy drove him round the world. And everywhere he went no other person of his times was better equipped than him to shake our people out of their slumber.

It is our misfortune that like another Vedantic sage Sankaracharya, Vivekananda also died prematurely. Just as in his short span of life Sankaracharya travelled the length and breadth of India attempting to breathe a new life force into the Indians, similarly in the nineteenth century Swamiji toured like a hurricane throughout India and the western countries preaching the new gospel of the concord of all religions — the message of Ramakrishna. It was this new religious message, so devoid of dogma, that took the people by storm at Chicago and later elsewhere in the West.

In Vivekananda we find an unprecedented synthesis of the scientific temper and spirituality. India needed this sort of education in his times. The need for such education persists even today. In the thirty-nine years of his life Vivekananda could only partially achieve the task he had set for himself. After he passed away, his disciples derived their inspiration from his ideals and dedicated their lives to this cause. Perhaps a major part of that task stands completed today. But we are now faced with new problems arising from unforeseen quarters. For every nation in every situation there is always the need for strong men with strong wills. In our present period of crisis all the more specially we recall Vivekananda. Today if we fail to regenerate his ideals, then it is pointless for us to feel proud of his heritage. The observance of his birth centenary becomes meaningful only in the revival of his ideal. Vivekananda said: Plunge into work like an avalanche. Only if we can dedicate ourselves to his ideals of vigour and sacrifice can we really honour him.

Published in Bengali for the first time from his original manuscript, in Rachanā Sankalan; *obviously written in 1963, on the occasion of the centenary of the birth of Swami Vivekananda (1863-1902), saint and missionary, founder of the Ramakrishna Mission. Translated.*

8

Netaji

Nazrul [Quazi Nazrul Islam (1898-1976), eminent Bengali poet, with strong radical, bohemian strain] sings of crossing 'insurmountable peaks and stretches of desert' in order to reach our destination. But the aim of the mariner must remain fixed. At one time Subhaschandra Bose had wanted to achieve the impossible — to strike at the roots of the century old British rule. He took advantage of the time when in the Second World War Great Britain and the Allied forces were involved in a combat with Hitler. He eluded the strict surveillance of the security, fled the country and crossed thousands of miles clandestinely. We have come to know a lot about that now, though the details are not yet entirely clear — particularly, the way in which he reached Europe and forged an alliance with Hitler and Mussolini is shrouded in mystery. All the people of Europe did not accept Hitler's dictatorship. Interested readers can go through the diary of Romain Rolland where he has recorded his conversation with Subhas in this regard.

The Congress in this country was then following the Gandhian policy. Everybody thought that the British empire would be crippled through non-cooperation during the course of the war. The communists, on the other hand, were vocal against Hitler's dictatorship. This led to their differences in opinion and bandying of accusations with the followers of the Congress — a fight which has not yet ceased.

Subhas wanted to forge an alliance with Japan. Before that one heard of Rashbehari Bose compaigning in favour of this. He thought Japan would play a significant role as the future saviour of Asia. Subhas arrived in Singapore, and within a short time gathered together an army of Indian soldiers, leading them to a confrontation with the British and finally managing to land in India and cross Kohima — all these will remain

immortalized in the annals of Indian history.

Through centuries foreign invaders have crossed the boundaries and attacked India and weakened still further the warring powers in this country involved in internal feuds. Time and again the entire nation has been subjected to the humiliation of loss of freedom. This was the first time that a son of India had dared to stand up against an extremely powerful enemy, to break the shackles of mother India. His act will be remembered for ever, though what happened after that remains unknown. This brave son of India appeared like a comet and disappeared mysteriously. There are many stories about what happened to him but most of it is mere surmise. My feeling is that Subhas would have come back if he were alive — it is contrary to his nature to be in hiding only to avoid persecution.

A statue of Gandhiji in an advancing posture has been put up in Calcutta. A bronze statue of Netaji has been installed at the five-point crossing at Shyambazar. Both symbolize fearless adventure — I was telling a friend that they seem like pilgrims on their march. Subhas' statue is an incarnation of the very spirit of a self-sacrificing saint. I do not know if these two leaders have any true followers any more. Most of the leaders who are so voluble now have forgotten the ideals of Gandhi and Subhas. We have heard many empty slogans like democracy and universalism. In this old age when I venture into the streets, I find different slogans written in red and black. To me everything looks blurred — my sight is dimmed, and I can no longer perceive the future of the country.

Published in Bengali for the first time in Rachanā Sankalan, *from undated manuscript. Translated.*

9

Kumar Haritkrishna Deb

Kumar Haritkrishna Deb expired suddenly on Friday, 22 July 1966. We had always derived inspiration and sympathy from him ever since the foundation of the Bangiya Vijnan Parishad. He had always been with us at the various ceremonies and meetings that the Parishad organized to popularize science over these many years. He was intimate with many employees of the Science College. We feel his death has deprived us of a sincere friend. He was born in the famous Raj family of Shovabazar in January 1894.

Although much of their old wealth and grandeur was gone, the family still retained an ambience of music and literary culture in their ancestral home. His grandfather, Upendrakrishna, had written social novels in Bengali and they were much appreciated at one time. Haritkrishna and other grandchildren, in their childhood, had listened to his many amusing stories. That is how probably Harit's sense of irony and humour was nourished and developed from childhood. His father, Ashimkrishna, had an excellent collection of books on the ancient history of many countries of which Harit made good use. He had formed his own new ideas about Greece, Rome, Arabia, India and Egypt. From time to time he would present his views to his friends in his salon, but he had no taste for the hard work and research required to earn a reputation among the cognoscenti. The Shovabazar house had a long tradition of private concerts of music and dance, and discussions on literature and drama. Ashimkrishna was a great connoisseur of music — he was an excellent harmonium player himself, and many eminent singers and musicians used to call on him and famous musicians occasionally held concerts in the house.

Born and bred in this cultural ambience, Haritkrishna could sing very well. He had sound training in Tappa, Thumri and Tagore songs. He had a reputation for being a connoisseur among his contemporaries.

He studied in the Scottish Churches College first and then went on to Presidency where he did his masters in English. He also attended law classes in the university, where Pramatha Chaudhury used to teach — Harit was one of his pupils.

Chaudhury had already published his articles under the pseudonym 'Birbal'. A controversy over colloquial versus chaste Bengali was raging in full swing. *Sabujpatra* was being published already. There were weekly literary meets at Chaudhury's Bright Street residence. Rabindranath was an occasional visitor, Dilip Roy and others participated in concerts. Many of the younger generation also frequented the house along with the mature, learned and famous personalities. Many of them later made names for themselves in various fields. Among the younger people Dhurjatiprasad was a great favourite of Pramatha Chaudhury; his friend Haritkrishna used to accompany him there. Mr Chaudhury wanted the younger generation to come forward, to express their own views about history, philosophy, poetry, science and so on in their essays. It is he who inspired Harit and some others to write in Bengali.

Around this time postgraduate classes in ancient Indian and world history were started in Calcutta. Devdutta Bhandarkar joined as the Carmichael Professor. The first batch of students included such people as Prabodh Bagchi and Noni Majumder. Haritkrishna, who had already studied some ancient history in the regular company of his father, joined them With the assistance of investigators he now delved into his own research but not for a degree. He went to Rakhal Banerjee, Haraprasad Shastri and Bhandarkar to acquire new knowledge and to master the fundamentals of the methodology. He stopped going to the law classes. He was curious to investigate the historical sources of many of the fascinating views held by his father. He started studying Sanskrit, Pali, Philology, the deciphering of old inscriptions on stones and copper and identification of old coins. He worked very hard and soon achieved success and fame. He came to new conclusions about Vikramaditya, Aśoka and Udayan. We find him discussing the ancient Indian method of reckoning time, astronomy, the Scythians and the Greeks, and seeking to determine the international character of *Pani-kapardi* and *Poulastya* as mentioned in the Vedas.

All his life he was busy with his own researches. During his last years he wanted to put some of his theories on a firm foundation and got involved in his studies and discussions with renewed vigour. However, it is a matter of great regret that various troubles and anxieties held him back — he could not finish the work. Handsome since his teens, he was courteous, well-mannered, and had a pleasant voice and a keen sense of humour — all these qualities had greatly endeared him to his friends. He had a natural ability to mix with all classes of people. He could be seen at the Hedua Swimming Club, the plays put up at the University, in the literary and musical gatherings of the Banga Sanskriti Sammelan, in the Kishore Kalyan Parishad and many such places. His grief-stricken friends recall today his pleasant manners, his easy sympathetic nature, his harmonium recitals, and his wit and humour.

He began his life studying science, logic and mathematics — later he came in contact with the *Sabujpatra* and *Parichay* groups and realized the stupendous need to popularize science through the mother tongue. That is why he had always been such a sincere friend of our Vijnan Parishad.

He was a bachelor and served the goddess of learning all his life. He never compromised his ideals for the sake of money. Totally unattended, he breathed his last on 22 July on the balcony of his ancestral home in Shovabazar.

Published originally in Bengali in Jnan O Vijnan, *August 1966, and included in* Rachanā Sankalan. *Translated.*

10

Prabodhchandra Bagchi

I first met Dr Prabodhchandra Bagchi in Paris in 1924. His name was familiar to me. I had heard of him as a brilliant student. But I met him for the first time abroad. Soon our acquaintance developed into a friendship that lasted until the end.

I went abroad for the first time in 1924 with the support of the University of Dhaka. I was worried about where to stay, whom to meet and how to adjust myself to the ways of a foreign land. I set out with many such questions in mind. But as soon as I reached Paris, all my problems were resolved. I found help and friendship when I was introduced to Dr Bagchi. He had been living in Paris for a while. He had not received his doctorate yet and was working with Professor Sylvain Levi. He was around twenty-six then and I was approaching thirty. Having found a friend more or less my own age, I felt at home.

The Indian students in Paris had a club called the Indian Students Association. Dr Bagchi was its Secretary. It sheltered Indian students who were branded as terrorists by the British Government. It had branches in various European cities with its headquarters in Paris, at 17 Rue de Sommerard. We lived in this house and had our meals together. So our friendship matured fairly quickly.

Dr Bagchi was a favourite student of Professor Levi. Both Professor and Madame Levi had great affection for him and referred to him as their own son. They were deeply involved in research in Indology. We watched them from a distance, for although I had interest in the subject, I had no knowledge of it. My own subject was science — physics.

With science for my subject, it was only natural for me to be drawn to scientists. I had a great urge to meet the famous scientists of the time. But I could hardly approach

them. Dr Bagchi came to my help. He introduced me to Professor Levi who gave me a letter of introduction to Madame Curie. That was how I met her. Later I went to Germany and met Einstein with whom, of course, I had been corresponding from India.

I came back to India before Dr Bagchi did. He brought back with him some books I had left behind.

Back home, we had occasion to meet a number of times. The more I saw him, the more grew my admiration for his integrity and scholarship.

Then I moved to Dhaka. I continued to meet him during my occasional visits to Calcutta. I found evidence of his deep knowledge of Buddhist and Yoga scriptures, and the intricacies of Tantra. From my discussions with him I felt he had a profound understanding of these subjects. The many books he wrote to disseminate the knowledge he acquired from his researches have an enduring value.

He was a simple, kind and liberal person. He always came forward to help Indian students when they had difficulties abroad, and found money for them. There was no end to his earnestness that Indians should go abroad to acquire knowledge. He had collected money from the Indian business community abroad, and established a fund.

I recall his farsightedness. The relationship between India and China, built over ages, had a temporary setback. Not only had he not forgotten this kinship between India and China; he actually took the initiative to re-establish the ties. He was the first to edit a Sanskrit-Chinese Dictionary which went a long way in rebuilding the ancient relationship.

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11

Sisirkumar Mitra

Arrangements for teaching the various disciplines of science have now been made at the University College of Science on Acharya Prafullachandra Road (Upper Circular Road). Initially when this three-storeyed building was established, there was a general feeling that it would provide facilities only for research and higher studies in chemistry. This was still the notion circulating in scientific circles when Acharya Ray began his research in this building after retiring from Presidency College. Of course, there was the directive that the donations which the University had received from Taraknath Palit and Rashbehari Ghose should be utilized in teaching the several disciplines of science. The University also wanted that. But it was the troubled times of the World War. Hence many of the professors who were selected for the various disciplines of science could not join. Professor Raman was still working in the government department. Dr Debendramohan Bose and Professor Agharkar were interned in Germany. Yet Sir Asutosh decided that classes in different science subjects should be started without delay. Ways were found to overcome the lack of apparatus. Some instruments were acquired from the National Council for Education and a few were brought from Krishnanath College in Baharampur. There were also some sophisticated instruments at Sibpur, which were bought long ago but were not required for general education. The German Professor Brühl had wanted the native students to be trained in intricate measurement work. So the theodolite, balance etc. that he had acquired evoked awe and wonder among the postgraduate candidates at examination times those days.

Through the efforts of Sir Asutosh a new system of teaching was introduced in the University in 1916. Physics classes also began. He appointed a few young degree

holders as lecturers and started the postgraduate classes in 1917. Dr Sisirkumar Mitra came and joined then. Sir C V Raman was still devoting all his spare time to research at the Indian Association at 210 Bowbazar Street after his official work in the government department. His enthusiasm brought hope for many. Many dared to get involved in higher research. So they started doing research at the Indian Association under Professor Raman.

Meanwhile Dr Ganesh Prasad had joined the University as Rashbehari Ghose Professor of higher mathematics. To assist Sir Asutosh he engaged the students in research. Almost all of them obtained their doctoral degrees and many from this group later became teachers at the University Science College.

Both Dr Sisirkumar Mitra and Phanibhusan Ghosh [actually P N Ghosh] were engaged in investigating how light waves, when obstructed, scatter and create a wonderful ambience of light and shadow in the darkness. Dr Mitra's work at the Indian Association was in this direction. Dr Mitra used to study and experiment on how light would diffract on being incident on the wavy edges of the one-anna and two-anna coins of the time and enter the shadowy region as tiny wavelets to create the wonderful ambience. The young scientists tried to understand the new facts revealed by these experiments with the help of Maxwell's electromagnetic wave theory. This was a totally new kind of work. Acharya Jagadischandra Bose had just moved away from physics and was engrossed in unravelling the mystery of the varied manifestations of life in the world of plants. Hence he did not have much time for physicists. Professor Raman ushered in a new era by charting out the path for the more ambitious students of Calcutta University.

Postgraduate teaching began in the Science College in a routine manner. After a while Dr Raman joined as the Palit Professor. But he also continued his own research work at the Indian Association. His research students always worked in Bowbazar even though all the modern instruments had been bought with money from the Palit Fund. After a while Dr Raman's earnest efforts were successful and he won scientific laurels. He was awarded the Nobel Prize for discovering the effect that bears his name. Following his footsteps, Dr Mitra and other young students continued both research and teaching.

After some time, Dr Mitra went to France and started a new kind of work. In the post-war period, the development of sophisticated electronic valves made it easier to send electrical signals. It was during the First World War that for the first time it was discovered that messages can be transmitted to relatively distant places by creating small vibrations in ether. Earlier scientists believed that powerful transmitters would be required to send messages across thousands of miles. Now it was seen that messages could indeed be sent to far-off places with the help of tiny waves, and they could be amplified to be easily audible by means of electronic valves. Mathematicians had proved that the waves generated in research laboratories located on the curved surface of the earth could not go very far because of the special shape of the earth. Scientists now realized that these small waves travel in a different way. This is how the upper layers of the ionosphere conceived by Heavyside and Appleton attracted notice. The

tiny waves first travel upwards and then revert back towards the earth after hitting the ionosphere. The ionosphere then acts almost like a mirror. The scientists realized that passage through the enormous atmosphere does not diminish the energy of these waves, making the transmission of messages possible. Dr Mitra was one of those scientists and the first Indian who came forward to work in this newly discovered area.

It was through his efforts that a few attractive programmes were broadcast from the Science College even before Akashvani [All India Radio] came into being. There should be many people who would recall these programmes.

After his return from France Dr Mitra devoted himself to the task of building a radiophysics department and found many enthusiastic students around him. Later after Dr Raman left he became the Rashbehari Ghose Professor and continued his research in several directions. Rakshit [Hrishikesh Rakshit], Bhar [Jatindranath Bhar] and some other successful students who were trained by him became famous working in radiophysics. Dr Mitra carried out extensive research on how the ionosphere was formed in the upper atmosphere. Collaborative work by the Indian scientists led to the publication of an authoritative book on the ionosphere for which Dr Mitra won wide acclaim from different countries.

For a long time Dr Mitra had been trying to establish a full-fledged department of radiophysics and electronics. The government as well as the University authorities realized in the post-independence period that this subject ought to have a separate department rather than be a part of the physics department. As a result, a new research centre was created within the University's Science College campus. The work that Dr Mitra initiated there is being developed further and used in different spheres.

His lifelong endeavour resulted in the establishment of an active department. His students are working hard to put his plans into practice. Today he is no more with us. But his lifelong devotion to science will serve as an illuminating guide and enlighten all.

Dr Mitra was one of the few accomplished writers of his time who popularized science in the Bengali language. He had very cordial and friendly relations with the Bangiya Vijnan Parishad since its inception. Tributes to his memory were paid in a special issue of the journal *Jnan o Vijnan.*

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12

In Memoriam : Prasantachandra Mahalanobis

Since this morning I have been feeling rather weak. Yesterday I ventured out, ignoring the doctor's advice. With the stabs of pain that I had to endure, I don't feel confident any more. But I can hardly ignore your kind invitation which has prompted me to write something about my dear friend, the late Prasantachandra. I hope somebody in the meeting will read it out and help me honour my commitment at least partially.

We were more or less the same age, Prasantachandra being older by six or seven months. But we came to know each other only after our apprenticeship in education was over and we had chosen our careers. It was the high tide of nationalism. Most of those who won the hallmark of the university as the best students of our batch rushed to the laboratory of Acharya Jagadis Bose or of Acharya Ray. We persuaded Sir Asutosh to start postgraduate classes at the Science College, although it was nearly impossible to import any equipment during the First World War. We went on a hunt to laboratories where expensive equipment lay idle. Sir Asutosh was kind enough to procure them for us, and the young men started work in real earnest.

Prasanta had an advantage. He had come back from Cambridge, crowned with honours during the war. He came with the idea of going back to Britain to do research in physics and mathematics for a while. But it so happened that Presidency College was looking for good teachers; there were no expatriates coming. So the government got hold of him and offered him a professorship. Prasanta got busy with many new problems here. He was fascinated by the latest methods of science. It set him thinking about whether statistical methods could throw light on many of our long-standing problems. He started applying scientific methods. He felt that in spite of caste divisions

between the Brahmans and the Shudras in Hindu society and the fear of crossbreeding enjoined by Manu, statistically speaking the upper caste Bengalis seemed to come from more or less the same strata of population. According to statistical theory their ancestors must be regarded as belonging to the same anthropological group. To determine this, lots of measurements were taken — various characteristics of the skull, the lengths of limbs and so on.

Meanwhile he went on with his physics teaching at Presidency College. Later, as was customary, he also had to head the Alipore Meteorological Office. Mahalanobis collected a band of able students seriously interested in the study and application of statistical methods. Many of them later became famous as leading statisticians. The credit for this Indian achievement goes to Prasanta. He convinced the foreign rulers of its utility — for economic development of the country it was necessary first to have a clear idea of our predicament and the darkness that surrounds us. To accomplish this, he made arrangements to collect data from all over the country and was able to convince the government that in estimating the effects of introducing special measures, the statistical method afforded the only evidence, and made arrangements for the teaching of the new science.

It all started during the British rule — in Suhrawardy's [H S Suhrawardy, 1893-1963, political leader and chief Minister of Bengal in 1946] time. After a series of developments the country became independent. Jawaharlal came and gave his blessings to this effort. The untiring efforts of Mahalanobis resulted in an elaborate centre for teaching and research around Amrapali. Soon it had branches all over India like a banyan tree. Other institutions came up in other provinces. Today ISI has branches all over India. The experts who sit on the Planning Commission to advise the government on how to implement its future plans, in accordance with the Mahalanobis plan, had recourse to the advice and resources of this network.

Saha and I taught pure physics to start with. The war was over. News of the latest developments in physics — the latest results of the theory of relativity and the quantum theory published during the war by Planck, Einstein, Bohr — now household names, had only just started trickling in.

Dr Deben Bose had come back from Germany where he had been interned during the war and started working at 92 Upper Circular Road. I took German lessons from him and translated Einstein's new article on the curved path of light rays in a gravitational field. Dr Saha had already mastered German. He translated the purely mathematical formulations of relativity published by Minkowski. Prasantachandra also taught these subjects. He wrote a scholarly introduction. Our combined effort, a set of articles on relativity, was published as a book by the Calcutta University. It had a good market here and abroad. Probably it is no longer available.

Saha went to Allahabad, I moved to Dhaka. After a long career of teaching Prasanta retired from his government job. Later he introduced the teaching of statistics in the University. For twenty more years he went on serving the motherland. It was his firm conviction that only statistical measures could reveal to us the real nature of our problems and point to their solutions. His talent has been recognized throughout the

country and abroad. He travelled widely, and his wisdom has been acknowledged by international bodies.

He thought about statistics until the last day of his life. When I went to see him before he was admitted to the nursing home, he spoke to me about management problems in the ISI and turned to a new and easy method to compare various economic models — how easily one could make out their merits or demerits. But he was still not very sure. He sought the cooperation of his colleagues from his death bed. I heard later that his colleagues have obtained the results that he had anticipated, following his ideas, and that they will be published shortly.

I have been talking mostly about his scientific contributions. Everybody knows about his long and close association with Gurudeva Rabindranath, and his travels with the poet. When Gurudeva met Einstein, Prasanta was with him. He was Secretary of Visva-Bharati for several years at its inception; he withdrew when Rathindranath took over. Prasanta always remained concerned about every effort beneficial to the country. He showered enthusiastic praise whenever talent was honoured. He was a great admirer of Acharya Brajendra Seal.

Many of his contemporaries would remember the help they received from him with gratitude. Financial help from him made Saha's trip to Europe possible. People like Rajchandra Bose or the late Samar Roy were his pupils. There are so many more examples one can recollect.

In his later years he had to spend a good deal of his time outside India, though he was firmly committed to the progress of science in India. He was elected President of the Indian Science Congress, and was its Treasurer for a long time.

One can worship the goddess of learning, but not many are fortunate enough to be able to express their talents in creating new avenues of employment for the masses in the country. When I think of my contemporaries Saha and Mahalanobis, and their great contributions, my head bows down in reverence. Bengalis and Indians now have received international recognition in the field of statistics — the ISI is a premier international organization. Calcutta's Amrapali is well known throughout Asia. I pay my respectful homage to the great man who has made the impossible possible in this way.

Read in the original Bengali at a memorial meeting for P C Mahalanobis held at the Brahmo Mandir, Calcutta, and published for the first time in Jnan O Vijnan, *December 1972; included in* Rachanā Sankalan, *under the title* Smritikathā *(lit. Memoirs). Translated.*

S N Bose : Miscellaneous Pieces : Science and Scientists

1

Galileo

Galileo was born on 15 February, 1564 in Pisa. His name is well known to scientists everywhere. Today, 400 years after his birth, his life and works are being discussed at meetings and seminars all over the world.

His family name was Galilei. His father was a master of mythological literature; he was also proficient in music and mathematics. He could play the lute quite well. He also wrote a few books on musicology. The thirteen year old Galileo first went to the Seminary of the Benedictine sect of Vallombrosa. He studied literature, logic and religion for two years there. But he had to leave the Seminary. His father said that his son's eyesight was poor, and too much study could be harmful. Perhaps, he was also afraid that the son might become an ascetic — then there would be nobody to look after the family after him. He was no longer well off. The son would have to make an effort to retrieve the family's lost prosperity. Today it is difficult to find out all about Galileo, but we know that he was very fond of music and art. Left to himself he would have perhaps become an artist. But that did not happen. In 1581 at the age of seventeen he went to Pisa University to study medicine. His father thought that there was a lot of money in the medical profession. At that time every university student had to study philosophy. It was an Artistotelian era when the Greek philosopher's positions were uncritically accepted by all concerned. All knowledge and science had their basis in his philosophy. However, since childhood Galileo was inclined to test things for himself, and for this reason he often got into arguments with other students. Sometimes he also got involved in debates with his teachers. His propensity for logical arguments was evident throughout his life and was the cause of great misery in old age. One wonders

how this workaholic turned to pure mathematics. The story goes that a family friend was a great mathematician. He was very famous then and everyone flocked to him. Galileo went to his house one day on some errand. The son of the ruler of Tuscany was then taking lessons from him. So Galileo had to wait at the door for quite some time — he listened attentively to the mathematical discourse. This triggered a radical change in his mind. He lost interest in medicine. The urge to study mathematics became so overpowering that he gave up his medical studies. He did not get a University degree.

For various reasons the family moved to Florence. His father did not have the means to let his son stay elsewhere for education. So Galileo went to Florence where he began to study mathematics and physics under the court scholar. So extraordinary was his perseverence that he managed to surpass his teacher within a very short time. He established himself as a mathematician and researcher. Soon his fame spread to other countries. By the time he was twenty-four he was engrossed in his subject, designing instruments and performing various experiments with them. Initially the budding scientist had to face financial hardship. He tried to meet his needs by offering tuitions, but one could not earn much from tuitions those days. But in 1588 we find him teaching at Pisa University. The income was a paltry 60 scudi which by today's reckoning would be equivalent to an annual income of Rs 900–1000. It was quite inadequate to make both ends meet. It was the age of joint families in Italy, like we have in India. Unfortunately, his father died in 1591, and Galileo became head of the family — he had to support his mother and two sisters. His younger brother, Michaelangelo (who probably spent most of his time with music) went to Poland and became an artist in the King's Court. His wife and seven children stayed back in Florence. Galileo had to look after them too. This is why we find that all his life Galileo was, on the one hand, magnanimous and concerned about others, and on the other hand, anxious to earn a lot of money. He tried his hand at business, solicited jobs in many places, and ran around and changed jobs though his heart yearned for Florence. He wanted to live there all his life. Anybody who knows Florence would understand why his artistic mind was so attached to this grand city.

The family was further impoverished after his father's death. He left his homeland Tuscany and went to Padua University in 1592. It is here that his life as a real scientist began. Of course, pressure of work also increased considerably. Not only did he have to teach, he also became an advisor on different defence matters. But he could not forget Florence. He used to visit Florence every summer. The Duke's son, Cosmo, was his favourite student. His mother believed in applied astrology, in forecasts based on the Zodiac signs. From time to time Galileo also indulged in it to please her, although it is difficult to say if Galileo himself had any faith in it. He had deep faith in the Copernican system of the universe. Of course, Ptolemy's ideas were still in vogue everywhere. All the stars and planets in the applied astrologer's Zodiac chart circled round the motionless earth — astrological analyses and calculations were done from the positions of planets within this Ptolemaic system. Galileo was, on the other hand, taken up with a new theory. He delivered lectures at Padua University in favour of Copernicus' theory.

Many came to hear these captivating lectures.

Eighteen years passed by in the same University. The Venetian government was very pleased with him. In 1604 his term as a professor was extended by another six years. No one had then opposed his revolutionary ideas.

A new development took place in 1609. While fiddling with lenses, a man in Holland had put two of them on two sides of a tube and noticed that distant objects looked larger and seemed to come closer. When Galileo heard of it, he sketched plans on paper and began analysing the path of light rays. The problem was soon solved. He was also able to build a telescope which was better and more powerful. Everything appeared upside down in the Dutch man's telescope. But through Galileo's instrument everything appeared the right way up — there were no inversions. This endeared him more to the Venetian authorities. The merchant navy of Venice then roamed the seas, collecting merchandise from many countries and trading in them at European markets. This led to great prosperity. The wealth of Venice was then unbelievable — it was like a fairy tale city. From time to time the merchant navy had to protect itself against enemy attacks. It was easier to prepare for defence if the enemy could be sighted in advance. So, the authorities decided to equip all ships with these telescopes. Galileo was required to supply these telescopes. He agreed and his house was soon turned into a workshop. Hoards of telescopes were sold from there and he devised improved and more powerful instruments.

Galileo now had the right instrument to survey the universe. He turned it towards the sky and witnessed many new sights that had been beyond man's imagination till then. His eyes caught the sight of lunar mountains, the assembly of millions of stars in the Milky Way as well as many new satellites. The earth has only one moon that goes round it. Galileo found four satellites circling the planet Jupiter. The religious pundits of the time were not prepared to believe all this. They thought that Galileo was doing injustice by collecting evidence in favour of the Copernican system. They claimed that the pictures of the moons of Jupiter were cast on the telescope by some magic; if what appears to be substantiated by an instrument cannot be seen, it must be a trick of the instrument. The religious community did not change their opinion and even refused to look through the telescope lest their faith was swayed. Galileo felt amused by this. When a high priest who did not want to make use of a telescope and did not believe in Jupiter's satellites, died, Galileo mockingly said that the priest would perhaps notice the 'moons' on his way to heaven. Galileo had become a celebrity in many countries. He also received a lot of money from the Venetian government. But so burdened was he with work that he did not have a moment to spare for science. Many new ideas kept coming to him and he wanted to probe into many new areas, but he found little time to think more deeply about them. A lot of money was still needed for the family. So, when the old Duke of Tuscany died in 1609 and his student, Cosmo, inherited the dukedom, Galileo thought that if he could go to him, he would find both the desired leisure and money. He began secretly to solicit favour from the new Duke. A few lines from a letter written to a friend in Florence at that time are quoted below:

I don't think I would get more leisure to do my work if I were to move from

> here to elsewhere, because I will have to earn my living by giving lectures. For many reasons I don't feel like teaching in any city other than Padua. Yet, my work will not progress if I don't get respite.
>
> There is democracy in Venice, and however broad-minded or magnanimous they may be, it is futile to expect any grant from them without performing routine duties. I have to continue to lecture and do my research as long as possible — that is what the people here want. If I accept a salary, I won't have leisure. In other words, the leisure and financial support I am looking for can only be had from a king of an independent country.

In another place he wrote, 'I am able to invent many new things every day. I would be able to do more experiments and make more discoveries if only I had the leisure and support.'

He negotiated for a year with the trusted ministers and officials of the Duke. At last in the autumn of 1610, the new Grand Duke of Tuscany engaged his old tutor on an annual salary of 1000 scudi. In addition, he was honoured with a gold medal as court scholar and philosopher. Galileo left Padua and went to Florence.

Now he had plenty of leisure to pursue his scientific studies. But the new theories he propounded about astronomy in particular, created a big stir among the European pundits. Many began to oppose him. Moreover, for a different reason his discoveries and theories did not remain confined within the academia. He adopted a new method of propagating his ideas among the educated public. He gave up writing in Latin, the language of the elite, and began to write about his theories and discoveries in Italian so that all literate Italians could read them. He wrote in a letter in May 1612 :

> I find that young people are attending the Universities and are becoming doctors, philosophers or something else — as though any sort of degree would do. Then they land themselves with jobs for which they are total misfits. On the other hand, those who are really qualified cannot cultivate knowledge while remaining engrossed in work and in daily worries. They are talented but they do not understand Latin. Hence throughout life they bear the deep-seated belief that the huge books which are repositories of vast knowledge will forever remain beyond their reach. But I would like them to realize that nature has given eyes to man to observe its activities and also intelligence so that everyone can understand their essence and use them for their own purpose.

Galileo made many new discoveries with his own telescope — the lunar mountain ranges, the satellites of Jupiter, sunspots, the phases of Venus like those of the moon, the rings of Saturn and so on. This way he could see with his own eyes many characteristics of the planets, which anyone could verify with the help of a telescope. The Copernican theory appeared indisputable to him. Being a rationalist, Galileo thought that he could win everybody over to his side by making all these facts known. He therefore also wrote a book on the subject. Despite this, the orthodoxy continued to oppose him. On the one side there were the Dominican priests of Florence, and on the

other side were the University teachers and students who did not either agree with these theories or were jealous of his fame. In the beginning, Galileo used to make fun of his colleagues' attitudes. But this provoked more animosity. The priests began to proclaim that Galileo's teachings were contrary to religious beliefs and directly contradicted many teachings of the Bible. They waged a secret campaign, alleging that Galileo was spreading anti-religious sentiments and was trying to destroy people's faith in the Bible.

Conspiracy against him began to brew secretly. First, the Inquisition ruled that the idea of the sun being the centre of the universe was illogical and contrary to religious doctrines because this idea did not tally with many parts of the Bible which priests and scholars had been teaching so long. At the same time they also declared that the idea of the earth's annual and diurnal motions was contrary to religious belief. In March 1616 they banned Copernicus' book and two others on the same subject. They also conveyed this news to the holy Pope

The Pope ordered that Cardinal Bellarmine should summon Galileo and persuade him to give up his mistaken belief. And, if he did not comply, he should be ordered, in keeping with rules, to stop propagating and discussing his theory. If he refused to obey, he should be put behind bars. Galileo was summoned to Rome in 1616. Bellarmine was a well-wisher and friend. Galileo had by then earned fame for many of his new discoveries beyond astronomy. He was thinking about the hydrostatics of floating objects. He had also begun saying new things about kinematics. Cardinal Bellarmine called Galileo to his own palace. He tried to persuade him not to argue with the priests about the Copernican theory or quote passages from the Bible and try to explain them in his own way. Galileo agreed, but he thought that it would still be possible to discuss Copernicus' theory as a mathematical idea or that the merits and demerits of Copernicus' and Ptolemy's world systems could still be logically debated. Hence even after that he wrote a few other books on science — about kinematics and hydrostatics — and at the same time sought permission to write and publish a book about the two systems in the form of a dialogue.

But many things had happened by then. Both the Pope and Bellarmine were dead. There was a new Pope. At one stage Galileo had thought that the new Pope admired science and would grant permission to publish his book. But what happened was just the contrary. He had earned the wrath of the Pope for various reasons. Investigations were started against him. Galileo was finally summoned and imprisoned on 12 April. He was not allowed to meet friends. On 30 April Galileo was compelled to confess that whatever he had written in the form of a dialogue was proof of his ignorance, vanity and indiscretion.

This was not the end of his persecution. He was forced to recant his faith in the Copernican theory. He kneeled before the judges in the white penitential robe. The judges said : 'Your mistakes have done great harm to the country. You must be punished for it, and your book will be banned. You will remain imprisoned by our order as long as we want. Besides, you will have to offer prayer in penitence once every week for three years.' Two days later, the Inquisition sent him to the Embassy in Florence. At

first he was interned in Siena under the observation of the Archbishop. Then he was interned in his own house in a suburb of Florence.

The last nine years of Galileo's life were spent in great misery. Even then he tried to think about new things in science but life had become distasteful. The girl who looked after him also died in the middle of this misery. He was turning blind. The restrictions were slightly relaxed during the last five years, by the grace of the Pope. People from many countries used to come to meet him, for his books and writings had been pirated to other countries and published. He had a reputation and some sympathy among the Christians who were not Roman Catholics. Finally, he died on 8 July, 1642 at the age of seventy-seven.

It is said that when he stood up from his kneeling position before the judges of the Inquisition, he had whispered, 'Despite this the earth still moves!' But this may well be a story. The religious orthodoxy tried to gag and suppress science as usual. As a result, it was Italy which lagged behind other countries. Galileo's lifelong endeavours bore good results in France, England and in other countries.

Galileo had tried at first to propagate the tested truths of science against fallacious traditional theories. He counselled that one must examine, judge and test all facts oneself — it would be folly to build one's life on presumptive truths. As a result, he got crushed under the grindstone of superstition and religious bigotry. But that did not stop human progress.

The tributes of countless men four hundred years after his birth proclaim the victory of that truth. May truth triumph.

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2

Einstein (1935)

The paper in which Einstein introduced the theory of relativity thirty years ago has brought about a new age in physics. The main point of this new theory is that the canvas of space and time which a scientist conceives in order to describe the events of the material world and determine their causal relations, is not independent of the observer, and that there is a close and invariant relationship between the properties of the conceived space-time canvas and the velocity of the observer. Although the theory may not sound very new to a philosopher, that physics remains a science even after one has accepted such a position is not quite self-evident or axiomatic. Hence no scientist before Einstein had thought it possible. The belief that it is possible to measure distances and time intervals in a scientific and objective manner, in total disregard of the observer, and that based on such measurements and with the help of mathematics it is possible to predict the positions and velocities of material objects as well as the results of events, lies at the foundation of science. Hence Newton and the other masters laid the foundation of physics on the assumption that the properties of space-time and their measures were objective and absolute. They had recourse to only Euclid's geometry to measure distances. They believed without a shred of doubt that the measuring rod was the same for all observers. That there could be changes in it brought about by alterations in its motion or position was beyond their ken.

They were equally convinced about the invariance of the time marked by a clock. The objectivity of space and time was treated as an axiom in the mechanics of the Principia and the theory of gravitation. The mathematics and physics that were built on the view that the canvas of space-time was the same for all observers and that its

properties were not affected by the state of the observer, were accepted by all for two centuries. With improvements in instruments the field of application of science expanded, and there continued constant checking as to whether calculations and observations matched in every sphere. As a result, it became impossible for the old science to reconcile the minute discrepancies that began to be noticed between predictions and observations.

Towards the end of the nineteenth century it became clear to everyone from the discussions and experiments concerning the effect of the observer's motion on light waves, that there was an inconsistency between the old theory and direct experience. Einstein first proposed the theory of relativity to remove this inconsistency. Earlier it was the ether, the carrier of light waves, and the frequencies of its vibrations that were regarded by scientists as the embodiment of Newton's absolute space-time. Einstein was able to explain very simply and beautifully that this idea was wrong and responsible for all the inconsistencies of the theory of light.

In order to demonstrate that science was possible even if one relinquished the absolute character of space-time, Einstein in his first paper began his examination of the principle of relativity by introducing observers in uniform relative motion. He did not take into account the effects of gravity at this stage. Until then no effect of gravity on light had been discovered. As a result, although the sphere of applicability of the relativity principle was restricted as above, the reconciliation that was required was achieved. Moreover, impartial scientists also became convinced that it was possible to build a science based on such a new principle. However, Einstein did not rest content with his success. After ten more years of relentless effort he was able considerably to extend the applicability of the relativity principle and lay the foundation of a new mathematical theory of gravitation which rivalled Newton's theory. Relativity did not any longer remain confined within the limits mentioned above. Einstein was able to show that it was valid for all kinds of variable motion.

In order to understand the difference between the new theory and that of Newton, we have to recall certain basic concepts of the theory of gravitation. Newton advocated the theory of gravitation in order to explain the variety of motion of material bodies. No matter how great the distance is between any two material objects, according to Newton there is a force of attraction between them. The effect of such a gravitational force varies as the inverse square of the distance. But in his theory there is no room for a limitless material medium to transmit the influence of one body on another. In other words, unlike optics the theory of gravitation is not a wave theory. Many found the idea that material objects could influence one another across distances even while remaining detached from one another in every way, mysterious and difficult to understand. It seems Newton himself regarded his theory to be incomplete in this respect. Newton's successors introduced a similar notion of attraction between electrically charged particles. But Faraday and Maxwell established with the help of experiments and logical arguments that in this case the influence spread in the form of waves from one object to the other. Although many found the idea of action-at-a-distance unsatisfactory, nobody before Einstein was able to propose an alternative theory. His theory

was able to determine the characteristics of the orbits of planets and satellites and the variations in their motion in a simple and beautiful manner. The remarkable success of this theory ushered in a new era of astronomy. Copernicus banished Ptolemy. The modern astronomers produced ample evidence that the effect of gravity spread far beyond the solar system and reigned unimpaired in the world of very distant stars.

One must remember that the mechanics consistent with the theory of gravitation was based on the concept of objective space-time. While trying to advocate the principle of relativity instead of this concept, Einstein found that there was no room for the Newtonian concept in his new theory. Consequently, there was a need for a different explanation of the motions of matter, planets and stars. The explanation that he found after ten years of labour is in a way incomprehensible without the help of higher mathematics. But he was able to give every natural phenomenon that had an explanation in terms of the old theory of gravitation a different explanation in the new theory. In order to do this Einstein has abandoned Euclid's geometry and adopted Riemann's perception of space. To express it mathematically, one has to say that the reason behind the diverse motions of material objects is the curvature of the space-time canvas conceived by an observer.

The concept of the universe in this theory is completely new and startling. In the old theory Euclidean geometry was the only perception of space. The universe was infinite in extent and unbounded in such a theory. It was impossible to conceive of its magnitude. In the theory of relativity, although the universe is admitted to be unbounded, it is no longer legitimate to call it infinite or unmeasureable. Once the curvature of space-time is admitted, according to Einstein it is no longer impossible to give a measure of its extent.

Like the motion of material objects, the motion of light is also affected by the curvature of space-time. Hence, the so-called effect of gravity on a ray of light is an indispensable part of Einstein's theory. Nearly all the predictions he made mathematically from this theory have been borne out by experiments. Einstein published the results of his new calculations during the First World War. Immediately after the truce English astronomers tested these conclusions during the solar eclipse of 1919 and announced their veracity. That was the beginning of Einstein's international fame. His ideas began to be discussed even in ordinary newspapers. Even laymen became curious and were eager to discuss the content of his relativity theory. As a result Einstein's name is now well known to scientists and non-scientists alike. There is no limit today to people's curiosity to find out about his life and personality.

Long before he became known to common people, his many-sided genius had amazed lovers of science. The significance of many a complicated problem of science is today easy to grasp as a result of his acute and uncommon insight. Each one of his investigations — on the light-quantum hypothesis, the scientific explanation of the ceaseless agitation of microscopic particles discovered by Brown, etc. — spearheads the victorious march of modern science. Einstein's position in the vanguard of all this is universally acknowledged today.

In order to understand the personality of such extraordinary scientists, it is necessary to take into consideration the post-World War I national and international political situation over the last few years. Those who have faith in the future of human civilization, those who regard human life as invaluable irrespective of race, those who are not averse to acknowledging the axiomatic right of every nation to live on this earth irrespective of colour or religion, were alarmed at the all-engulfing, cruel and frightening countenance of the last World War. After the Treaty of Versailles put out the fire that was about to destroy European civilization, true lovers of humanity in every country became anxious and eager to take steps to prevent its recurrence with a vengeance. This feeling lay at the root of international cooperation. Einstein attended the Geneva Convention with the idea and hope that with the efforts of educated and true lovers of humanity in every country, it should be possible to ensure enduring political peace — that for the welfare of the whole human race man's united efforts, knowledge and experience will ever remain engaged against disease, poverty, blind racial hatred, cruel social oppression and all evils. The history of the last few years has been cruel indeed. Those who dreamt of the end of chauvinism, savage warmongering, jealousy, oppression and imperialism and welcomed the international Geneva Convention as the harbinger of a great age of humane civilization, are having a difficult time.

In a state of frustration after many years of untiring hard work, Einstein refused to attend the Geneva Convention again in 1932. With the rise of Hitler in Germany and the beginning of anti-semitism in 1933, he left his country voluntarily, oblivious of any regard for his personal prestige, comfort or happiness. Like many Jews who held on to their religion, he is today an émigré. Even a scientist of his vision has been caught against his own will in the whirlpool of malice and hatred.

The World As I See It is an English translation of the German book *Mein Weltbild*, a collection of Einstein's lectures on various topics, essays and letters, which was published in 1933. The book has five sections separated subjectwise; the essays concerning science fill nearly half the book. The rest of the articles give us a glimpse of his philosophical position and his opinions on current international affairs and the problems of the Jews in Germany. Most of the essays were written at different times. As a result, the consistency of his opinions expressed in different contexts is not always clear, but there is no scope for misunderstanding the author from the simple and lucid style in which every article has been written. Even if the solutions he offers to overcome the current political and economic crises do not appear logical, compulsive or well-founded enough to all, yet the reverence for mankind that surfaces in his essays will touch and move everyone. The essays on science are the most valuable in the whole book. Among these, many essays on the different problems of modern physics, the history of the theory of relativity, the ideals of a scientist and the scientific method, etc. especially deserve the close attention of serious readers.

I have already given a short introduction to the theory of relativity. I will conclude this essay with a short account of Einstein's views on the nature and method of science. Although a complete understanding of the infinite beauty and pure consciousness that our minds can sense just a little and feebly in the boundless mysteries of creation, is

beyond the capacity of the human mind, its relentless pursuit is, according to him, the true sign of a scientific attitude. The scientist tries to comprehend at least partially the entire external world beyond the narrow limits of his personality. Hence he is ever busy building to his own satisfaction a mental representation of the external world reflecting its causal connections. He attempts to grasp the true nature of the external world by reaching beyond the sense impressions coloured by his personality with the help of such a causal representation. Since it is beyond the capacity of human intelligence to analyse every complicated law of nature, a scientist has to remain satisfied with the analyses and syntheses of the comparatively simpler events in that representational domain. It is essential that such analyses and syntheses be logical and correct. Hence a physicist has to use the methods and terminology of mathematics. Consequently, although incomplete, there is no room for errors, vagueness and uncertainties in the scientist's representational domain. Although it is beyond the capacity of the finite human mind to explain the totality of events in the world, Einstein believes that it is possible to find explanations of every material and organic event using the method of logical analysis. There is no logical method, however, of making a discovery within the method of logical analysis that a scientist follows to build his world-picture, starting from a few simple and universal principles. They reveal themselves to the scientist's mind as a result of its intimate acquaintance with the events of the external world. Of course, we have to test the reasonableness and correctness of these principles by checking the predictions that follow from them against our experience.

Like Einstein old-fashioned scientists such as Newton also believed that it was possible to explain the events of the world, starting from a few fundamental axioms and using the analytical method. But many of them thought that it was possible for man to extract these axioms from particular natural experiences, using the method of induction by simple enumeration. They believed that the axioms were logically strung to experience. They also considered it impossible to arrive at an explanation of the universe by choosing the fundamental laws of nature freely.

They used to cite the examples of Euclid's geometry and Newtonian mechanics in support of this view. The theory of relativity proved this view of the scientists to be wrong. Although the axioms of the relativity theory were of a different nature from the concepts of the old mechanics, this theory was also able to provide satisfactory explanations of the same class of events. Therefore, there cannot be any invariable logical connection between the fundamental concepts and definitions on which science is built and man's experiences.

This view has however raised a new problem. If there is no invariable logical connection between the concepts and definitions which are linked together to build up the scientist's world-picture and man's experience, and if the right to choose the concepts freely is admitted, then would it be true to say that there is no relation whatsoever between the world imagined by the scientist and the external world? Is it then a futile attempt on the part of man to understand the real nature of the external world by means of a causal picture? Is utility for man then the final justification for science?

Einstein does not believe that. Einstein firmly believes that although there is no logical connection, the external world uniquely determines the concepts and axioms of our world-picture. He also believes that it is possible for man to discover these unique principles. He maintains that we will be able to grasp the true nature of the external world and the inherent and immutable relationships connecting events with the help of the comparatively simpler concepts of mathematics. But such concepts cannot be found by means of logic alone. We have to depend on our intuition for their discovery. We have to find out whether we have found the right concepts by comparing the predictions that follow from the union of the concepts that arise in our minds and seem probable as a result of our experiences, with the nature of the external world. The science that we create by assuming correct concepts and using mathematics is pure and free of inner contradictions. Though it is free of ambiguities, there is no immutable relation between it and the external world. Thus it is an empty construction. We have to ascribe appropriate correspondences between its concepts and the properties of external objects and see whether we are able to comprehend the events of the external world with the help of the theory.

Einstein has tried to explain with the help of his various attempts in the context of the relativity theory that it is neither futile nor a mere daydream to try and comprehend the nature of the external world following the above method.

It should be clear from the above discussion that like his predecessors, Einstein too believes in the independent existence of an external world and the possibility of a causal scientific description. Many have lost faith in causality as a result of the latest quantum theory in physics. Consequently, they consider the consideration of the probabilities of events rather than their inevitability as the proper aim of science. Although this latest theory is able to solve many complicated problems of the world of atoms and molecules, Einstein firmly believes that causality will be reinstated in the future in science. All his endeavours are entirely devoted to such a reinstatement.

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3

Einstein (1965)

All of you have surely heard the name of the great genius Einstein. His biography has been written in various languages and some have also been translated into Bengali. If you are interested, look for them and read them. Einstein was born on 14 March, 1879 in a Jewish family in the city of Ulm in Southern Germany. His father, Hermann, started his business with a small capital. After trying his luck in various places, he set up a small shop of electrical goods first in Munich and then in Milan, Italy. The income from it was meagre and gradually his savings were all gone to pay off the debts. Therefore when son Albert finished his school in Munich, the family found it beyond their means to bear the expenses of his higher studies in a German University. Einstein then applied for admission to the famous technical college in Zurich in Switzerland. His application was rejected the first time. He got excellent marks in mathematics in the entrance examination. But because of his inadequate knowledge of biology and foreign languages, he had to study these subjects in a Swiss school for one more year. Finally he was admitted in 1896 and studied in the Polytechnic for four years. He read what he liked in physics, like the original works of the great Maxwell, Boltzmann, Hertz and Kirchhoff and mastered them. He completed his studies at the Polytechnic in the year 1900. Initially, he had to do various odd jobs, like giving private tuitions, to earn a living. Eventually he got a job in the Patent Office in Berne in July 1902 on an annual salary of 3500 Swiss francs.

Writing about this small job, Einstein remarked towards the end of his life :

> This job at the beginning of my career seems to me now to have been a blessing in disguise. It proved to be of better value to me than teaching in

a school or college, because I had a lot of free time to think independently about different areas of science.

Young scientists of my age who got the opportunity to do scientific research in schools and colleges right after getting degrees had to remain occupied with inconsequential problems in order to produce quick results. The firmness of character and will power that are necessary to resist the temptation of easy success and to dedicate oneself wholeheartedly to the search of the fundamental and profound theories of science, are indeed rare among young scientists. So they get lost in small things.

Einstein devoted all his free time to the pursuit of science. The goddess of learning was pleased by his silent and devoted work for three long years. He published his first paper on the relativity theory in 1905 at the age of twenty-six. A certain problem in the theory of light was then baffling scientists all over the world. His original paper threw new light on the subject from a new standpoint and revolutionized the world of science. Veteran scientists like Poincare, Lorentz and Planck had the highest praise to offer to this budding young scientist. He could not live in the small city of Berne for long. The offer of a professorship first came from Prague. There his original papers on various subjects elevated him to the rank of world-famous scientists, and he received an invitation to teach in Berlin just before the First World War.

Einstein had many new things to say. He showed how it was possible to give a causal description of the events of the external world independent of the state of motion and position of the observer. He also said many novel things about the new standpoint that a scientist requires in order to investigate the world on the assumption that a description of the external world independent of the observer is possible. We have learnt from Einstein many new facts about matter, energy, force and gravitation, and following the path charted out by him, several scientists of the new era have progressed further and earned fame. The new hypothesis that Planck proposed in 1900 about the interaction of light and matter and about exchange of action-energy, has gone through various modifications and refinements to evolve into the modern quantum theory. In the papers that he wrote in support of his theory Einstein added a new element to Planck's idea — he introduced the idea of light-quanta or photons. With its help several mysteries were clearly and easily solved. Although it had become quite impossible to reject the wave theory and to think of light, as Newton did, as a collection of light corpuscles, Einstein thought deeply about how these completely contradictory properties could be present in light.

These new theories surfaced when we were still in college, but the First World War began before these theories got well circulated in our country. We learnt about these fascinating theories much later.

After discovering the relativity theory, he spent ten years thinking about gravity and how it could be placed in the context of his relativistic space-time theory. Finally, he explained gravity in a totally new way. However, scientists in different countries could not pay much attention to it because of the War. Einstein had predicted that according to his theory light rays would be slightly deflected when they graze past the

sun's surface. He even calculated the extent of this deflection. Germany was defeated in 1918, and peace was restored in the world. Then scientists the world over found time to test Einstein's theory of gravitation.

In 1919 a sensational news astounding all scientists appeared in newspapers all over the world. During the full solar eclipse that year a group of British scientists tried to find out whether light from a distant star got slightly deflected by the gravitational pull of the sun while passing close to it. The extent of the deflection was expected to be 1.75 second of arc according to Einstein's new theory. The scientists were amazed to find that the results of their observation almost fully supported Einstein's calculation. Einstein's fame spread all over the world from this event.

We had only just begun to teach at that time. The Calcutta University had started postgraduate classes in science. Three of us, Dr Meghnad Saha, Professor Prasanta Mahalanobis and I selected the main papers on the new theories and translated them from the original German into English, and the Calcutta University published them as a book. Initially our book was the only one in which English translations of the main papers on relativity were available. Other translations followed and our book is no longer available.

The Dacca University was established in 1921. I had to go there as a Reader in the Physics Department. I had to explain many new ideas to the higher classes. During that time I wrote a paper synthesizing the basic principles of Planck's theory and Einstein's light-quantum (photon) hypothesis. I wrote it in English and sent it to England for publication. Meanwhile, I was also curious to know Einstein's opinion of the new idea. So I also sent a copy of the unpublished paper to him. I did not then hope for any unexpected result — in fact, I had very little hope that I would be able to draw his attention to this matter at all.

Yet an amazing thing happened. Einstein sent his reply on a postcard within a few weeks of receiving my paper. He acknowledged that this helped to take his theory further. He added further that my views were so valuable that he would himself translate the paper into German and get it published. His translation was published (in 1924), long before anything was heard from London.

The possibility of my going abroad was then being discussed. Professor Einstein's letter put an end to the controversy within the Executive Committee of Dacca University regarding sending me abroad on deputation.

I got both leave and travel expenses from the University to go abroad for two years. I got my visa for Germany quite easily, thanks to the little postcard from Einstein. This letter opened up a new phase in my scientific career. Later when I arrived in Europe, I found that practically everybody had read and was discussing my paper. Einstein himself had applied my statistical method to material particles and extended the sphere of its application considerably. As long as I was in Germany, I had easy access to all the scientific institutions. In addition, like German professors I enjoyed the privilege of borrowing books from the government libraries and read them at home without depositing any money — all this was because of the guru's blessings.

I arrived in Berlin in 1925. The Einsteins then lived in a small flat on 5 Heberland

Strasse. I was fortunate to be able to visit him often and have long discussions with him on many matters. I will conclude by relating an interesting conversation I had with him one day. After the World War the Jews got an opportunity to establish a new State in the land of their ancestors. A new University was set up in Jerusalem at that time. Einstein went to deliver an address at this new and obscure University. He also went to Japan more or less at the same time. He had an invitation to visit India but for some reason could not come here. The Jewish State of Israel was then growing under British protection. Many thought that this link with the British would be entirely for the good of the Jews. Einstein also held the same view. On the other hand, many did not fully support the British policy. A lot of Muslim Arabs used to live in Palestine then. They did not welcome the arrival of the Jews. The British rulers sought to establish their domination on a divide and rule policy just as they did in our country. Everybody in India is quite familiar with this policy, and we are still suffering from its evil effects.

The time I am talking about was when the movement to drive the British out had already started in this country. The revolutionaries were making futile attempts to drive the British away by armed struggle, and the Congress was raising the slogan of non-cooperation. All these appeared rather strange to Einstein.

One day he confided to me : 'I think Englishmen are better than other Western colonial nations, and I feel that they are by far better than the French and the Dutch. You should not be surprised at a German like me praising the English (after the World War). Now, tell me, do you really want that the British should quit your country?' I said, 'Of course, we all want to determine our destinies ourselves.' He was not quite convinced. He raised a hypothetical question. He said, 'Suppose, there was a button near you and all the Englishmen would quit India if you were to press it. So, would you press that button?' I smiled and said, 'If God were to grant me such an opportunity, I will not hesitate even for a moment to press it.' 'Really?,' he said, and kept quiet for a while. I asked him, 'Well, why do you Jews then want to establish a new Israeli state? Even you seem to be fairly inclined in its favour.' He said, 'I have, of course, now understood what you are saying — it is an emotional matter and cannot be explained by logic.' Then, on my way back to India I met another world-famous man in France. He had visited our country. He said : why create yet another kind of nationalism led by all these emotions? It could only aggravate conflicts and unrest. The Jews will separate as a different race and establish an independent nation — it is difficult to say what the result will be. It was clear that opinions differed among great men. Of course, other French Jews did not agree with this French pundit. That was forty years ago.

Afterwards, both India and Israel became independent States almost at the same time. Unfortunately, the Englishman's divisive policy has sown the seeds of fatal racial conflict in the wombs of these two young States. Nobody can say what the final outcome will be.

I left Germany in the middle of 1926. Then anti-Semitism took a serious turn in Germany. Einstein had to leave his own land and take shelter in Princeton in America before the start of the Second World War. It was from there that he propagated his latest theory. He stood for world peace, and he prayed for it in unambiguous terms. His

immortal words about Gandhi will remain indelible in our history.

I was not destined to meet Einstein again. We corresponded from time to time. I was present in Berne in 1955 on the occasion of the Golden Jubilee of the relativity theory. But a few days before that, Einstein had suddenly expired in a hospital. The Berne celebration had to take place without him.

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4

In the Company of Madame Curie

It was October 1924. That was my first opportunity to visit Europe. The University of Dacca had granted me study leave for two years. I had arrived in Paris. A number of friends like Dr Mukherjee and Dr Bagchi who were already there insisted that I stay on with them. A short time ago the paper regarding photon statistics had been published in German. It had attracted the attention of scientists because Einstein himself had translated and published it with a word of appreciation. This had happened while I was in India. On my arrival in Europe I found myself welcome wherever I went. My first stop was Paris. I hoped to learn something new so that on my return I could be of some use to my students.

Paris had many sights worth seeing. I was keen to see them all. My friends came forward as guides.

Langevin, a student of Pierre Curie, was then the Director of the Municipal School where radium was discovered. He had read my paper and welcomed me warmly. He asked me what I wanted to learn, how long I wished to stay, etc. I went to Madame Curie with a letter of introduction from him. I wanted to join her Institute and learn to work with radioactivity. I was allowed entry to her small chamber. The great elderly lady sat there, dressed in black. I could recognize her from pictures of her that I had seen. I handed her the letter of introduction. She greeted me affectionately and said that there was no way she could disregard a recommendation from such a person. You will certainly get an opportunity to work with me, she said. But not right now, after three or four months. Get to know the language, otherwise you will find it difficult to work in the laboratory. You are in no hurry, I presume.

She spoke at ease in chaste English for about ten minutes. I had no opportunity to tell her that I knew a French of sorts already. I had been at it for the last ten years at home. I came away, resigned to carrying out her instructions. I was in Paris for the next five or six months and had a chance to work at the Radium Institute for some time.

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5

Jacques Hadamard, Mathematician

The world-famous French mathematician Jacques Hadamard passed away on 17 October, 1963. He was born in 1865. His father taught literature in a school. He often warned his son against taking up mathematics as it was a very abstruse subject. Hence, Jacques started studying mathematics a little late. But he displayed his extraordinary proficiency in his first competitive examination (1884). His marks in this examination surpassed all previous records. His record still remains unbroken in France. He was admitted to the Ecole Normale Supérieure which has always attracted the best students in France. Jacques's thesis in 1892 on the general properties of functions led to a specific direction in the subject. Within four years after his doctorate he solved a complicated problem of the theory of numbers. The French Institute awarded him a prize for this work.

Hadamard spent all his time in research and teaching. Until 1937 he was a professor in College-de-France of the Sorbonne University. His enduring contributions to various aspects of mathematics earned him worldwide recognition and acclaim.

He suffered immensely in his personal life. He lost two of his sons in the First World War. He expected much from one of them. He used to say that in comparison to his son, he was himself no mathematician at all. He lost another son in the Second World War. To avoid being persecuted as a Jew, he had to stay in America for four years (1940-44). Notwithstanding his misfortune and sorrow his intellect showed no sign of decline. Even at the age of ninety-one he wrote original papers. At the fag end of his life he lost his wife. A grandson also died in an accident. Despite all these afflictions, his patience was boundless. He had the enthusiasm of a young man and also the determination to

protest against injustice.

He used to say that one should never tolerate or fail to protest against injustice with one exception — when the person himself becomes the target of injustice.

During the many turbulent times in France, he never hesitated to speak out. He held the view that struggle against injustice was not hostility against an individual. It was a battle for justice, truth and idealism. His sterling qualities have been recognized the world over. His native Institute acclaimed his papers on many occasions. After Poincare's demise in 1912 he was elected a member of the Institute. On the occasion of its anniversary in 1962, the Institute awarded him a gold medal. His friends made a selection of his papers and published a book of four hundred pages to celebrate his seventieth birth anniversary. The Government honoured the old man of ninety with the 'Grand-croix de la legion d'honneur'.

He had toured many countries, even India on one occasion to participate in the Indian Science Congress. Many noted Indian scientists were his students. All through his life he sought truth. To him the aesthetics of science was primary and its application secondary. In a small booklet published in 1959 he attempted to analyse the psychological factors behind scientific discoveries.

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6

Otto Hahn : an obituary

Born 8 March 1879
at Frankfurt – on – Maine

Died 28 July 1968
at Göttingen

On the 28th of July died at the ripe old age of 89 years, Professor Otto Hahn, the celebrated discoverer of uranium fission which led afterwards to the development of the atom bombs which first fell over Hiroshima in August 1945.

Hahn himself had been deeply moved by this development — the first news of the havoc of death reached him in Godmanchester where he was then in detention. He had almost wanted to commit suicide at the thought that his discovery had such a destructive use and had killed so many innocent beings.

Hahn had begun his life as an organic chemist in Marburg; however, his professor Zincke sent him to Sir William Ramsay in London where he could stay for some time, do some work and pay for his stay, but principally learn English which was regarded then as an essential qualification for a German industrial chemist. Sir William Ramsay asked Hahn to purify pure radium compounds from an impure sample extracted from thorianite, a mineral rich in thorium found in Ceylon. Hahn discovered in it also radiothorium. This brought him name and fame and he decided to devote himself completely to the study of radioactivity. He then went to Canada to work with Sir Ernest Rutherford, who was at first sceptical about his discovery of radio-thorium. But he soon convinced Rutherford by his work in the laboratory, where he discovered another radioactive element — radio-actinium. He returned to Germany, got a modest accommodation in the cellar of the Chemical Institute as Emil Fischer's personal assistant

on the personal recommendation of Sir William Ramsay.

Radioactivity was not then much studied in Germany. In his modest corner Hahn discovered yet a third element, Mesothorium — a short-lived isotope of radium which was very much in vogue as a substitute for the much more expensive and rarer radium in hospitals and in industry. This brought him fame, and attracted more German workers in this new branch of science. Lise Meitner, then a young physicist from Vienna, came to work with him at that time. She remained his close associate and co-worker for thirty years and together they published many important papers. In 1918 Meitner and Hahn discovered Protoactinium (91), a long-lived element missing in Mendeleev's table.

After the discovery of the neutron and the claim of the Italian physicist, Enrico Fermi, to have obtained evidence of transuranium elements in the process of bombardment, Hahn and Meitner worked for five years on the subject. Towards the end of 1938 Meitner had to flee from Germany for fear of Nazi persecution, as she happened to be of Jewish origin. The work, completed six months after her flight, was published early in 1939. However, it was Lise Meitner who found the right explanation of the origin of barium, the presence of which was unquestionably established by the work of Hahn and Strassmann. Hahn continued his celebrated experiments in Berlin during the Second World War. Here he was concerned principally with the unravelling of a very complicated phenomenon with the modest means at his disposal. Hahn had noted with modest pride that he could unravel the presence of about a hundred radioactive isotopes in the process of encounter of neutrons with the nucleus of uranium.

In his book, *A Scientific Autobiography*, he has told in a charming way about his important work which had placed his name for ever among the immortals.

In 1945 he obtained the Nobel Prize for Chemistry which was announced in 1944 over the radio as he was till then a *detenu* in England, and his whereabouts were unknown.

He took over the direction of the old Kaiser Wilhelm Institute from Max Planck in 1946. Planck died soon after. From the depleted and miserable state of poverty, by a steady work of fifteen years, Hahn restored the Institute (now named Max-Planck Institute) to its former preeminent position in the world of science. He formally retired from the Presidentship of the Institute at the age of eighty, but remained its honorary President till the end of his long life. He never missed going to his office and was almost every morning at his President's desk at Göttingen.

On 8th March this year Hahn had an accident while getting down from his car in front of the Göttingen Institute. He was quickly removed to the hospital where he died after a short illness and of heart failure on Saturday morning of 28th July. Hahn was deeply respected by his countrymen, and was respected the whole world over for his firm pacifist stand against the utilization of atomic energy for destructive purposes. But he was all for development and progress — he believed in the future of mankind. I will conclude this account quoting his own words, which explain Hahn's attitude.

> I am not afraid of the future of this domain of work. With the utilization of energy within the nucleus of the atom we have obtained artificial radioac-

tive stuff — the so-called radio-nuclides. This process of atom-building is now in full development. New domains of application in pure research and industry are being steadily found every day. It is my earnest wish for the future that physicists would be able to work out a controlled fusion of hydrogen to helium. Then we would have the possibility of getting artificial elements without the utilization of the atomic reactor and Uranium 235, or the utilization of Plutonium 239, both of which had been used in the atomic bomb for destructive purposes. The reaction heat for fusion could perhaps be used for producing electric current exactly as the heat out of the atomic reactors.

We can then imagine a world in not a very distant future, where the inexhaustible supply of water in the seas will be utilized for atomic technology for the blessing of mankind. Up to the present moment, the technology has remained tied to the dangerous transformations of uranium in the reactor.

Hahn died a convinced believer in the utilization of atomic energy for peaceful purposes.

The whole world respects him; in West Germany the first ship manufactured for steering in the ocean with the utilization of atomic power has been named 'Otto Hahn'.

7

The Japanese Scientist Tomonaga

Dr Tomonaga has received the Nobel Prize for physics this year. His father, Zunjiro, was a distinguished professor of philosophy at Kyoto University where he taught for years. His book *Renaissance in Modern Japan* is still widely read in Japan. Dr Tomonaga's younger brother, Jojiro, is a professor of geography in a University. His uncle and maternal uncle were also teachers in Kyoto. The Tomonagas are well known for their scholarship in Japan.

He was born in Tokyo but moved to Kyoto with his family at an early age. Weak and gaunt, he suffered from chronic ill-health from his childhood. He has bright intelligent eyes set in a disproportionately large head, but he is all skin and bones. From his childhood he has been passionately fond of science. He read children's science magazines regularly and occasionally even contributed articles to them. Collecting plants and insects was his hobby. He was also skilful in making ships and other odds and ends from paper. When in class III in a Japanese high school, he was fascinated by physics. Professor Einstein happened to visit Japan at that time to give a series of lectures. Young Tomonaga attended all his lectures. That made him decide to devote all his life to physics. Shortly afterwards Nishina, a well known scientist, came back from Copenhagen where he had studied under Niels Bohr. In 1932 Nishina was a professor of physics at Kyoto. Yukawa and Tomonaga (both of them were to win the Nobel Prize later) were doing their research under him. There was also Sakata who has also become famous now. Yukawa published his paper on mesons in 1936. A joint paper by Nishina, Tomonaga and Sakata on the origin of electron-pairs was published in 1937. That was Tomonaga's introduction to the world of science. Thereafter he spent two years in

Germany (1938-40) with Heisenberg.

He used to spend his leisure in fun and frolic. He wrote a comic skit on his return from Germany and also acted in it. His interest in such theatricals remains undiminished even today.

Then the Second World War broke out. The war also engulfed the Pacific region. Tomonaga was kept busy in research for the development of radar and wireless equipment necessary for the war. Occasionally, on invitation from Fujioka, he visited Kyoku University in Tokyo to discuss various scientific problems. Japan was suffering from a serious shortage of food in those days — even researchers did not have enough food. Tomonaga became so weak that he often fainted while walking along the road. At last the war was over, but both his laboratories were ravaged by bombing. Tomonaga sat by the broken windows, engrossed in his thought, carrying on his research along his own ideas of atomic particles and their actions and reactions, and the role of many special time scales in such processes.

His ideas took a final shape in 1948. After some time the American scientists Feynman and Schwinger also published their paper in which they arrived at the same position and conclusion but using a different method. Their new ideas have found universal acceptance now and all three have been awarded the Nobel Prize. When one takes note of the fact that long after the war no scientific information or journals were available in Japan, and Tomonaga built his new theory in complete isolation from the world of science, the more one appreciates his erudition and creative brilliance.

In 1950 Tomonaga spent a year in Princeton on an invitation from R Oppenheimer. When he came back, he would jokingly tell his friends that he managed to have his teeth properly treated. I have already mentioned his love of theatricals. He could explain the mysteries of the quantum theory clearly to people with the help of metaphors.

One day Tomonaga said to Professor Nakamura : 'Somebody is searching for something under the street light. Have you lost something? Yes, says he, my keys! Where? Over there, in the dark. But it is difficult to find anything in the dark. So I am trying to find it under the light. This exactly describes our quantum theory today, doesn't it?'

Tomonaga is a wonderful storyteller. He has tried to popularize various aspects of modern science with the help of jokes. He is still engrossed with theatricals. Simple as he is, he spends his leisure entertaining his relatives and friends. To keep himself fit, he needs to go to bed early. So he takes an early dinner; his doctor has advised him twelve hours' sleep every day. Many make fun of him for his drinking habits. As soon as the news of his Nobel Prize arrived, the Science Academy sent him a few bottles of imported drinks at its own cost, with the inscription, 'For a distinguished connoisseur'.

He has no love for money. He is able to make both ends meet with whatever little he gets in addition to his fixed salary. He is perhaps not even aware of how much he earns. His wife (Riyoko) has to face all the problems. His sons go to school. One of his daughters got married recently. But Tomonaga is not bothered about all this. He played with them when they were kids. Now that they are grown up, they must fend for themselves — they have to do their own studies. His wife jokingly says — there is a

Japanese proverb : the cobbler's son goes bare-footed. Tomonaga is busy teaching others. Where is the time for him to think about the education of his own children?

Journalists had asked Mrs Tomonaga : what will you do with the Nobel prize money? She said, 'We have not thought about it yet. Maybe the entire amount will go to repay the house-building loan.'

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8

A Flash-back

Last monday (29 August) I had to visit the Physics Department of Jadavpur University. I saw Dr Chatterjee [Shyamadas Chatterjee] collecting rain water for his routine experiments. He said that he had been doing this for the last few years. Once the sky is flushed, drops of water of many kinds come down with the rain. Spectral analyses reveal the presence of radioactive particles.

We are now scared of the Chinese bogey. I have read in newspapers that they have exploded two or three atom bombs over the last few months. So I had a question for Chatterjee. After every explosion the upper atmosphere is expected to remain polluted with radioactive dust for a long time. The north wind is bound to disperse them over the Indian skies, and eventually they are bound to come down with the rain. Have these experiments at Jadavpur given us any information about the materials used in the bombs? Chatterjee said that an isotope of Uranium, U-235, was detected in the rainwater after the explosions. So there must be quite a lot of U-235 in the Chinese bombs. A very small proportion of U-235 is found in common Uranium. I understand that China has Uranium ores in many places. They have been able to extract a lot of U-235 with much effort and the correct use of advanced technology. This has enabled them to make atom bombs in China this year.

I came home thinking of many things. My mind went back to a week twenty-four years ago.

The Second World War was on full steam. The Japanese were advancing towards India. Singapore and Burma had fallen; on the other side, a large part of China was also under their control for quite a few years. Chiang Kai-Shek had fled to the hilly

tracts of South China. The University of Chungking had also had to shift to that region to save themselves from the air raids. China was going through a very hard time. Food was scarce. Even ordinary medicines were not available in the villages; they had to import almost everything. Warm clothes were also in short supply. The blackmarketeers were selling them to the villagers at exorbitant prices. Of course, these were all pilfered from relief materials sent by friendly countries out of compassion for the common people. The American pilots made regular trips to Chungking from Dhaka. They used to say that they made a lot of money selling torn stockings and underwear

I used to teach then in Dacca University. One of my students was busy doing research with sulphonamide. In the organic chemistry laboratory Dr Kalipada Basu was analysing various kinds of rice and other cereals, calculating the percentage of protein and fatty carbohydrates in each. This also included information about the amount of vitamins present and a method to identify them. The Americans had an air base near Dhaka. Many hospitals for American, British and Indian soldiers had been set up within the city of Dhaka. Many foreign doctors and scientists came and lived in Dhaka. We used to get a lot of information from them when they visited our science departments.

It was then that we heard of an expert in organic chemistry from a Chinese University who was visiting India. He had come to find out how vitamin tablets were being made in various places in India. At Tuticorin in South India he saw Vitamin D being extracted from fish oil and other vitamins from African red palm oil. Chinese children were suffering from acute vitamin deficiency. He had come to find a remedy. After visiting Rajasthan, Bombay, Punjab and Delhi he arrived at Dhaka in the eastern part of India. He had heard of the analysis and classification done by Kalipada Basu. He had himself done similar research, so he was curious to know how it was being done in India.

We welcomed the Chinese scientist. A room in the Dhaka Hall was furnished for him. He spent more than a week with us.

We discussed various things. He told us how he had located vitamins B and C in Chinese plants. The chemical industry had just been established in China. There were only a few sulphur factories; other necessary chemicals and medicines were still being imported. The ordinary Chinese led an old-fashioned life, although already a few years ago the revolution led by Sun Yat-Sen had led to the fall of the Manchu Empire. At first he did not believe that we had made sulphonamide and other compounds in Dhaka. We invited him to come and see for himself every step of the process my student followed to obtain the pure substance. In return we wanted to find out from him how milk was made from soyabean in China. Kalipada had carefully stored a lot of soyabean obtained from Kashmir. We followed his procedure and extracted milk from it, and from milk we made cottage cheese. The Chinese confectioners apparently made many sweets from cottage cheese extracted from soyabean. (Thanks to the sweetmeat control measures of the government now, we can feel in our bones how useful such knowledge is.)

There was a great famine in Bengal. It was a common sight to find hordes of skeletal beggars begging for 'phan' (water strained from boiled rice) from door to door, and a few dead bodies in the streets. Millet porridge was being served in the common kitchens. We were trying to find out if it was possible to make some tasty food from tree leaves, comparable to the cutlets Biresh Guha had succeeded in making from grass. From time to time the Chinese scientist used to sit in my office. A couple of cows grazed in the adjoining field covered with green grass.

Someone would talk about the famine and the Chinese scientist would say that he failed to understand our people. He could see plenty of cattle here, and yet people were dying of starvation. In his country, he would say, people would kill wild animals for food. As a result, China does not have the cattle-wealth of India, and babies who cannot get mother's milk have to depend on milk from soyabean. Soyabean milk contains plenty of protein though it does not have much calcium — this deficiency is made up by adding powdered egg shells. The day the Chinese scientist helped us extract milk from soyabean, I boasted and insisted on cooking a meal of cottage cheese extracted from it. The consequence was, I suffered from indigestion and could not sleep the whole night. I came to the conclusion that the Chinese children were omnivorous. I was afraid if ever we were to come into conflict, they would be able to swallow us, the non-violent Indians.

Apparently, beri-beri has been endemic in Southern China from old times. The traditional doctors of that region used the seed of a particular flower and found it efficacious. The yellow wild flowers of the plantagenus species grow all over the place. The Chinese name for it, when translated, means 'yellow colour in front of the car'. In other words, it grows profusely in the fields in every village. Our Chinese scientist had tested them himself and actually found vitamin B in these flowers. The same medicine is being used now all over India as an antidote to beri-beri. The Chinese scientist had also found plenty of vitamin C in the stems of dried roses. When we mentioned our traditional Ayurvedic system, he said that traditional methods were also in vogue in China. He said jokingly that in order to maintain a high rate of cures in government hospitals, patients who were considered to be incurable were sent home with consolations. Once in a while a few of them have been found to move around in an apparently healthy state. Surprised, they found on enquiry that quite often they had benefited more from the treatment of traditional doctors. I pointed out to him that such cases were not infrequent in our country as well.

Initially we had trouble understanding the English of our newly arrived Chinese guest. But eventually we got used to his accent. We became great friends, and shared our experiences. Our friend came from the city of Mukden in North China. He left the city when the Japanese occupied it. He joined the Chinese army at war and had been trying to serve it as best as he could. He moved to Chungking. The Chinese had been fighting a losing battle with the Japanese for thirty years. They had been pushed down to the

hilly tracts of the south, but he had not given up the hope of driving the Japanese out and getting back to his motherland.

About a week later the scientist left Dhaka. Then we lost touch of him. Those were the days we felt that the Chinese race was close to us (Hindi-Chini bhai bhai!) [The Indians and Chinese are like brothers]. Both nations have a great heritage and were perhaps a little conservative, but the Chinese lagged behind us a little. Twenty-four years later today I feel that the Chinese scientist's heart-felt desire has been fulfilled. They have got back Mukden. They have abandoned their ancient tradition and are marching ahead speedily, this powerful nation. They may be an ancient race, but they are young and revolutionary. They are opportunists, so they have severed the bonds of fosterbrotherhood. Chiang has, of course, had to leave the mainland and settle in Formosa. The victorious president Mao continues his triumphant march. His ability to swim across a turbulent river in old age has become the symbol of the indomitable courage of a newly emerging Chinese nation.

One cannot win by underestimating one's enemy. When I compare our progress with that of China during the last twenty-four years, I cannot find anything to be proud of. To what extent are Indian scientists responsible for this?

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9

What Next?

Diary entry dated 22 July 1969

When the American astronauts reached the moon, it was the dead of night in our country. But here too many enthusiastic friends kept awake with their radios tuned in and they heard the news as the rocket carrying human beings touched the surface of the moon. Elsewhere viewers watched Armstrong on their television screens as he climbed down the ladder to step on the moon. The world is rejoicing at this tremendous achievement of technology in this age of the machine. Human beings have reached the moon after years of effort by thousands of scientists. This project has unveiled many mysteries of chemistry, physics and the life sciences, as a result of which rockets have been able to penetrate the atmosphere with ease and reach the outer space.

Someone had said that many precious stones lay scattered on the moon's surface and the explorers will return with bags full of them. And the money earned by selling them on the market will help recover the cost of the venture. But photographs show how the astronauts walked on the moon's surface for two and a half hours and filled their bags with stones, pebbles and soil, which will be analysed by scientists for unknown elements. Those who ponder over the origin of the universe hope to find out about the nature of our planet at the beginning of its being by studying the moon's soil. Traces of the original form have been erased from the face of the earth by natural calamities, the march of life and its hazards.

But they do not expect to learn much only by analysing the moon's soil. For when the many meteors and stones that have fallen from the sky were collected and examined, they were found to be made of the same familiar elements that are found on

the earth itself. So elements in the moon may not reveal anything new. The finding may not be significant but the tremendous energy spent over these years and the great mountain of knowledge scaled have themselves enriched the storehouse of science. The American and Russian scientists have not yet brought all the information out in the open, but maybe they will after a hundred years.

23 July

The Russians have always been in the race for the moon. They have sent Luna-15 almost simultaneously and, according to information received today, their rocket too has touched the moon's shores. Of course, the Russian operation is wholly remote-controlled — there are no human beings on the Russian trip. Maybe it is collecting information, taking photographs, maybe it too will return with soil from the moon. Some British scientists think that Apollo-11 might take some time to bring back its people. If Luna-15 returns before it, then it will cast a shadow on the American achievement.

24 July

The two superpowers are rivals in space exploration. But this time the Americans seem to have won the laurels. They are being congratulated from all over the world. People are saying that the names of the moon explorers will go down in history. Some want the year to start from 21 July, instead of 1 January. Today everybody is eagerly awaiting the astronauts' return. The American President himself is on his way to welcome them — they are supposed to land in the middle of the Pacific Ocean tonight. Then they will remain quarantined and under observation for some time, so that any alien germ that they may bring back with them does not spread among the people of the earth — a safety measure, perhaps, to ensure the continuity of human life for many centuries to come.

Even if they are not imported from the moon, there are enough weapons of destruction on the earth itself. Be it bomb or poison gas, there is plenty. Besides, man can even spread germs of disease in enemy territory, and there are rumours that such experiments have been carried out from time to time. However, these rumours have drowned under the drumming for world peace. In poor countries like India the illiterate people are thinking — what good will this great leap of science be for them. The astronauts have, of course, planted their star-studded flag on the moon in the belief that they were working for enduring world peace, and that the flag itself will stand there as a symbol of the combined human effort to ensure world peace.

Old people in this country whose memories have not failed will recall hearing of Pax Britannica as children in school. They will also recall the British aspiration in the nineteenth century for establishing world peace under the Union Jack. We still hear about equality, fraternity and liberty; slogans of brotherhood rend the air at conferences. But they should be supplemented with a timely note from the Upanishads — these things are not for the weak to achieve.

Technology has reached such a stage that automatic machines are giving people

relief from a lot of worries. To surrender oneself to the power of the machine, to sacrifice one's personality and plunge without fear into the known seas is the motto of the times. This does bring about good results, as the achievement of 21 July shows. There is a lot of speculation going on about the future. Our astrologers have made some predictions; I have seen similar predictions in American papers also. Of course, one cannot wish the astrologers away.

Scientists are the soothsayers of the modern times. They say that they have found the way now — another trip to the moon is being planned by the end of the year. Besides, it is also being said that human beings might even reach Mars by the end of this century.

We used to think in the past that this beautiful earth of ours — in its dust is blended the ashes of our forefathers — is loved by all. We used to think that people are attached to this beautiful green earth. We thought that with the progress of science and the efforts of mankind, heaven will be created on earth. The imaginative scientist keeps thinking of how through the ages life has tried to express itself on this earth and sought its gratification in various life forms. It has perhaps reached the final stage of evolution in the human being. It is time now to seek answers to the eternal questions through explorations in science. Perhaps science will now be able to satisfy our daily needs, and so the mind will be at peace. There will be no more superstition, we will overcome the discriminations of race, religion and colour. Man will discover his true essence in the full light of wisdom. I am not sure if such a day will ever come. But I am convinced that the voyage to the moon has not given us any clue to any route leading to that direction. That makes one wonder : what next?

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S N Bose : Biographical Resumé

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1894 1 January. S N Bose is born in the family residence, 22, Iswar Mill Lane, Goabagan, Calcutta.

Bose is the only male child of his parents (to be followed by six sisters), Surendranath (1868-1964), an accountant in the Executive Engineering Department of the East India Railway, who later founded the Indian Chemical and Pharmaceutical Works, a small unit dealing in chemicals; and Amodini (d. 1939).

1899 Bose is admitted to the Normal School in north Calcutta.

1900 7 October. Max Planck discovers what later comes to be known as Planck's Radiation Law.

14 December. Planck gives his original derivation of the law, and lays the foundation of the quantum theory.

1905 The discovery of special relativity by Albert Einstein, that 'brought clarity to old physics and created new physics, in particular Einstein's derivation of the relation $E = mc^2$.' In 1905, Einstein produces six papers, including one on the light-quantum and the photo-electric effect, completed 17 March, leading to his Nobel Prize in physics; his doctoral thesis on a new determination of molecular dimensions, completed 30 April; one on Brownian motion, received by the *Annalen der Physik* on 11 May; the first two papers on special relativity, received by the *Annalen* on 30 June and 27 September respectively; and a second paper on Brownian motion, received 19 December.

16 October. The Partition of Bengal. The day is declared a day of mourning throughout Bengal, with a day's fast, a closure of shops and institutions, people tying *rākhis* on each other's hands, demonstrations, and two huge rallies; giving a fillip to the Swadeshi movement.

1906 31 March. Sir Asutosh Mookerjee (1864-1924) becomes Vice-Chancellor, University of Calcutta, to continue till 30 March 1914 on his first term, running a second term, 4 April 1921–3 April 1923. Under the new Indian Universities Act which came into force on 1 September 1904, and under the leadership of Sir Asutosh, the University ceases to be a purely examining body, comes to exercise control over the schools and colleges, and 'provide for postgraduate teaching, study and research in the Faculties of Arts and Science'. The same year the National Council of Education, conceived as a parallel non-governmental University on the Swadeshi ideals, comes into being under the presidentship of Rashbehari Ghose (1845-1921).

1907 Bose changes school again — and joins the Hindu School, Calcutta, one of the city's oldest and best known schools.

1908 Down with an attack of chicken pox, Bose fails to appear for the school-leaving examination, then known as the Entrance Examination.

1909 Bose appears for the Entrance Examination of the Calcutta University, and stands fifth in order of merit. The same year he joins the intermediate classes in science at the Presidency College, where his classmates include Jnanchandra Ghosh (1894-1959), Jnanendranath Mukherjee (1893-1983), and Nikhilranjan Sen (1894-1963).

1910 'Perhaps the neatest derivation of Planck's radiation formula' is given by P Debye, in the *Annalen der Physik*, 33, 1427.

1911 L Natanson draws attention to the 'indistinguishability' of Planck's energy quanta implied by Planck's permutation measure. Planck adapts statistical mechanics to suit the requirements of the quantum theory by introducing elementary phase cells of volume h^3. Solvay Congress at Brussels on The Theory of Radiation and Quanta.

Bose stands first in the intermediate science examination of the Calcutta University from Presidency College, and is awarded the Duff Scholarship for the best performance in Physics. He joins the B Sc classes at Presidency College the same year, and meets M N Saha (1893-1956) as classmate, beginning years of scientific collaboration and a lifelong friendship.

7 March. Rutherford presents his discovery of the atomic nucleus, at a meeting of the Manchester Literary and Philosophical Society. His definitive paper on the subject appears in *Phil. Mag.*, May 1911.

1912 Taraknath Palit (1831-1914), an eminent lawyer, executes two Trust Deeds in favour of the University of Calcutta, the effect of which is to vest in the University, lands and money of the aggregate value of 1 million 500 thousand rupees in aid of the foundation of a University College of Science and Technology and the maintenance of two Professorships, one in Chemistry and the other in Physics, and scholarships for advanced students in science to enable them to carry on research or investigation abroad, with the stipulation that the 'chairs shall always be filled by Indians.'

1913 Bose stands first in the B Sc Examination of the Calcutta University, with Honours in Mixed Mathematics, from Presidency College, and is awarded the Harischandra Prize and the Herschel Gold Medal for being the Best in Mathematics and the Manmathanath Bhattacharya Gold Medal for standing first in the B Sc Examination. M N Saha stands second, and Nikhilranjan Sen third.

8 August. Rashbehari Ghose comes forward with an offer of one million rupees 'in furtherance of the University College of Science' and for the maintenance of four Professorships, one each in Applied Mathematics, Physics, Chemistry, and Botany with special reference to Agriculture; with the stipulation that the chairs 'must be always filled by Indians.'

Niels Bohr's paper on the quantum theory of the hydrogen atom, completed 5 April, and appearing in *Phil. Mag.*, 26, 1, in July, is the next major step in quantum theory after Planck's introduction of energy quanta and Einstein's hypothesis of light quanta. On 20 December, in a lecture before the Danish Physical Society, Bohr presents the germs of the correspondence principle, a powerful method of calculating the energies of the states of an atom.

1914 Bose marries Ushabati Ghosh, aged eleven. They would have nine children, of whom seven survive.

27 March. The foundation stone is laid for the building designed to house the University College of Science.

Ehrenfest and Kamerlingh Onnes distinguish between Planck's energy quanta ('indistinguishable') and Einstein's hypothetical light quanta ('distinguishable').

1 August. World War I breaks out.

1915 Bose stands first in the M Sc Examination, Calcutta University, in Mixed Mathematics,

from Presidency College, and is awarded the Hemchandra Gossain Prize and Gold Medal. M N Saha stands second.

25 November. Einstein presents to the physics-mathematics section of the Prussian Academy of Science his paper, 'The Field Equation of Gravitation', in which 'finally the general theory of relativity is closed as a logical structure.'

1916 Prafullachandra Ray joins the University College of Science as the first Palit Professor of Chemistry, a post which he holds till 1937. Teaching in the postgraduate departments in the sciences begins in July. Bose joins the University College of Science as a research scholar.

20 March. The *Annalen der Physik* receives 'Grundlage der Allgemein Relativitätstheorie', the first systematic exposé of general relativity, to be published later in the year as Einstein's first book.

July. Einstein returns to the quantum theory, and in the next eight months publishes three overlapping papers on the subject, containing the coefficients of spontaneous and induced emission and absorption, a new derivation of Planck's law, and the first statement in print by Einstein that a light-quantum with energy $h\nu$ carries a momentum $h\nu/c$.

December. Einstein completes *Über die Spezielle und die Allgemeine Relativitätstheorie, Gemeinverständlich*, to be published early next year, to remain his most widely known book.

1917 Bose becomes a Lecturer in Physics and Applied Mathematics at the University College of Science, Calcutta University. C V Raman joins the same institution as the first Palit Professor of Physics, to continue in the post till 1934.

1918 Bohr formulates the correspondence principle in the lengthy memoir 'On the quantum theory of line spectra', part I appearing in April, part II in December, and part III in 1922.

Bose's first important contribution to theoretical physics, a paper (jointly with M N Saha) 'On the Influence of the Finite Volume of Molecules on the Equation of State', published in *Phil. Mag.*, Ser. 6, 36, 199-203. The equation of state proposed in the paper is generally known as the 'Saha-Bose equation of state'.

1919 6 April. Bose reads 'The Stress-Equations of Equilibrium' at the Calcutta Mathematical Society. The paper is published in the Society's *Bulletin*, 10, 117-21.

31 August. Bose reads 'On the Herpolhode' at the Calcutta Mathematical Society. The paper is published in the Society's *Bulletin*, 11, 21-22.

'On the Equation of State', another joint paper by Bose and Saha, is published in *Phil. Mag.*, Ser. 6, 39, 456.

Calcutta University publishes *The Principle of Relativity*, containing the original papers by Einstein and H Minkowski, translated from German by Saha and Bose, with an historical introduction by P C Mahalanobis.

6 November. At a joint meeting of the Royal Society and the Royal Astronomical Society in London, it is announced that the 29 May observations of the total solar eclipse, conducted under the supervision of Eddington on the island of Principe and under Crommelin in northern Brazil, confirm Einstein's predictions about the bending of light made in 1915, and wildly enthusiastic press announcements mark the beginning of the perception by the general public of Einstein as a world figure. Pais, Einstein's biographer, describes it as 'the

day on which Einstein was canonized.'

1920 Bose's paper 'On the Deduction of Rydberg's Law from the Quantum Theory of Spectral Emission', published in *Phil. Mag.*, 40, 619-27.

1921 15 July. The University of Dhaka (then spelt Dacca) starts functioning. Bose joins the new University as a Reader in the Department of Physics, with J C Ghosh as his colleague in the Department of Chemisty. He starts teaching quantum theory and acutely feels the lack of a logically satisfactory derivation of Planck's law. He attempts to provide one in his own way.

1922 January. Einstein completes his first paper on unified field theory.

9 November. The Nobel Prize for physics for 1921 is awarded to Einstein while he is en route to Japan.

1923 The general acceptance of Einstein's light-quantum hypothesis comes through the theoretical work of Peter Debye and the experimental studies of A H Compton. Pauli shows that free electrons and radiation can be in thermal equilibrium (described by Planck's law), provided the probability of collisions between electrons and light-quanta satisfies a certain condition. Einstein and Ehrenfest generalize Pauli's work to elementary processes involving more than two light-quanta.

1924 M N Saha, then Professor of Physics, Allahabad University, meets Bose in Dhaka, and draws his attention to the papers of Pauli (1923) and Einstein and Ehrenfest (1923), published in *Zeitschrift für Physik*. Bose studies them and produces his celebrated paper, 'Planck's Law and the Light-Quantum Hypothesis'. He sends the paper to *Phil. Mag.*, but receives no response.

4 June. Bose sends his paper to Einstein, with an accompanying letter, asking him for his opinion and to 'arrange for its publication in *Zeitschrift für Physik*.'

15 June. Bose sends another paper, 'Thermal Equilibrium in the Radiation Field in the Presence of Matter', to Einstein.

2 July. In a postcard Einstein informs Bose that he has 'translated your work [the first paper] and communicated it to *Zeitschrift für Physik* for publication', and tells him that his work 'signifies an important step forward and I liked it very much . . . You are the first to derive the factor quantum theoretically, even though because of the polarization factor 2 not wholly rigorously. It is a beautiful step forward.' The journal receives the paper on the same day and publishes it in its August issue under the title, 'Plancks Gestez und Lichtquantenhypothese', with a note by Einstein at the end : 'Bose's derivation of Planck's law signifies, in my opinion, an important step forward. The method used here gives also the quantum theory of an ideal gas, as I shall show elsewhere.'

7 July. *Zeitschrift für Physik* receives Bose's second paper translated and communicated by Einstein, and publishes it in its September issue.

10 July. Einstein presents his first paper on Bose's counting method to the Prussian Academy, in which he extends Bose's method to ordinary material atoms.

September. Bose sails from Bombay on board a steamer of the Lloyd Triestine Line, for Paris.

18 October. Bose reaches Paris, where he stays at 17 Rue du Sommerard, Paris 5, with his Indian friends, Prabodh Bagchi (1898-1956) and others.

Bagchi introduces him to Sylvain Levi, the well-known French Indologist, who in his turn introduces Bose to Paul Langevin, Professor titulaire at the College de France. On Langevin's recommendation, Bose has the opportunity of working on X-ray spectroscopy at the laboratory of Maurice de Broglie, and in a letter tells P J Hartog, Vice-Chancellor, Dhaka University, that 'Madame Curie also has given me hopes of allowing me facilities for work in the Radium Institute from the beginning of the new year.'

25 November. Louis de Broglie defends his thesis, which is a development of 'the idea, during the year 1923, that the discovery made by Einstein in 1905 should be generalized by extending it to all material particles and notably to electrons.' Einstein receives a copy of the thesis from Langevin, who was one of de Broglie's examiners, and considers it, in a letter to Lorentz (in December), 'a first feeble ray of light on this worst of our physics enigmas.'

1925 8 January. Einstein presents his second paper on Bose's counting method to the Prussian Academy, in which he discusses the differences between the counting methods of Boltzmann and Bose, and draws the inference that material particles obeying Bose's statistics should have a wave-like character. He draws attention to Louis de Broglie's doctoral thesis in which the latter had attached wave properties to ponderable matter.

27 January. Bose writes to Einstein, forwarding his 'third paper' to the latter 'under separate cover,' with the information that 'Langevin . . . seems to think it interesting and worth publishing.' This paper remains unpublished and untracable even in the Einstein archives.

29 January. Einstein presents his third paper to the Prussian Academy.

July. Heisenberg sends a paper to *Zeitschrift für Physik*, giving a preliminary account of matrix mechanics.

Pauli discovers the exclusion principle.

8 October. Bose, in Berlin, writes to Einstein seeking an appointment.

November. First Bose-Einstein meeting in Berlin, followed by several meetings and encounters with the leading German scientists : 'I was very friendly with Franck, Einstein, Born, Ewald, Szilard and Mark,' Bose would later recall.

1926 Bose, in Berlin, studies X-ray crystallography at Polanyi's laboratory, conducts a physical experimental investigation about the refractive index of Roentgen rays in the Kaiser Wilhelm Institute, and engages in theoretical studies with Gordon (of Klein-Gordon equation fame) and visits the radioactivity laboratories of Hahn and Meitner. He visits Göttingen where he meets Max Born and Erich Hückel.

Schrödinger writes four papers in which he develops wave mechanics and proves its equivalence with matrix mechanics. Fermi publishes his paper on the statistics of particles, obeying Pauli's principle. Dirac links the Bose and Fermi statistics of particles to the symmetry properties of their wave functions and names them 'bosons' and 'fermions'.

Bose applies for a Professor's post at Dhaka University, with recommendations from Einstein, Langevin, and Mark. Later in the year, he returns to Dhaka, and is appointed Professor and Head of the Department of Physics, when D M Bose, the original appointee, declines the offer. In his Dhaka years, he reorganizes the Physics Department, developing special facilities for research work in X-ray spectroscopy, X-ray diffraction, magnetic properties of matter, optical spectroscopy including Raman spectra, wireless etc.

1927 February. Dirac invents 'second quantization' of the electromagnetic field, and establishes a logical quantum theory of the interaction of light and matter in which the light field is represented by quantized harmonic oscillators.

Bose is appointed Dean, Faculty of Science, Dhaka University.

Bose and Sushilchandra Biswas publish a joint paper 'Measurements of the Decomposition Voltage in Non-Aqueous Solvents', in *Z. Phys. Chem*, 125, 442-51.

1928 February. Dirac announces his discovery of the relativistic wave equation for the electron, incorporating the spin; followed by several applications.

October. Arnold Sommerfeld, visiting Calcutta to receive a doctoral degree, honoris causa, writes to the Vice-Chancellor, Dhaka University, expressing a desire to meet Bose : 'It would be a great pleasure for me indeed if Professor S N Bose could decide to see [me] at Calcutta.' Bose comes down to Calcutta to see Sommerfeld, and attends Raman's talk on the discovery of the new radiation.

1929 Bose presides over the Physics and Mathematics section of the Indian Science Congress held at Madras, and devotes his Presidential address to 'Tendencies in the Modern Theoretical Physics'.

March. General quantum field theory outlined by Heisenberg and Pauli.

Bose contributes (jointly with S K Mukherjee, Assistant Lecturer in Physics, Dhaka University) 'Beryllium Spectrum in the Region λ 3367-1964', to *Phil. Mag.*, Ser.7, 7, 197-200.

K S Krishnan joins Dhaka University as Reader in Physics.

1931 Bose contributes his first article in Bengali, *Vijnaner Sankat* (Crisis in Science) to the first issue of *Parichay*, a major Bengali periodical, beginning his lifelong campaign for the dissemination and teaching of scientific ideas through 'the mother tongue'.

May. Dirac declares the 'holes' to be new unknown particles, 'anti-electrons', which had the same mass as electrons, but with charge + *e*, and postulates the positron.

1932 February. Discovery of the neutron by Chadwick.

June. First nuclear process produced by an accelerator : Cockcroft and Walton become first men to accelerate protons and split lithium nuclei.

2 August. Experimental observation of the positron for the first time, by Carl D Anderson, in a cloud chamber exposed to cosmic radiation.

1933 October. 'The canonization of the new developments,' at the seventh Solvay Conference, Brussels, in which the evidence for both the positron and the neutron is discussed by Bohr, Curie, Dirac, Heisenberg, Pauli, and others.

Dirac is awarded the Nobel Prize.

30 January. The Nazis come to power in Germany.

20 March. While Einstein is on a visit to the USA, the Nazis raid his summer house in Caputh to look for weapons allegedly hidden there by the Communist Party.

28 March. Einstein returns to Europe, and settles temporarily in Belgium, never to set foot in Germany again; and sends his resignation to the Prussian Academy.

7 April. The *Beamtengesetz* (the civil service law) authorizes German Universities fo fire

staff on grounds of politics and/or race.

10 May. Burning of books in Germany.

1934. January. Iréne and Frèdéric Joliot-Curie first produce artificial radioactivity.

March. Fermi uses slow neutrons to bombard nuclei. 'In rapid succession, 40 of the 60 elements the [Fermi] group irradiated revealed the existence of at least one new isotope. A new one was found every few days' (G Holton, in *Minerva*, 12, 159, 1974, quoted by Pais, *Niels Bohr's Times*, Oxford 1991).

1935 7 January. Bose is nominated a Foundation Fellow of the National Institute of Sciences that has its inaugural meeting at the Senate Hall of the Calcutta University; and is also one of the five members representing the Indian Science Congress on the first general committee of the Institute.

Bose delivers the Adharchandra Memorial Lecture at Calcutta University, on 'Recent Progress in Nuclear Physics'.

Chadwick receives the year's Nobel Prize in physics, and the Curies in chemistry.

1936 January. Bohr describes for the first time his theory of the compound atomic nucleus. 'The compound nucleus dominated the theory of nuclear reactions at least from 1936 to 1954 . . . At Los Alamos . . . the compound model could explain many phenomena.'

February. Bose lectures on 'Science in Education' at a conference held in connection with the Bengal Education Week.

June. Bose makes his first contribution to statistics — a paper 'On the Complete Moment-Coefficients of the D^2-statistic', in *Sankhyā*, 2, 385-96.

1937 Rabindranath Tagore dedicates his first book on science— *Visva-Parichay* (Introduction to the Universe) — to Bose.

Bose contributes his second paper in statistics to *Sankhyā*, 3, 105-24 'On the Moment-Coefficients of the D^2-Statistic and Certain Integral and Differential Equations Connected with the Multivariate Normal Population'. 'Recent Progress in Nuclear Physics', the text of his Adharchandra Memorial lecture, appears in *Science and Culture*, 2, 473-79. *Science and Culture*, 3, 335-7, carries 'Anomalous Dielectric Constant of Artificial Ionosphere'.

1938 28 January. Bose writes to Syamaprasad Mookerjee, Vice-Chancellor, Calcutta University, in response to a feeler sent by the latter, stating that 'it will not be wise for me to go in for change at this age.'

5 April. *Indian Journal of Physics* receives Bose's 'On the Total Reflection of Electromagnetic Waves in the Ionosphere', and carries it in issue no. 12, 121-44.

10 December. Fermi receives the Nobel Prize in physics 'for his demonstration of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought out by slow neutrons.'

1939 January. Hahn and Strassmann in Germany, Meitner and Frisch in Denmark in cooperation with Bohr and Fermi establish the practicability of fission and chain reaction. Joliot, Halban and Kowarski in Paris prove that uranium and thorium could perhaps be burned to yield energy.

March. Fermi meets US Navy officials and points out the possibility of obtaining atomic power with fission by fast neutrons; but the Navy officials are unimpressed.

July. Szilard and Wigner take up the subject of the bomb with Einstein, and the three subsequently talk with Alexander Sachs, who had served as an advisor to President Roosevelt on several occasions.

2 August. Einstein sends a letter to Roosevelt drawing the latter's attention to the military implications of atomic energy.

1 September. Germany invades Poland, and sparks off World War II.

11 October. Sachs carries Einstein's letter to the White House in person.

21 October. The Advisory Committee on Uranium, headed by Lyman J Briggs, holds its first meeting — the three-man committee was appointed on the same day Roosevelt replied to Einstein's letter. Einstein would later regret having written the letter to Roosevelt, 'Had I known that the Germans would not succeed in producing an atomic bomb, I would not have lifted a finger.'

Bose publishes his paper in mathematical physics — 'Studies in Lorentz Group' — in the *Bulletin of the Calcutta Mathematical Society*, 31, 137-47.

16 November. Bose is appointed Provost of the Dhaka Hall at the University.

1940 June. With the Uranium Advisory Committee recommending funding for the procurement of uranium and graphite and the construction of a lattice, and the creation of the National Defence Research Committee, the US government enters the project that would lead to the making of the first atom bomb.

17 November. 'The complete solution of the Equation : $\nabla^2\varphi - \frac{\partial^2 \varphi}{c^2 \partial t^2} - k^2 \varphi = -4\pi\rho(xyzt)$', joint paper by Bose and S C Kar, received by the *Proceedings of the National Institute of Sciences*, India, to be published in issue 7, 93-102.

1941 Compton, Chadwick, and Cockcroft, among others, prove the feasibility of the atomic bomb.

1942 July. Bose is nominated member of the Agricultural Research Sub-committee of the Indian Central Jute Committee.

2 December. Under Fermi's supervision, the first nuclear reactor comes into operation in Chicago, producing the first self-sustaining chain reaction, and thereby initiating the controlled release of nuclear energy.

1943 March. J R Oppenheimer arrives at the site of the Los Alamos Scientific Laboratory and takes charge as director, to be soon joined by Bethe, Fermi, Chadwick, Bohr, Wilson, Kennedy, Smith, Parsons, Kistiakowsky, Bacher, and others, on a project of making the atomic bomb.

Bose is nominated Chairman, Weights and Measures Committee, Government of Bengal.

Bose contributes paper on 'Reaction of Sulphonazides with Pyridine : Salts and Derivatives of Pyridine-Imine', to *Science and Culture*, 9, 48-9; and 'A Note on Dirac Equations and the Zeeman Effect' (jointly with K Basu) to the *Indian Journal of Physics*, 17, 301-08.

1944 3 January. In his presidential address at the Thirty-first Indian Science Congress, Bose speaks on 'The Classical Determinism and the Quantum Theory'.

30 April. Hitler takes his own life under the ruins of his Chancellery in Berlin.

7-8 May. Germans surrender in Reims and Berlin.

19 July. Bose addresses the annual convocation of the College of Engineering and Technology, Jadavpur University.

2 September. The Japanese surrender, bringing World War II to an end.

11 December. As member of the Bengal Industrial Research Committee, Bose attends the first meeting of the Committee.

1945 15 May. *Bulletin of the Calcutta Mathematical Society* receives Bose's paper 'On an Integral Equation associated with the Equation for Hydrogen Atom', to be published in the *Bulletin*, 37, 51-61.

16 July. The first atomic explosion at the Alamogordo Air Base in the New Mexico desert takes place at 5.30 a.m.

6 August. The first atomic bomb is dropped on Hiroshima at 8.15 a.m. (Japanese time) from a B-29 aircraft, Enola Gay.

9 August. The second atomic bomb is dropped on Nagasaki at 11.02 a.m. (Japanese time) from a B-29 aircraft, the Great Artiste.

Bose returns to Calcutta as Khaira Professor of Physics at Calcutta University, and becomes President of the Indian Physical Society (in the latter position he would continue till 1948).

10 December. Einstein delivers an address in New York, 'The War is Won but Peace is Not'.

1946 Einstein agrees to serve as chairman of the Emergency Committee for Atomic Scientists.

October. Einstein writes an open letter to the general assembly of the United Nations, urging the formation of a world government.

1947 15 August. India gains independence.

1948 25 January. Bose launches the Bangiya Vijnan Parishad (Science Association of Bengal), and its monthly organ, *Jnan O Vijnan* (Knowledge and Science), to popularize science in and through Bengali.

Bose becomes President of the National Institute of Sciences of India (to continue in this position till 1950).

1949 27 October. Bose sends a communication on 'Germanium in Sphalerite from Nepal' to the *Journal of Scientific and Industrial Research*, to be published in its issue no. 9B, 52-53, 1950.

1950 The *Journal of Scientific and Industrial Research*, New Delhi, carries 'Extraction of Germanium from Sphalerite Collected from Nepal' by Bose and R K Dutta of the University College of Science, in two parts in 9B, 251-52, and in the following issue.

1951 As a Special Representative from India, Bose attends a meeting, sponsored by the UNESCO, at Paris, to consider the establishment of an international statistical centre. He visits Germany and meets Walther Bothe, Otto Haxel, J H D Jenson and H Meier Leibnitz in Heidelberg, and Otto Hahn, Werner Heisenberg and Houtermans in Göttingen.

1952 Bose addresses the All India Bengali Literature Conference at Cuttack, as President of the Science Section.

20 September. Bose writes to Einstein — what amounts to his first contribution to Einstein's 'new' unified field theory.

29 September. Bose's paper 'The Affine Connection in Einstein's New Unitary Field Theory', is received by the *Annals of Mathematics, USA*, which publishes it in issue 59, 171-76, 1954.

4 October. Einstein acknowledges Bose's communication, appreciating his 'interest' in the theory, and his 'so much work and penetration to the solution of the equations' of the unitary field, but does not find 'the solution of those equations . . . of great help'.

1953 Bose attends the World Congress for General Disarmament and Peace at Budapest; and visits the USSR, Denmark, Czechoslovakia and Switzerland; meeting Wolfgang Pauli in Zurich and Niels Bohr in Copenhagen.

Bose publishes a Note on 'The Identities of Divergence in the New Unitary Theory' in the *Comptes rendus de L'Academie des Sciences*, Paris, 236, 1333-5, 'presented by Louis de Broglie'.

18 July. *Le Journal de Physiqueet le Radium*, Paris, receives Bose's papers 'A Unitary Field Theory with $\Gamma\mu \neq O$', and 'Certain Consequences of the Existence of the Tensor g in the Affine Relativistic Field' which it publishes in its issue no. 14, 641-47.

22 October. Einstein writes a long letter to Bose, commenting on the paper 'A Unified Field Theory . . .'

1954 26 January. The Republic Day Honours List shows Bose as Padmavibhushan, one of the highest national honours conferred by the President of India.

3 May. Bose is sworn in as a member of the Rajya Sabha, the upper house of the Indian parliament, on a special nomination made by the President of India.

Bose attends the International Crystallography Conference at Paris, where he presents the constructional details of his Spectrophotometer, subsequently adopted in several well-known laboratories in Europe.

13 April. The trial of Oppenheimer begins.

Einstein sides with the overwhelming majority of atomic scientists who publicly condemn the United States government's actions in the security case against Oppenheimer.

30 November. Bose delivers the sixteenth Acharya Jagadischandra Bose Memorial Lecture at the Bose Institute, on 'Search for New Sources of Power'.

1955 9 February. 'A Report on the Study of Thermoluminescence', a joint paper by Bose, J Sharma, and B C Dutta, is received for publication by the *Transactions of the Bose Research Institute*, which carries it in its issue 20, 177-80.

Bose's last scientific paper, 'Solution of a Tensor Equation Occurring in the Unitary Field Theory', appears in the *Bulletin de Société Mathématique de France*, 83, 81-85.

11 April. Einstein's last signed letter (to Bertrand Russell), in which he agrees to sign a manifesto urging all nations to renounce nuclear weapons. That same week, Einstein writes his final phrase in life, in an unfinished manuscript : 'Political passions, aroused everywhere, demand their victims'.

18 April. 1.15 a.m. Death of Einstein.

Bose visits Paris, on an invitation from the Council of National Scientific Research of France.

July. Bose attends the international conference commemorating Fifty Years of Relativity,

at Berne.

1956 Bose retires from the post of Khaira Professor at Calcutta University.

July. Bose takes over as Vice-Chancellor of the Visva-Bharati University.

Bose attends as a special invitee the annual meeting of the British Association for the Cultivation of Science.

1957 15 January. Bose delivers the Vice-Chancellor's formal address at the Convocation of Visva-Bharati, also addressed by Jawaharlal Nehru, then Prime Minister of the country and Chancellor of the University. Calcutta University, on the occasion of its centenary, confers a D Sc, honoris causa, on Bose; followed by honorary doctoral degrees from the Allahabad and Jadavpur Universities.

1958 Bose is elected Fellow of the Royal Society, and visits London via Paris for the occasion.

Bose is nominated Professor Emeritus at Calcutta University.

Bose presides over the year's session of the All India Bengali Literature Conference, at Jabbalpur.

1959 Bose is appointed National Professor.

10 August. Bose's resignation from the Rajya Sabha is reported in the House.

1960 January. Bohr visits India to attend the Indian Science Congress session at Bombay, where he gives two lectures; and comes to Calcutta to deliver the M N Saha Memorial lecture at the M N Saha Institute of Nuclear Physics, Calcutta University College of Science; the session is chaired by Bose.

1961 Bose receives the title of Deshikottama — equivalent to a doctorate, honoris causa — from the Visva-Bharati University.

March. Bose speaks at the Indian Association for the Cultivation of Science at a meeting held to commemorate the death anniversary of Mahendralal Sircar.

1962 May. Bose participates in the deliberations of the Organizing Committee of the World Peace Council, held in Stockholm.

August. Bose attends and addresses a seminar on 'Science and Philosophy' held in Tokyo.

September. Bose attends the World Peace Conference in Moscow.

October. Bose inaugurates and addresses the Angrezi Hatao (Banish English) Conference, held at Hyderabad.

Bose receives an honorary D Sc degree from the Indian Statistical Institute.

Bose delivers the convocation address at the Calcutta University Convocation.

1963 11 May. Bose delivers the convocation address at Ranchi University.

July. Bose visits Cairo as a member of a delegation of Indian scientists.

1964 1 January. Bose is felicitated on his seventieth birthday, at the Mahajati Sadan auditorium, with Prafullachandra Sen, then Chief Minister of the State, as President of the Celebration Committee.

Bose receives an honorary D Sc from the Delhi University, and the title 'Vijnan-Bhaskaram' from Sanskrit College, Calcutta.

1965 1 January. Bose inaugurates the 51-52nd session of the Indian Science Congress.

Bose receives the Jagattarini Medal from Calcutta University in recognition of his contribution to the Bengali language.

7 April. Bose delivers the convocation address at the Indian Institute of Technology, Kharagpur.

1968 Bose elected President of the Asiatic Society.

1969 Bose inaugurates the building of the Vangiya Vijnan Parishad, at P 23, Raja Rajkrishna Street.

1 March. Bose delivers the convocation address at the Indian Agricultural Research Institute, New Delhi.

1970 Bose is awarded a D Litt, honoris causa, by Rabindra Bharati University.

1971 12 March. Bose delivers the Krishnan Memorial Lecture at the Nationál Physical Laboratory.

10 May. Bose delivers the convocation address at Rabindra Bharati University.

1973 16 June. Bose delivers the convocation address at Calcutta University.

29 December. Bose speaks at the inaugural session of a Seminar on the Scientific Contributions of Professor S N Bose, held at the Calcutta Mathematical Society.

1974 1 January. Bose's eightieth birth anniversary is celebrated throughout the country.

The fiftieth year of Bose Statistics is commemorated at the Bose Institute, Calcutta.

4 February. Death of S N Bose.

Books by S N Bose (in Bengali) : in chronological sequence

Vijnaner Sankat O Anyanya Prabandha (Crisis in Science and Other Essays), Lekhak Samabaya Samiti, Calcutta, 1371 [Bengali era, corresponding to 1964], 176 pages, and prelims, including a dedicatory note by Bose, dated 24 May 1964.

Satyendranath Bose Rachana Sankalan (A Selection or Satyendranath Bose's Writings), Vangiya Vijnan Parishad, Calcutta, 1387 [Bengali era, corresponding to 1980], 422 pages, and prelims, including a prefatory note, a life sketch of Bose, and a memoiral piece by Gaganbehari Banerjee. Enlarged second edition, published as centenary edition, 1993, 450 pages, with additional articles, letters and photographs.

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